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$\left(\frac{G'}{G}\right)$-Expansion Method for ZK-BBM Equation

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ABSTRACT

In this paper, we apply $\left(\frac{G'}{G}\right)$-expansion to find travelling wave solution of the ZK-BBM equation. The traveling wave solutions are expressed by three types of functions which are hyperbolic, trigonometric and rational functions. Numerical results clearly reflect the efficiency of proposed scheme.

Keywords: $\left(\frac{G'}{G}\right)$-expansion method; Travelling wave solutions; ZK-BBM equation

1. Introduction

There has been an unprecedented development in nonlinear sciences during the last two decades. In the similar contexts, several numerical and analytical methods have been developed and implemented to solve nonlinear problems of diversified physical nature. Abbasbandy [1] applied variational iteration method (VIM) using Adomian's polynomials for solving quadratic Riccati differential equation; Abdou and Soliman [2] used VIM for nonlinear problems, Wazwaz [3] obtained solitons and kink solutions for nonlinear parabolic equations, Wang, X. Li and J. Zhang [4] applied (G'/G) for travelling wave solutions of nonlinear evolution equations in mathematical physics, Ozis and Aslan [5] used $(G'/G)$to Kawahara type equations using symbolic computation, Zayed, and Al-Joudi [6] implemented an extended (G'/G)-Expansion Method to find exact solutions of nonlinear PDEs arising in Mathematical Physics, Naher et. Al. [7] used (G'/G)-expansion method for abundant traveling wave solutions of Caudrey-Dodd-Gibbon equation, Feng et. al. [8] seek traveling wave solution of Kolmogorov-Petrovskii-Piskunov equation by (G'/G)-expansion method, Zhao, Yang and W. Li [9] applied improved (G'/G)-expansion method for the Variant Boussinesq equations, Liu, Tian Wu [10] used (G'/G)-expansion method to two nonlinear evolution equations, Mohyud-Din, Noor and Noor [11] used a verity of some relatively new techniques to solve nonlinear boundary value problems of diversified physical nature, Biswas [12, 13] obtained 1-Soliton solution of the generalized Zakharov-Kuznetsiov equation with nonlinear dispersion and time-dependent coefficients and with dual-power law nonlinearity, Ma et al. [14, 15] developed soliton solutions of some special types of PDEs and Wazwaz [16] developed Adomian's polynomials for Fisher's equations. The basic inspiration of this paper is the extension of (G'/G)-expansion method for ZK-BBM equation of the form

$$u_t + u_x - a(u^2)_x - (bu_{xt} + ku_{yt})_x = 0,$$

and arise very frequently in mathematical physics and applied sciences.

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2. $(\frac{G'}{G})$-expansion method

We assume the given nonlinear partial differential for $u(x,t)$to be in the form

$$P(u, ux, ut, uxx, uxt, utt, ...) = 0, \tag{2.1}$$

Where $P$is a polynomial in its arguments. The essence of the $(\frac{G'}{G})$-expansion method can be presented in the following steps:

Step 1. Find solution of eq.(2.1) by taking $u(x,t) = u(\eta), \eta = x + y - ct$ and transform eq.(2.1) to the ordinary differential equation

$$Q(u, u', -cu', u'', ...) = 0, \tag{2.2}$$

Where prime denotes the derivative with respect toη.

Step 2. If possible, integrate eq.(2.2) term by term one or more times. This yield constant(s) of integration. For simplicity, the integration constant (s) can be set to zero.

Step 3. Introduce the solution $u(\eta)$of eq. (2.2) in the finite series form

$$u(\eta) = \sum_{i=0}^{n} ai(\frac{G'(\eta)}{G(\eta)})^i, \tag{2.3}$$

Where $G = G(\eta)$satisfies equation

$$G''(\eta) + \lambda G'(\eta) + \mu G(\eta) = 0, \tag{2.4}$$

Step 4. Determine$n$. This usually, can be accomplished by balancing the linear term(s) of highest order with the highest order nonlinear term(s) in eq. (2.2).

Step 5. Substituting (2.3) together with (2.4) into eq.(2.2) yield an algebraic equation involving power of $(\dfrac{G'}{G})$. Equating the coefficients of each power of $(\dfrac{G'}{G})$to zero gives a system of algebraic equations for $ai(i = 0, ..., n), \lambda, \mu$. Then, we solve the system with the aid of Maple, to determine these constants.

3. Solution Procedure

The ZK-BBM equation is presented as

$$u_t + u_x - a(u^2)_x - (bu_{xt} + ku_{yt})_x = 0, \tag{3.1}$$

We make the transformation $u(x,t) = u(\eta)$,where $\eta = x + y - ct$. Then we get

$$(-cu' + u' - a\left(u^2\right)' - (-bcu'' - kcu''\Big)' = 0, \tag{3.2}$$

Integrating it with respect to η once, it yields

$$(1-c)\,u - au^2 + (b+k)cu'' = 0, \tag{3.3}$$

Where $u = u(\eta)$ satisfies eq. (2.4). By balancing u^2 and u''in eq. (3.3) we get $n = 2$. From (2.3) and (2.4) we get that

$$u(\eta) = a2(\frac{G'}{G})^2 + a1(\frac{G'}{G}) + a0, a2 \neq 0 \tag{3.4}$$

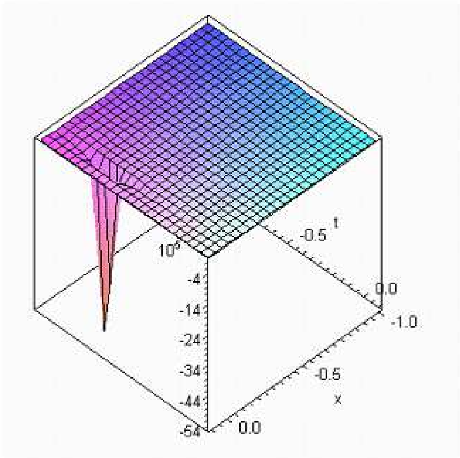

Figure 1: The soliton-like solution of $u_2(x,t)$.

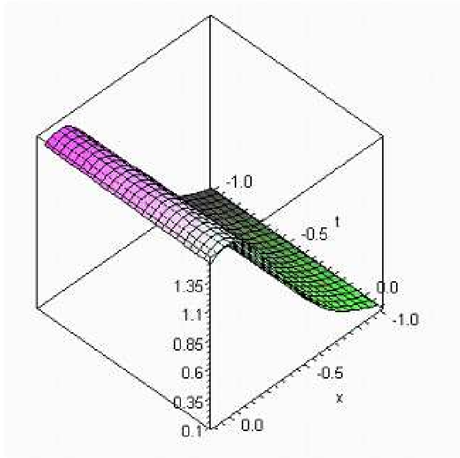

Figure 2: The soliton-like solution of $u_3(x,t)$.

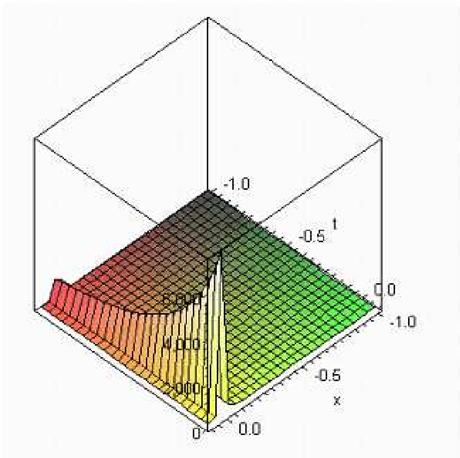

Figure 3: The periodic-like solution of $u_5(x,t)$.

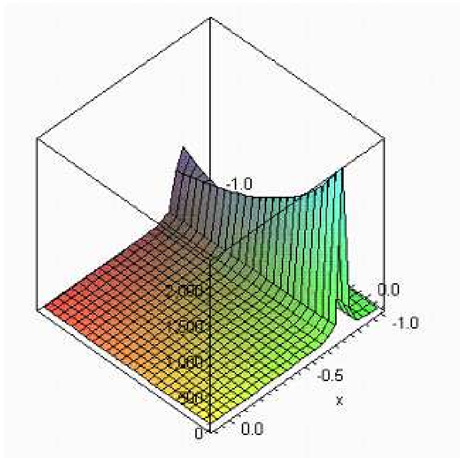

Figure 4: The periodic-like solution of $u_6(x,t)$.

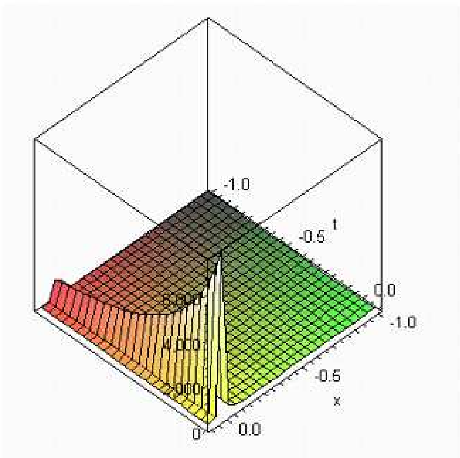

Figure 5: The soliton-like solution of $u_2(x,t)$.

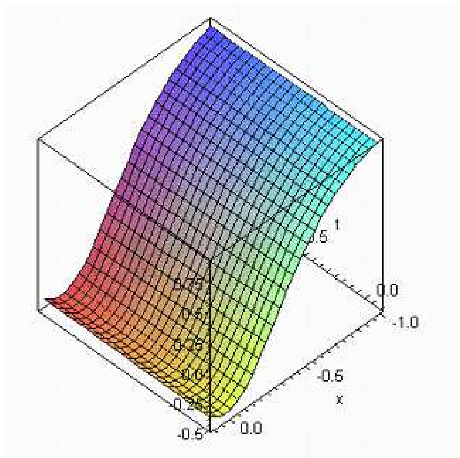

Figure 6: The soliton-like solution of $u_3\ (x,t)$.

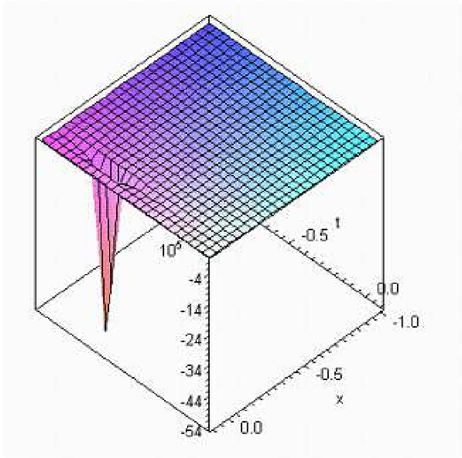

Figure 7: The periodic-like solution of $u_5\,(x,t)$.

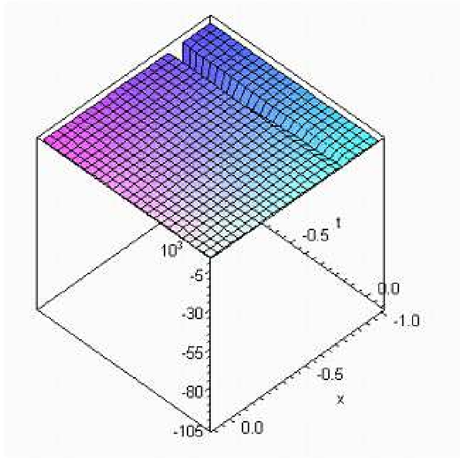

Figure 8: The periodic-like solution of $u_6\,(x,t)$.

$$u^2(\eta) = a2^2(\frac{G'}{G})^4 + 2a1a2(\frac{G'}{G})^3 + (2a0a2 + a1^2)(\frac{G'}{G})^2 + 2a0a1(\frac{G'}{G}) + a0^2 \tag{3.5}$$

$$\begin{aligned} u''(\eta) \;=\; & 6a2(\frac{G'}{G})^4 + (2a1 + 10a2\lambda)(\frac{G'}{G})^3 + (8a2\mu + 3a1\lambda + 4a2\lambda^2)(\frac{G'}{G})^2 \\ & +(6a2\lambda\mu + 2a1\mu + a1\lambda^2)(\frac{G'}{G}) + 2a2\mu^2 + a1\lambda\mu \end{aligned} \tag{3.6}$$

Substituting eqs (3.4)-(3.6) into eq. (3.3), setting the coefficients of $(\frac{G'}{G})^i(i = 0, 1, 2)$to zero, we obtain a system of algebraic equations as follows

$$(\frac{G'}{G})^4 \;:\; 6cka_2 - aa_2{}^2 + 6cba_2 = 0,$$

$$(\frac{G'}{G})^3 \;:\; 10cka_2\lambda - 2aa_1a_2 + 2cba_1 + 10cba_2\lambda + 2cka_1 = 0,$$

$$(\frac{G'}{G})^2 \;:\; a_2{+}3cba_1\lambda{+}4bca_2\lambda^2{-}aa_1{}^2{-}ca_2{+}\,8cba_2\mu{+}{+}\,8cka_2\mu{+}3cka_1\lambda\,-2aa_2a_0{+}4cka_2\lambda^2 = 0,$$

$$(\frac{G'}{G})^1 \;:\; 2cka_1\mu - ca_1 + a_1 + cba_1\lambda^2 - 2aa_1a_0 + 2cba_1\mu + 6bca_2\lambda\mu + cka_1\lambda^2 + 6cba_2\lambda\mu = 0,$$

$$(\frac{G'}{G})^0 \;:\; -aa_0{}^2 + cba_1\,\lambda\mu\, - ca_0 + a_0 + 2cba_2\mu^2 + cka_1\,\lambda\mu + 2cka_2\mu^2 = 0.$$

Solve the above equations by Maple gives
Solution 1.

$$a_0 \;=\; -\frac{6\mu(b+k)}{(-4b\mu - 4k\mu - 1 + k\lambda^2 + \lambda^2 b)\,a}, a_1 = -\frac{6\lambda(b+k)}{(-4b\mu - 4k\mu - 1 + k\lambda^2 + \lambda^2 b)\,a},$$

$$a_2 = -\frac{6(b+k)}{(-4b\mu-4k\mu-1+k\lambda^2+\lambda^2 b)\,a},$$

$$c = -\frac{1}{(-4b\mu-4k\mu-1+k\lambda^2+\lambda^2 b)}. \tag{3.7}$$

Therefore, substitute (3.7) to (3.4), and we can obtain that

$$u(\eta) = \tag{3.8}$$
$$-\frac{6(b+k)}{(-4b\mu-4k\mu-1+k\lambda^2+\lambda^2 b)a}\left(\frac{G'}{G}\right)^2 - \frac{6\lambda(b+k)}{(-4b\mu-4k\mu-1+k\lambda^2+\lambda^2 b)a}\left(\frac{G'}{G}\right) - \frac{6\mu(b+k)}{(-4b\mu-4k\mu-1+k\lambda^2+\lambda^2 b)a}.$$

Where $\eta = x + y + \frac{1}{(-4b\mu-4k\mu-1+k\lambda^2+\lambda^2 b)}t$,$\lambda$and $\mu$are arbitrary constants.

Case 1.

When $\lambda^2 - 4\mu$>0, $C1, C2 \neq 0$,we obtain the general hyperbolic function solutions of Eq. (3.1)

$$u_1(x,t) =$$
$$-\frac{6\mu(b+k)}{(-4b\mu-4k\mu-1+k\lambda^2+\lambda^2 b)a}$$
$$-\frac{6\lambda(b+k)}{(-4b\mu-4k\mu-1+k\lambda^2+\lambda^2 b)a}\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\eta\right)+C_2\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\eta\right)}{C_1\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\eta\right)+C_2\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\eta\right)}\right)-\frac{\lambda}{2}\right)$$
$$-\frac{6(b+k)}{(-4b\mu-4k\mu-1+k\lambda^2+\lambda^2 b)a}\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\eta\right)+C_2\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\eta\right)}{C_1\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\eta\right)+C_2\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\eta\right)}\right)-\frac{\lambda}{2}\right)^2$$

where $C1, C2$ are arbitrary constants.

When $\lambda^2 - 4\mu$>0, $C1 = 0$, $C2 \neq 0$,then

$$u_2(x,t) =$$
$$-\frac{6\mu(b+k)}{(-4b\mu-4k\mu-1+k\lambda^2+\lambda^2 b)a} - \frac{6\lambda(b+k)}{(-4b\mu-4k\mu-1+k\lambda^2+\lambda^2 b)a}\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\coth\frac{\sqrt{\lambda^2-4\mu}}{2}\eta - \frac{\lambda}{2}\right)$$
$$-\frac{6(b+k)}{(-4b\mu-4k\mu-1+k\lambda^2+\lambda^2 b)a}\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\coth\frac{\sqrt{\lambda^2-4\mu}}{2}\eta - \frac{\lambda}{2}\right)^2$$

When $\lambda^2 - 4\mu$>0, $C1 \neq 0$, $C2 = 0$,then

$$u_3(x,t) =$$
$$-\frac{6\mu(b+k)}{(-4b\mu-4k\mu-1+k\lambda^2+\lambda^2 b)a} - \frac{6\lambda(b+k)}{(-4b\mu-4k\mu-1+k\lambda^2+\lambda^2 b)a}\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\tanh\frac{\sqrt{\lambda^2-4\mu}}{2}\eta - \frac{\lambda}{2}\right)$$
$$-\frac{6(b+k)}{(-4b\mu-4k\mu-1+k\lambda^2+\lambda^2 b)a}\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\tanh\frac{\sqrt{\lambda^2-4\mu}}{2}\eta - \frac{\lambda}{2}\right)^2$$

Case 2.

When $\lambda^2 - 4\mu$<0, $C1, C2 \neq 0$, we obtain the general trigonometric function solutions of Eq. (3.1)

$$u_4(x,t) =$$
$$-\frac{6\mu(b+k)}{(-4b\mu-4k\mu-1+k\lambda^2+\lambda^2 b)a}$$

$$-\frac{6\lambda(b+k)}{(-4b\mu-4k\mu-1+k\lambda^2+\lambda^2b)a}\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\eta\right)+C_2\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\eta\right)}{C_1\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\eta\right)+C_2\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\eta\right)}\right)-\frac{\lambda}{2}\right)$$
$$-\frac{6(b+k)}{(-4b\mu-4k\mu-1+k\lambda^2+\lambda^2b)a}\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\eta\right)+C_2\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\eta\right)}{C_1\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\eta\right)+C_2\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\eta\right)}\right)-\frac{\lambda}{2}\right)^2$$

When $\lambda^2-4\mu<0$, $C_1=0$, $C_2\neq 0$,then

$$u_5(x,t)=$$
$$-\frac{6\mu(b+k)}{(-4b\mu-4k\mu-1+k\lambda^2+\lambda^2b)a}-\frac{6\lambda(b+k)}{(-4b\mu-4k\mu-1+k\lambda^2+\lambda^2b)a}\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\cot\frac{\sqrt{4\mu-\lambda^2}}{2}\eta\quad-\frac{\lambda}{2}\right)$$
$$-\frac{6(b+k)}{(-4b\mu-4k\mu-1+k\lambda^2+\lambda^2b)a}\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\cot\frac{\sqrt{4\mu-\lambda^2}}{2}\eta-\frac{\lambda}{2}\right)^2$$

When $\lambda^2-4\mu<0$, $C1\neq 0, C2=0$,then

$$u_6(x,t)=$$
$$-\frac{6\mu(b+k)}{(-4b\mu-4k\mu-1+k\lambda^2+\lambda^2b)a}-\frac{6\lambda(b+k)}{(-4b\mu-4k\mu-1+k\lambda^2+\lambda^2b)a}\left(-\frac{\sqrt{4\mu-\lambda^2}}{2}\tan\frac{\sqrt{4\mu-\lambda^2}}{2}\eta\quad-\frac{\lambda}{2}\right)$$
$$-\frac{6(b+k)}{(-4b\mu-4k\mu-1+k\lambda^2+\lambda^2b)a}\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\tan\frac{\sqrt{4\mu-\lambda^2}}{2}\eta-\frac{\lambda}{2}\right)^2$$

Solution 2.

$$a_0=\frac{\lambda^2b+2b\mu+k\lambda^2+2k\mu}{(-4b\mu-4k\mu+1+k\lambda^2+\lambda^2b)\,a},a_1=\frac{6\lambda(b+k)}{(-4b\mu-4k\mu+1+k\lambda^2+\lambda^2b)\,a},$$
$$a_2=\frac{6(b+k)}{(-4b\mu-4k\mu+1+k\lambda^2+\lambda^2b)\,a},c=\frac{1}{(-4b\mu-4k\mu+1+k\lambda^2+\lambda^2b)}. \tag{3.9}$$

Therefore, substitute (3.9) to (3.4), and we can obtain that

$$u(\eta)=\frac{6(b+k)}{(-4b\mu-4k\mu+1+k\lambda^2+\lambda^2b)a}\left(\frac{G'}{G}\right)^2+\frac{6\lambda(b+k)}{(-4b\mu-4k\mu+1+k\lambda^2+\lambda^2b)a}\left(\frac{G'}{G}\right)$$
$$+\frac{\lambda^2b+2b\mu+k\lambda^2+2k\mu}{(-4b\mu-4k\mu+1+k\lambda^2+\lambda^2b)a} \tag{3.10}$$

Where $\eta=x+y-\frac{1}{(-4b\mu-4k\mu+1+k\lambda^2+\lambda^2b)}t$ $\lambda$and $\mu$are arbitrary constants.
Case 1.
When $\lambda^2-4\mu>0$, $C1, C2\neq 0$, we obtain the general hyperbolic function solutions of Eq. (3.1)

$$u_1(x,t)=$$
$$\frac{\lambda^2b+2b\mu+k\lambda^2+2k\mu}{(-4b\mu-4k\mu+1+k\lambda^2+\lambda^2b)a}$$
$$+\frac{6\lambda(b+k)}{(-4b\mu-4k\mu+1+k\lambda^2+\lambda^2b)a}\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\eta\right)+C_2\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\eta\right)}{C_1\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\eta\right)+C_2\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\eta\right)}\right)-\frac{\lambda}{2}\right)$$
$$+\frac{6(b+k)}{(-4b\mu-4k\mu+1+k\lambda^2+\lambda^2b)a}\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\left(\frac{C_1\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\eta\right)+C_2\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\eta\right)}{C_1\cosh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\eta\right)+C_2\sinh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\eta\right)}\right)-\frac{\lambda}{2}\right)^2,$$

where $C1, C2$ are arbitrary constants.
When $\lambda^2 - 4\mu > 0$, $C1 = 0$, $C2 \neq 0$,then

$$\begin{aligned} u_2(x,t) &= \frac{\lambda^2 b+2b\mu+k\lambda^2+2k\mu}{(-4b\mu-4k\mu+1+k\lambda^2+\lambda^2 b)a} \\ &+\frac{6\lambda(b+k)}{(-4b\mu-4k\mu+1+k\lambda^2+\lambda^2 b)a}\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\coth\frac{\sqrt{\lambda^2-4\mu}}{2}\eta \quad -\frac{\lambda}{2}\right) \\ &+\frac{6(b+k)}{(-4b\mu-4k\mu+1+k\lambda^2+\lambda^2 b)a}\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\coth\frac{\sqrt{\lambda^2-4\mu}}{2}\eta \quad -\frac{\lambda}{2}\right)^2 \end{aligned}$$

When $\lambda^2 - 4\mu > 0$, $C1 \neq 0, C2 = 0$,then

$$\begin{aligned} u_3(x,t) &= \frac{\lambda^2 b+2b\mu+k\lambda^2+2k\mu}{(-4b\mu-4k\mu+1+k\lambda^2+\lambda^2 b)a} \\ &+\frac{6\lambda(b+k)}{(-4b\mu-4k\mu+1+k\lambda^2+\lambda^2 b)a}\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\tanh\frac{\sqrt{\lambda^2-4\mu}}{2}\eta -\frac{\lambda}{2}\right) \\ &+\frac{6(b+k)}{(-4b\mu-4k\mu+1+k\lambda^2+\lambda^2 b)a}\left(\frac{\sqrt{\lambda^2-4\mu}}{2}\tanh\frac{\sqrt{\lambda^2-4\mu}}{2}\eta -\frac{\lambda}{2}\right)^2 \end{aligned}$$

Case 2.
When $\lambda^2 - 4\mu < 0$, $C1, C2 \neq 0$, we obtain the general trigonometric function solutions of Eq. (3.1)

$$\begin{aligned} &u_4(x,t) = \\ &\frac{\lambda^2 b+2b\mu+k\lambda^2+2k\mu}{(-4b\mu-4k\mu+1+k\lambda^2+\lambda^2 b)a} \\ &+\frac{6\lambda(b+k)}{(-4b\mu-4k\mu+1+k\lambda^2+\lambda^2 b)a}\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\eta\right)+C_2\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\eta\right)}{C_1\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\eta\right)+C_2\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\eta\right)}\right)-\frac{\lambda}{2}\right) \\ &+\frac{6(b+k)}{(-4b\mu-4k\mu+1+k\lambda^2+\lambda^2 b)a}\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\left(\frac{-C_1\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\eta\right)+C_2\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\eta\right)}{C_1\cos\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\eta\right)+C_2\sin\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\eta\right)}\right)-\frac{\lambda}{2}\right)^2 \end{aligned}$$

When $\lambda^2 - 4\mu < 0$, $C1 = 0$, $C2 \neq 0$,then

$$\begin{aligned} u_5(x,t) &= \frac{\lambda^2 b+2b\mu+k\lambda^2+2k\mu}{(-4b\mu-4k\mu+1+k\lambda^2+\lambda^2 b)a} \\ &+\frac{6\lambda(b+k)}{(-4b\mu-4k\mu+1+k\lambda^2+\lambda^2 b)a}\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\cot\frac{\sqrt{4\mu-\lambda^2}}{2}\eta -\frac{\lambda}{2}\right) \\ &+\frac{6(b+k)}{(-4b\mu-4k\mu+1+k\lambda^2+\lambda^2 b)a}(\frac{\sqrt{4\mu-\lambda^2}}{2}\cot\frac{\sqrt{4\mu-\lambda^2}}{2}\eta -\frac{\lambda}{2})^2 \end{aligned}$$

When $\lambda^2 - 4\mu < 0$, $C1 \neq 0, C2 = 0$,then

$$\begin{aligned} &u_6(x,t) = \\ &\frac{\lambda^2 b+2b\mu+k\lambda^2+2k\mu}{(-4b\mu-4k\mu+1+k\lambda^2+\lambda^2 b)a}+\frac{6\lambda(b+k)}{(-4b\mu-4k\mu+1+k\lambda^2+\lambda^2 b)a}\left(-\frac{\sqrt{4\mu-\lambda^2}}{2}\tan\frac{\sqrt{4\mu-\lambda^2}}{2}\eta -\frac{\lambda}{2}\right) \\ &+\frac{6(b+k)}{(-4b\mu-4k\mu+1+k\lambda^2+\lambda^2 b)a}\left(\frac{\sqrt{4\mu-\lambda^2}}{2}\tan\frac{\sqrt{4\mu-\lambda^2}}{2}\eta -\frac{\lambda}{2}\right)^2 \end{aligned}$$

4. Conclusions

Three types of travelling solutions of the ZK-BBM equation are successfully found by using the $\left(\frac{G'}{G}\right)$-expansion method. Numerical results and graphical representations are explicitly reflecting the efficiency and higher accuracy of the proposed scheme.

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