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New Exact Travelling Wave Solutions for Generalized Zakharov Equations Using the First Integral Method

A. El Achab(✉)[a]

[a]Department of Mathematics, Faculty of Sciences,
University of Chouaïb Doukkali, B.P. 20, El-Jadida, Morocco

ABSTRACT

In this paper, we investigate the first integral method for solving the solutions of nonlinear Generalized Zakharov Equations. This idea can obtain some exact solutions of this equations based on the theory of Commutative algebra.

Keywords: First integral method; Generalized Zakharov Equations; Exact solutions.

1. Introduction

In recent years, the investigation of exact solutions to nonlinear partial differential equations has played an important role in nonlinear phenomena. Nonlinear phenomena appear in a wide variety of scientific applications such as plasma physics, solid state physics, nonlinear optics, quantum field theory and fluid dynamics. In order to better understand these nonlinear phenomena, many mathematicians and physical scientists make efforts to seek more exact solutions to them. Several powerful methods have been proposed to obtain exact solutions of nonlinear partial differential equations, such as the Bäcklund transformation method [1–3], Hirotas direct method [4], tanh-sech method [5], extended tanh method [6], the exp- function method [7, 8], sine-cosine method [9, 10], Jacobi elliptic function expansion method [11], F-expansion method [12] and so on .

The first integral method was first proposed in [13] in solving Burgers- KdV equation which is based on the ring theory of commutative algebra. This method was further developed by the same author and some other mathematicians. In this work, we use the first integral method to find the exact solutions of the Generalized Zakharov Equations.

This paper is organized as follows: Section 2 is a brief introduction to the first integral method. In section 3, we apply the first integral method to find exact solutions of nonlinear Generalized Zakharov Equations .

2. The First Integral Method.

Consider a general nonlinear partial differential equation (PDF) in the form

$$P(u, u_t, u_x, u_{xx}, , u_{xt}, u_{tt}, ...) = 0, \tag{2.1}$$

where $u(x, t)$ is the solution of nonlinear partial differential equation (2.1). By means of the transformation

$$u(x, t) = u(\xi), \quad \xi = k(x - \lambda t), \tag{2.2}$$

✉ Corresponding author.

where k, λ and p are arbitrary constants, we reduce eq (2.1)to an ordinary differential equation (ODE) of the form

$$P(u, u^{'}, u^{''}, u^{'''}, ...) = 0, \tag{2.3}$$

where $u = u(\xi)$ and the primes denote ordinary derivatives with respect to ξ. Next, we introduce a new independent variable

$$z(\xi) = u(\xi), \ \ \omega(\xi) = u^{'}(\xi), \tag{2.4}$$

which leads to a system of ODEs of the form

$$\begin{cases} z^{'}(\xi) = \omega(\xi), \\ \omega'(\xi) = H(z(\xi), \omega(\xi)). \end{cases} \tag{2.5}$$

According to the qualitative theory of differential equations [14], if we can find two first integrabls to system (2.5) under the same conditions, then analytic solutions to (2.5) can be solved directly. However, in genral, it is difficult to realize this even for a single first integral, because for a given plane autonomous system, there is no general theory telling us how ti find it's first integrals in a systematic way. A key idea of our approach here to find first integral is to utilize the division theorem . For convenience, first let us recall the Division theorem for two variables in the complex domain **C** [13].

Theorem 1 (Division theorem). Suppose that $P(\omega, z)$ and $Q(\omega, z)$ are polynomials in $\mathbf{C}[\omega, z]$, and that $P(\omega, z)$ is irreductible $\mathbf{C}[\omega, z]$. If $Q(\omega, z)$ vanishes at any zero point of $P(\omega, z)$, then there exists a polynomial $G(\omega, z)$ in $\mathbf{C}[\omega, z]$ such that

$$Q(\omega, z) = P(\omega, z).G(\omega, z). \tag{2.6}$$

It follows immediately from the following theorem in commutative algebra [15]:

Theorem 2 (Hilbert-Nullstellensatz Theorem). Let k be a field and L an algebraic closure of k. Then
i) Every ideal γ of $k[X_1, ...X_n]$ not containing 1 admits at least one zero in L^n
ii) Let $x = (x_1, x_2, ..., x_n)$ and $y = (y_1, y_2, ..., y_n)$ be two elements of L^n . For the set of polynomials of $k[X_1, ...X_n]$ zero at x to be identical with the set of polynomials of $k[X_1, ...X_n]$ zero at y, it is necessary and sufficient that there exists a k-automorphisms s of L such that $y_i = s_i$ for $1 \leq i \leq n$. iii) For an ideal α of $k[X_1, ...X_n]$ to be maximal, it is necessary and sufficient that there exists x in L such that α is the set of polynomials of $k[X_1, ...X_n]$ zero at x.
iv) For a polynomial Q of $k[X_1, ...X_n]$ to be zero on the set of zeros in L^n of an ideal γ of $k[X_1, ...X_n]$, it is necessary and sufficient that there exists an integer $m > 0$ such that $Q^m \in \gamma$.

3. Generalized Zakharov Equations

Let us consider the Generalized Zakharov Equations [16]

$$\begin{aligned} H_{tt} - H_{xx} = (|u|^{2m})_{xx}, \\ iu_t + u_{xx} = Hu + \alpha|u|^{2m}u + \beta|u|^{4m}u. \end{aligned} \tag{3.1}$$

where $m > 0$, α, β are constants.
We first introduce the transformations[17]

$$\begin{aligned} u(x, t) = U(\xi) \exp(i\eta), \ H(x, t) = V(\xi), \\ \eta = kx + \gamma t, \ \xi = (x - \lambda t). \end{aligned} \tag{3.2}$$

where $\gamma, \ k, \ and \ \lambda$ are constants to be determined later. Substituting (3.2) into (3.1) we obtain the $\lambda = 2k$ and $U(\xi)$ satisfy into ODE:

$$(4k^2 - 1)V^{''} = (U^{2m})^{''}, \tag{3.3}$$

$$U^{''} - (\gamma + k^2)U - UV - \alpha U^{2m+1} - \beta U^{4m+1} = 0. \tag{3.4}$$

In order to simplify ODEs (3.3) and (3.4), integrating Eq. (3.3) once and taking integration constant to zero, and integrating yields

$$V = \frac{1}{(4k^2 - 1)}U^2 + C_1, \;\; 4k^2 - 1 \neq 0 \tag{3.5}$$

where C_1 is constant. Substituting (3.5) into (3.4) results in:

$$U^{''} - (\gamma + k^2 + C_1)U - (\frac{1}{(4k^2 - 1)} + \alpha)U^{2m+1} - \beta U^{4m+1} = 0. \tag{3.6}$$

where $k^2(1 - p^2) \neq 0$, $\lambda = 2k$ and the prime denotes derivative with respect to ξ. Making the following transformation:

$$v = U^{2m}, \tag{3.7}$$

then (3.6) becomes

$$v^{''} - av + b\frac{(v^{'})^2}{v} + dv^2 + fv^3 = 0, \tag{3.8}$$

where

$$\begin{aligned} a = 2m(\gamma + k^2 + C_1), \; b = \frac{1 - 2m}{2m}, \\ d = -2m(\frac{1}{(4k^2 - 1)} + \alpha), \; f = -2m\beta. \end{aligned} \tag{3.9}$$

and $v^{'}$ and $v^{''}$ denote $\frac{dv}{d\xi}$ and $\frac{d^2v}{d\xi^2}$, respectively. We introduce new independent variables $v = z$, $\frac{dv}{d\xi} = \omega$. Then equation (3.8) can be rewritten as the two-dimensional autonomous system

$$\begin{cases} \frac{dz}{d\xi} = \omega, \\ \frac{d\omega}{d\xi} = az - b\frac{\omega^2}{z} - dz^2 - fz^3. \end{cases} \tag{3.10}$$

Assume that

$$\frac{d\xi}{z} = d\tau \tag{3.11}$$

thus system becomes

$$\begin{cases} \frac{dz}{d\tau} = z\omega, \\ \frac{d\omega}{d\tau} = az^2 - b\omega^2 - dz^3 - fz^4. \end{cases} \tag{3.12}$$

Now, we apply the Division Theorem to seek the first integral to (3.12). Suppose that $z = z(\tau)$ and $\omega = \omega(\tau)$ are the nontrivial solutions to (3.12), and $p(\omega, z) = \sum_{i=0}^{r} a_i(z)\omega^i$, is irreducible polynomial in $\mathbf{C}[\omega, z]$ such that

$$p(\omega(\tau), z(\xi)) = \sum_{i=0}^{r} a_i(z(\tau))\omega^i(\tau) = 0, \tag{3.13}$$

where $a_i(z)$ $(i = 0, 1, ..., r)$ are polynomials of z and all relatively prime in $\mathbf{C}[\omega, z]$, $a_r(z) \neq 0$. Equation (3.13) is also called the first integral to (3.12). We start our study by assuming $r = 1$ in (3.13). Note that $\frac{dp}{d\tau}$ is polynomial in z and ω, and $p(\omega(\tau), z(\tau)) = 0$ implies $\frac{dp}{d\tau}\ |_{(3.12)} = 0$. By the Division Theorem, the exists a polynomial $H(z, \omega) = h(z) + g(z)\omega$ in $\mathbf{C}[\omega, z]$ such that

$$\begin{aligned}
&\frac{dp}{d\tau}\ |_{(3.12)} = (\frac{\partial p}{\partial z}\frac{\partial z}{\partial \tau} + \frac{\partial p}{\partial \omega}\frac{\partial \omega}{\partial \tau}\ |_{(3.12)} \\
&= \sum_{i=0}^{1} a_i^{'}(z)\omega^{i+1}z + \sum_{i=0}^{1} i a_i(z)\omega^{i-1}(az^2 - b\omega^2 - dz^3 - fz^4) \\
&= (h(z) + g(z)\omega)(\sum_{i=0}^{1} a_i(z)\omega^i)
\end{aligned} \tag{3.14}$$

where prime denotes differentiating with respect to the variable z. On equating the coefficients of ω^i $(i = 0,\ 1,\ 2)$ on both sides of (3.14), we have

$$za_1^{'}(z) - ba_1(z) = g(z)a_1(z), \tag{3.15}$$

$$za_0^{'}(z) = g(z)a_0(z) + h(z)a_1(z), \tag{3.16}$$

$$h(z)a_0(z) = a_1(z)[az^2 - dz^3 - fz^4], \tag{3.17}$$

Since, $a_1(z)$ and $g(z)$ are polynomials, from (3.15) we conclude that $a_1(z)$ is a constant and $g(z) = -b$. for simplicity, we take $a_1(z) = 1$, and balancing the degrees of $a_0(z)$, and $h(z)$, we conclude that $deg\ h(z) = 2$ and $deg\ a_0(z) = 2$, only. Now suppose that

$$h(z) = Az^2 + Bz + C,\ a_0(z) = Dz^2 + Ez + F\ (A \neq 0,\ D \neq 0), \tag{3.18}$$

where $A,\ B,\ C,\ D,\ E$ and F are all constants to be determined. Using (3.18) into (3.16) we obtain

$$h(z) = ((2 + b)D)z^2 + ((1 + b)E)z + bF, \tag{3.19}$$

Substituting $a_0(z)$, $a_1(z)$ and $h(z)$ in (3.17) and setting all the coefficients of powers z to be zero, we obtain a system of nonlinear algebraic equations, and by solving it, we obtain the following solutions:

$$\begin{aligned}
&F = 0, D = \frac{1}{2+b}\sqrt{(-(2+b)\,f)}, \\
&E = \frac{1}{2+b}\sqrt{(-(2+b)\,f)}, d = \frac{f}{(2+b)}(3 + 2b)
\end{aligned} \tag{3.20}$$

$$\begin{aligned}
&F = 0, D = -\frac{1}{2+b}\sqrt{(-(2+b)\,f)}, \\
&E = -\frac{1}{2+b}\sqrt{(-(2+b)\,f)}, d = \frac{f}{(2+b)}(3 + 2b)
\end{aligned} \tag{3.21}$$

Using the conditions (3.20) in (3.13), we obtain

$$\omega = -\frac{1}{2+b}\sqrt{(-(2+b)\,f)}(z^2 + z) \tag{3.22}$$

Combining this first integral with (3.13), the second-order differential equation (3.8) can be reduced to

$$\frac{dv}{d\xi} = -\frac{1}{2+b}\sqrt{(-(2+b)\,f)}(v^2 + v). \tag{3.23}$$

Solving (3.23) directly and changing to the original variables, we obtain the complex exponential function solution to equation (3.1):

$$u(x,t) = \Big(\frac{1}{-1+\exp\left(\frac{2m}{2m+1}\sqrt{\beta\,(2m+1)}(x-2kt)\right)C_2}\Big)^{\frac{1}{2m}} \times \exp(i(kx+\gamma t)). \tag{3.24}$$

$$H(x,t) = \frac{1}{(4k^2-1)}\Big(\frac{1}{-1+\exp\left(\frac{2m}{2m+1}\sqrt{\beta\,(2m+1)}(x-2kt)\right)C_2}\Big)^{\frac{1}{m}} \times \exp(2i(kx+\gamma t)) + C_1. \tag{3.25}$$

where C_1 and C_2 are arbitrary constants. Similarly, for the cases of (3.21), we have anther complex exponential function solutions:

$$u(x,t) = \Big(\frac{1}{-1+\exp\left(-\frac{2m}{2m+1}\sqrt{\beta\,(2m+1)}(x-2kt)\right)C_2}\Big)^{\frac{1}{2m}} \times \exp(i(kx+\gamma t)). \tag{3.26}$$

$$H(x,t) = \frac{1}{(4k^2-1)}\Big(\frac{1}{-1+\exp\left(-\frac{2m}{2m+1}\sqrt{\beta\,(2m+1)}(x-2kt)\right)C_2}\Big)^{\frac{1}{m}} \times \exp(2i(kx+\gamma t)) + C_1. \tag{3.27}$$

where C_1 and C_2 are arbitrary constants. These solutions are all new exact solutions.

Now we assume that $r = 2$ in (3.13). by the Division Theorem, there exists a polynomial

$$\begin{aligned} \frac{dp}{d\tau}\mid_{(3.12)} &= \Big(\frac{\partial p}{\partial z}\frac{\partial z}{\partial \tau} + \frac{\partial p}{\partial \omega}\frac{\partial \omega}{\partial \tau}\mid_{(3.12)} \\ = \sum_{i=0}^{2} a_i^{'}(z)\omega^{i+1}z &+ \sum_{i=0}^{2} i a_i(z)\omega^{i-1}(az^2 - b\omega^2 - dz^3 - fz^4) \\ &= (h(z)+g(z)\omega)\Big(\sum_{i=0}^{2} a_i(z)\omega^i\Big) \end{aligned} \tag{3.28}$$

On equating the coefficients of ω^i $(i = 0,\ 1,\ 2,\ 3)$ on both sides of (3.12), we have

$$za_2^{'}(z) - 2ba_2(z) = g(z)a_2(z), \tag{3.29}$$

$$za_1^{'}(z) - ba_1(z) = g(z)a_1(z) + h(z)a_2(z), \tag{3.30}$$

$$g(z)a_0(z) + h(z)a_1(z) = 2a_2(z)[az^2 - dz^3 - fz^4] + za_0^{'}(z), \tag{3.31}$$

$$h(z)a_0(z)) = a_1(z)[az^2 - dz^3 - fz^4], \tag{3.32}$$

Since, $a_2(z)$ and $g(z)$ are polynomials, from (3.29) we conclude that $a_2(z)$ is a constant and $g(z) = -2b$. for simplicity, we take $a_2(z) = 1$, and balancing the degrees of $a_0(z)$, $a_1(z)$ and $h(z)$, we conclude that $deg\ h(z) = 2$ and $deg\ a_1(z) = 2$, only. Now suppose that

$$h(z) = Az^2 + Bz + C,\ a_1(z) = Dz^2 + Ez + F\ (A \neq 0,\ D \neq 0), \tag{3.33}$$

where $A,\ B,\ C,\ D,\ E$ and F are all constants to be determined. Using (3.33) into (3.30) and (3.31) we obtain

$$\begin{gathered} h(z)=((2+b)D)z^2+((1+b)E)z+bF, \\ a_0(z)=(\frac{1}{2}\frac{2f+2D^2+D^2b}{2+b})z^4+(\frac{2d+3ED+2EDb}{3+2b})z^3 \\ +(\frac{1}{2}\frac{E^2-2a+2DF+2DFb+E^2b}{1+b})z^2+bE\frac{F}{1+2b}z+\frac{1}{2}F\frac{E+bE+bF}{b}. \end{gathered} \tag{3.34}$$

Substituting $a_0(z)$, $a_1(z)$ and $h(z)$ in (3.32) and setting all the coefficients of powers z to be zero, we obtain a system of nonlinear algebraic equations, and by solving it, we obtain the following solutions:

$$\begin{gathered} D=\frac{-2f}{\sqrt{(-f\,(2+b))}}, \\ E=\frac{2d\sqrt{(-f\,(2+b))}}{f(3+2a)},\ F=0,\ a=-((1+b)\,d^2\frac{2+b}{f\,(9+12b+4b^2)}). \end{gathered} \tag{3.35}$$

$$\begin{gathered} D=\frac{2f}{\sqrt{(-f\,(2+b))}}, \\ E=-\frac{2d\sqrt{(-f\,(2+b))}}{f(3+2a)},\ F=0,\ a=-((1+b)\,d^2\frac{2+b}{f\,(9+12b+4b^2)}). \end{gathered} \tag{3.36}$$

Setting (3.35) in (3.13), we obtain that system (3.12) has two first integral

$$\omega=\frac{\sqrt[2]{f}(\sqrt[2]{2}-i)}{\sqrt[2]{2+b}}z^2-\frac{i(\sqrt[2]{2+b})d}{\sqrt[2]{f}(3+2b)}z,\ (i^2=-1), \tag{3.37}$$

$$\omega=-\frac{\sqrt[2]{f}(\sqrt[2]{2}+i)}{\sqrt[2]{2+b}}z^2-\frac{i(\sqrt[2]{2+b})d}{\sqrt[2]{f}(3+2b)}z,\ (i^2=-1). \tag{3.38}$$

Combining this first integral with (3.12), the second-order differential equation (3.8) can be reduced to

$$\frac{dv}{d\xi}=\frac{\sqrt[2]{f}(\sqrt[2]{2}-i)}{\sqrt[2]{2+b}}v^2-\frac{i(\sqrt[2]{2+b})d}{\sqrt[2]{f}(3+2b)}v, \tag{3.39}$$

$$\frac{dv}{d\xi}=-\frac{\sqrt[2]{f}(\sqrt[2]{2}+i)}{\sqrt[2]{2+b}}v^2-\frac{i(\sqrt[2]{2+b})d}{\sqrt[2]{f}(3+2b)}v. \tag{3.40}$$

Solving (3.39) and (3.40) directly and changing to the original variables, we obtain the exact solution to equation (3.1):

$$\begin{gathered} u(x,t)=\left(\frac{-\frac{(2m+1)(\alpha(4k^2-1)+1)}{2(1+i\sqrt[2]{2})\beta(m+1)(4k^2-1)}}{1-\left(\frac{(2m+1)(\alpha(4k^2-1)+1)}{2(1+i\sqrt[2]{2})\beta(m+1)(4k^2-1)}F_1\right)\exp^{-\frac{m(\sqrt[2]{2m+1})(\alpha(4k^2-1)+1)}{\sqrt[2]{\beta}(m+1)(4k^2-1)}(x-4kt)}}\right)^{\frac{1}{2m}} \\ \times\exp\left(i(kx+\gamma t)\right), \\ H(x,t)=\left(\frac{-\frac{(2m+1)(\alpha(4k^2-1)+1)}{2(1+i\sqrt[2]{2})\beta(m+1)(4k^2-1)}}{1-\left(\frac{(2m+1)(\alpha(4k^2-1)+1)}{2(1+i\sqrt[2]{2})\beta(m+1)(4k^2-1)}F_1\right)\exp^{-\frac{m(\sqrt[2]{2m+1})(\alpha(4k^2-1)+1)}{\sqrt[2]{\beta}(m+1)(4k^2-1)}(x-4kt)}}\right)^{\frac{1}{m}} \\ \times\frac{1}{4k^2-1}\exp\left(2i(kx+\gamma t)\right)+C_1. \end{gathered} \tag{3.41}$$

$$
\begin{aligned}
u(x,t) &= \left(\frac{-\frac{(2m+1)(\alpha(4k^2-1)+1)}{2(1-i\sqrt[2]{2})\beta(m+1)(4k^2-1)}}{1-\left(\frac{(2m+1)(\alpha(4k^2-1)+1)}{2(1-i\sqrt[2]{2})\beta(m+1)(4k^2-1)}F_2\right)\exp^{-\frac{m(\sqrt[2]{2m+1})(\alpha(4k^2-1)+1)}{\sqrt[2]{\beta}(m+1)(4k^2-1)}(x-4kt)}} \right)^{\frac{1}{2m}} \\
&\times \exp\left(i(kx+\gamma t)\right), \\
H(x,t) &= \left(\frac{-\frac{(2m+1)(\alpha(4k^2-1)+1)}{2(1-i\sqrt[2]{2})\beta(m+1)(4k^2-1)}}{1-\left(\frac{(2m+1)(\alpha(4k^2-1)+1)}{2(1-i\sqrt[2]{2})\beta(m+1)(4k^2-1)}F_2\right)\exp^{-\frac{m(\sqrt[2]{2m+1})(\alpha(4k^2-1)+1)}{\sqrt[2]{\beta}(m+1)(4k^2-1)}(x-4kt)}} \right)^{\frac{1}{m}} \\
&\times \frac{1}{4k^2-1}\exp\left(2i(kx+\gamma t)\right)+C_1.
\end{aligned}
\tag{3.42}
$$

where C_1, C_2, F_1 and F_2 are arbitrary constants. Similarly, for the cases of (3.36), we have another complex exponential function solutions:

$$
\begin{aligned}
u(x,t) &= \left(\frac{-\frac{(2m+1)(\alpha(4k^2-1)+1)}{2(-1+i\sqrt[2]{2})\beta(m+1)(4k^2-1)}}{-1-\left(\frac{(2m+1)(\alpha(4k^2-1)+1)}{2(-1+i\sqrt[2]{2})\beta(m+1)(4k^2-1)}F_3\right)\exp^{\frac{m(\sqrt[2]{2m+1})(\alpha(4k^2-1)+1)}{\sqrt[2]{\beta}(m+1)(4k^2-1)}(x-4kt)}} \right)^{\frac{1}{2m}} \\
&\times \exp\left(i(kx+\gamma t)\right), \\
H(x,t) &= \left(\frac{-\frac{(2m+1)(\alpha(4k^2-1)+1)}{2(-1+i\sqrt[2]{2})\beta(m+1)(4k^2-1)}}{-1-\left(\frac{(2m+1)(\alpha(4k^2-1)+1)}{2(-1+i\sqrt[2]{2})\beta(m+1)(4k^2-1)}F_3\right)\exp^{\frac{m(\sqrt[2]{2m+1})(\alpha(4k^2-1)+1)}{\sqrt[2]{\beta}(m+1)(4k^2-1)}(x-4kt)}} \right)^{\frac{1}{m}} \\
&\times \frac{1}{4k^2-1}\exp\left(2i(kx+\gamma t)\right)+C_1.
\end{aligned}
\tag{3.43}
$$

$$
\begin{aligned}
u(x,t) &= \left(\frac{-\frac{(2m+1)(\alpha(4k^2-1)+1)}{2(-1-i\sqrt[2]{2})\beta(m+1)(4k^2-1)}}{-1-\left(\frac{(2m+1)(\alpha(4k^2-1)+1)}{2(-1-i\sqrt[2]{2})\beta(m+1)(4k^2-1)}F_4\right)\exp^{\frac{m(\sqrt[2]{2m+1})(\alpha(4k^2-1)+1)}{\sqrt[2]{\beta}(m+1)(4k^2-1)}(x-4kt)}} \right)^{\frac{1}{2m}} \\
&\times \exp\left(i(kx+\gamma t)\right), \\
H(x,t) &= \left(\frac{-\frac{(2m+1)(\alpha(4k^2-1)+1)}{2(-1-i\sqrt[2]{2})\beta(m+1)(4k^2-1)}}{-1-\left(\frac{(2m+1)(\alpha(4k^2-1)+1)}{2(-1-i\sqrt[2]{2})\beta(m+1)(4k^2-1)}F_4\right)\exp^{\frac{m(\sqrt[2]{2m+1})(\alpha(4k^2-1)+1)}{\sqrt[2]{\beta}(m+1)(4k^2-1)}(x-4kt)}} \right)^{\frac{1}{m}} \\
&\times \frac{1}{4k^2-1}\exp\left(2i(kx+\gamma t)\right)+C_1.
\end{aligned}
\tag{3.44}
$$

where C_1, C_2, F_3 and F_4 are arbitrary constants. These solutions are all new exact solutions.

Notice that the results in this paper are based on the assumption of $r = 1,\ 2$ for the Generalized Zakharov Equations . For the cases of $r = 3,\ 4$ for these equations, the discussions become more complicated and involves the irregular singular point theory and the elliptic integrals of the second kind and the hyperelliptic integrals. Some solutions in the functional form cannot be expressed explicitly. One does not need to consider the cases $r \geq 5$ because it is well known that an algebraic equation with the degree greater than or equal to 5 is generally not solvable.

4. Conlusion

In this work, we are concerned with the Generalized Zakharov Equations for seeking their traveling wave solutions. We first transform each equation into an equivalent two-dimensional planar autonomous system then use the first integral method to find one first integral which enables us to reduce the Generalized Zakharov Equations to a first-order integrable ordinary differential equations. Finally, a class of traveling wave solutions for the considered equations are obtained . These solutions include complex exponential function solutions. We believe that this method can be applied widely to many other nonlinear evolution equations, and this will be done in a future work.

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