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Peculiarities of Double Differential Scattering Probability of Ultrashort X-ray Pulses by Free Electron

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ABSTRACT: The paper is devoted to the theoretical investigation of ultrashort X-ray pulses scattering by free electron. Using Klein-Nishina formula we obtained the expression for double differential scattering probability. The effect of scattered radiation spectral shift is studied for X-ray photons including soft X-ray range. Peculiarities of scattering probability dependence on duration of pulse are investigated.

INTRODUCTION

The development of generation of ultrashort electromagnetic pulses (USP) with a duration down to tens of attoseconds opens a new scientific field, attosecond physics [1, 2].

Ultrashort X-ray pulses are usually generated by X-ray free electron lasers. The durations of XFEL pulses of about 1 fs and less have already been obtained [3]. Tanaka [4] proposed a method for generation of single one-cycle X-ray pulses with a duration of 380 as on an carrier wavelength of 8.6 nm. In view of these achievements the theoretical analysis of the interaction of atto- and femtosecond X-ray pulses with various targets becomes topical. One of the simplest cases of such interaction is USP scattering on free electron

In the previous paper [5] we considered the USP scattering probability on free electron integrated over scattering angle and scattering frequency for various types of pulses in nonrelativistic limit. It was shown in particular that the total probability of scattering of ultrashort pulses for all carrier frequencies is a monotonically increasing function of the pulse duration.

In this paper we theoretically investigate the peculiarities arising in scattering of ultrashort X-ray pulses by free electron in terms of spectral-angular probability of the process during all time of the pulse action.

METHOD OF CALCULATION

The general expression for double-differential scattering probability during all time of the pulse action is given by formula: [6-7]:

$$\frac{d^2W}{d\omega' d\Omega'} = \frac{c}{4\pi^2}\int_0^\infty \frac{d\sigma_{scat}(\omega,\omega',\theta)}{d\omega' d\Omega'} \frac{\left|E(\omega,\omega_c,\tau)\right|^2}{\hbar\omega} d\omega \tag{1}$$

here θ is scattering angle, Ω' - solid angle, ω and ω' - frequencies of incident and scattered radiation correspondingly, c is light velocity, $\frac{d\sigma_{scat}(\omega,\omega',\theta)}{d\omega' d\Omega'}$ is spectral-angular cross-section of monochromatic radiation scattering. Using

Klein-Nishina formula [8] and entering Dirac δ-function we obtained expression for double differential scattering cross-section for free electron:

$$\frac{d\sigma_{scat}(\omega,\omega',\theta)}{d\omega' d\Omega'} = \frac{r_e^2}{2}\left(\frac{\omega'}{\omega}\right)^2\left(\frac{\omega}{\omega'}+\frac{\omega'}{\omega}-2\sin^2\theta\right)\delta\left(\frac{\omega}{1+\frac{\hbar\omega(1-\cos\theta)}{mc^2}}-\omega'\right) \tag{2}$$

$r_e = \dfrac{e^2}{mc^2}$ – classical electron radius.

$E(\omega,\omega_c,\tau)$ is Fourier transform of electric field strength in the laser pulse, ω_c is carrier frequency of the pulse, τ is pulse duration.

Using property of Dirac δ-function, we obtain from (1) and (2) the following expression:

$$\frac{d^2W}{d\omega' d\Omega'} = \frac{c\,r_e^2}{8\pi^2}\left(\frac{\omega_0}{\omega'}+\frac{\omega'}{\omega_0}-\sin^2\theta\right)\frac{\left|E(\omega_0,\omega_c,\tau)\right|^2}{\hbar\omega_0} \tag{3}$$

where

$$\omega_0 = \frac{\omega'}{1-\left(\hbar\,\omega'/m\,c^2\right)\left(1-\cos\theta\right)} \tag{4}$$

In the following we consider scattering of so called corrected Gaussian pulse [9]. Fourier transform of the electric field strength in this pulse has the form:

$$E(\omega,\omega_c,\tau) = i\,\tau\,E_0\sqrt{\frac{\pi}{2}}\left\{\frac{\omega^2\tau^2}{1+\omega_c^2\tau^2}\right\}\left\{\exp\left[-(\omega-\omega_c)^2\tau^2/2\right]-\exp\left[-(\omega+\omega_c)^2\tau^2/2\right]\right\} \tag{5}$$

Further we consider E_0 = 1 a.u.

Substituting (5) in (3) we obtain the calculation formula for double differential scattering probability of corrected Gaussian laser pulse by free electron in wide spectral range.

RESULTS OF THE CALCULATIONS AND THEIR ANALYSIS

Figure 1 illustrates the dependence of double differential scattering probability on frequency of scattered radiation at the different scattering angles. As it clearly seen from the graph, frequencies of maxima coincide and are equal to carrier frequency for different scattering angles in the case when ω_c = 150 eV. The scattering probability reaches its maximum at the tending of scattering angle to 0 or to π, and decreases monotonically at tending to π/2. According to our calculations, for carrier frequencies considered in this paper magnitudes of peaks are equal for scattering angles equidistant from θ = π/2. At carrier frequency on the order of 1 keV it is noticeable that frequencies of maxima are not equal to carrier frequency, but shifts in the area of lower frequencies at the increase of scattering angle. According to the (3), (5) the frequency of maximum $\omega'_{\max}$ could be found from the condition: $\omega_0 = \omega_c$. Hence:

$$\omega'_{\max} = \frac{\omega_c}{1+\frac{\hbar\omega_c(1-\cos\theta)}{mc^2}} \tag{6}$$

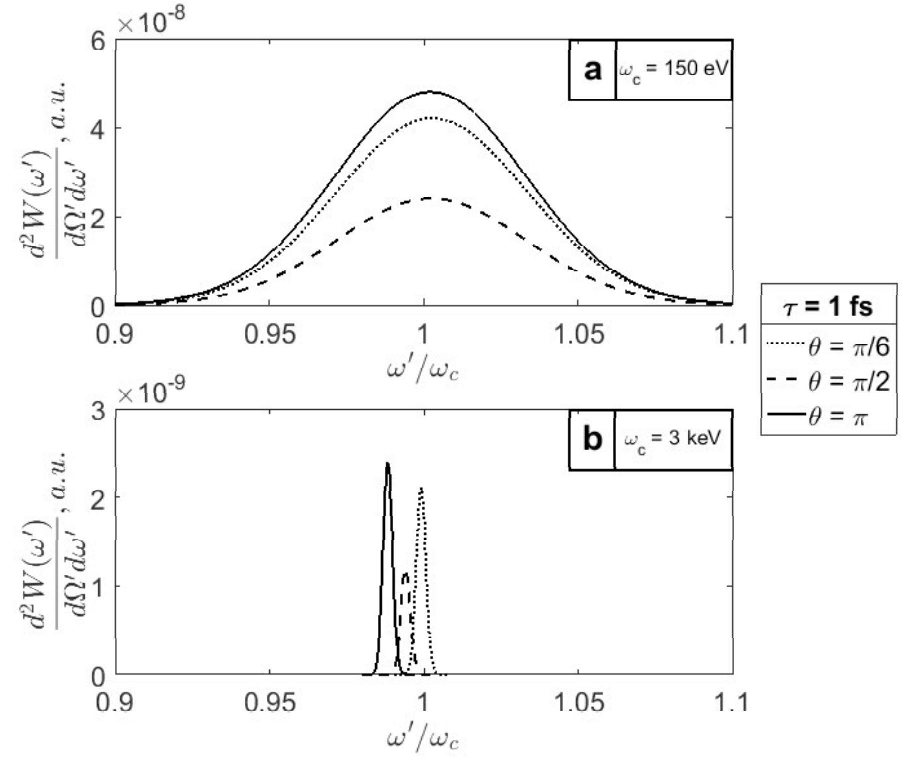

Fig.1. Dependence of double differential scattering probability on frequency of scattered radiation at the different scattering angles. Plot (a) corresponds to the carrier frequency ω_c = 150 eV, plot (b) – ω_c = 3 keV.

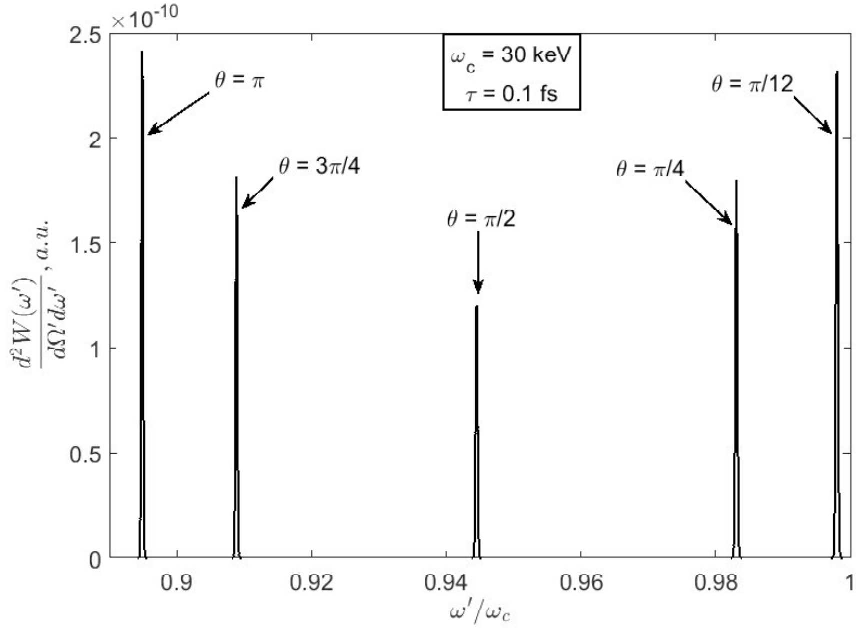

Fig.2. Dependence of double differential scattering probability on frequency of scattered radiation at different scattering angles, ω_c = 30 keV.

Fig.2 illustrates the same dependence as at Fig.1, but for sufficiently larger carrier frequency. In this case the energy of incident photons is about units of percent from the electron energy at rest. Nevertheless it is clearly seen that the value of shift is approximately 10% of carrier frequency. The shifting effect enhances at increase of carrier frequency. In contrast to the case of low carrier frequencies, the ratio $\omega_c/\Delta\omega$ is small enough to suppose the scattered radiation is quasi-monochromatic. Thus, important feature of scattering process at high carrier frequency is the strong dependence of scattered radiation on scattering angle. Fig.3 demonstrates this effect in detail. It can be seen that at higher carrier frequencies the influence of term $\frac{\hbar\omega'(1-\cos\theta)}{mc^2}$ is sufficient.

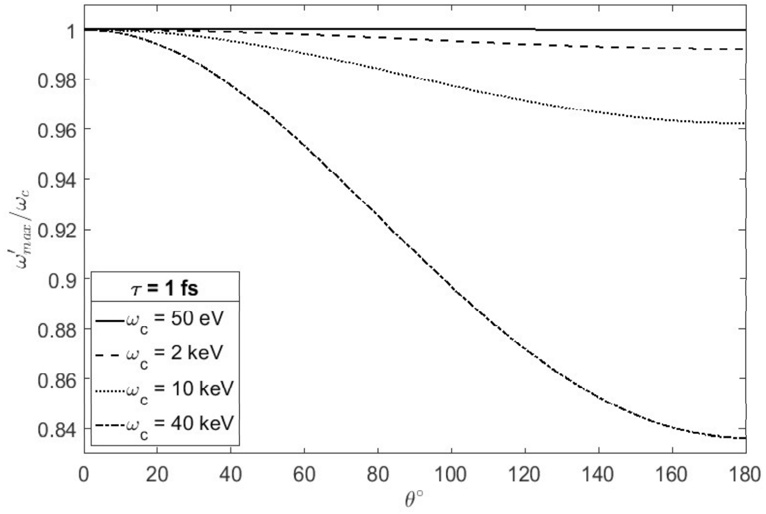

Fig.3. Dependence of frequency of scattering radiation on scattering angle at different carrier frequencies of incident pulse. Here we assume that scattered pulse is quasi-monochromatic.

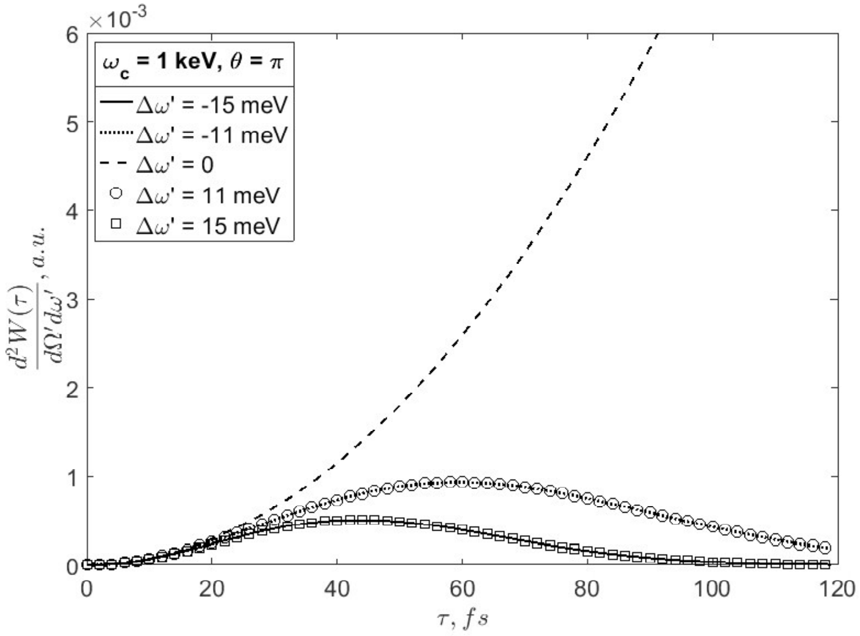

Fig.4. Dependence of double differential scattering probability on pulse duration at detuning of scattered radiation frequence from ω'_{max} by the value on the order of 10 meV.

Fig.4 illustrates a dependence of double differential scattering probability on pulse duration. Considered function has two different tendencies depending on value of detuning from frequency ω'_{max}. As it follows from (3) and (5), within this condition $\omega' = \omega'_{max}$ and $\tau\to\infty$ the dependence is parabolic. In case when $\omega' \neq \omega'_{max}$ the maximum in dependence appears. The limit of function at $\tau\to\infty$ is equal to 0. Moreover, at the increase of detuning $\Delta\omega'$ the considered dependency tends to zero quicker and the value of duration τ_{max} corresponding to the maximum of probability decreases. It is noticeable that the values of probability are equal for arguments $\omega' = \omega'_{max} \pm \Delta\omega$. The latter can be proved by evenness of graph illustrated at Fig.5, which demonstrates dependence of τ_{max} on $\Delta\omega'$.

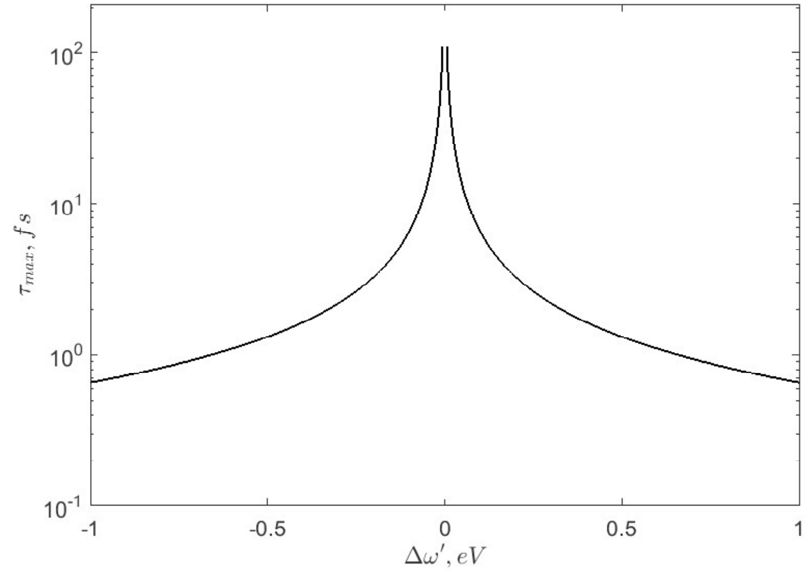

Fig.5. Dependence of pulse duration corresponding to the maximum of probability on value of detuning from frequency of maximum ω'_{max}.

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According to our calculations the value of τ_{max} is inverse to the value of detuning $\Delta\omega'$. Then using (6) we obtained:

$$\tau_{\max} = \left| \frac{\omega_c}{1 + \frac{\hbar\omega_c (1 - \cos\theta)}{mc^2}} - \omega' \right|^{-1} \tag{7}$$

Fig.6 reflects the same dependencies that the Fig.4, but for larger values of detuning. As it clearly seen, the value of double differential dependency in this case is sufficiently lower.

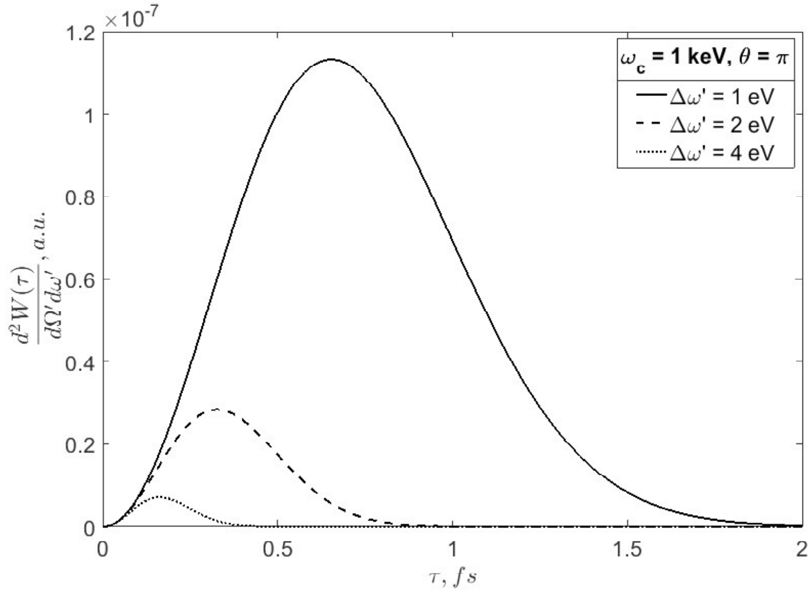

Fig.6. Dependence of double differential scattering probability on pulse duration at detuning of scattered radiation frequency from ω'_{max} by the value on the order of 1 eV.

CONCLUSION

We derived the formula for double-differential scattering probability of ultrashort X-ray pulse on free electron.

We analyzed the dependence of double-differential scattering probability on frequency of scattered radiation. It was shown that at high carrier frequencies the frequency of scattered radiation is strongly dependent on scattering angle in contrast to the case of low carrier frequencies when spectrum of scattered pulse differs from incident only by magnitude. The expression for frequency of scattered radiation was obtained.

The dependence of double-differential scattering probability on incident pulse duration was analyzed. We identified and described two trends of this dependence. In case of resonance when the frequency of scattered radiation is equal to carrier frequency, the dependence is parabolic. In other cases the function is not monotonic and asymptotically tends to zero. The formula for duration when the scattering probability is maximal was derived.

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