

# Design, Dynamic Modelling, and Control of a Two-wheeled Self-Balancing Robot (TWSBR)

Vinod Kumar P<sup>1</sup>, Dr. Kamala N<sup>2</sup>

<sup>1</sup>Research Scholar, Jain (Deemed to be University), Bengaluru, India

<sup>2</sup>Associate Professor, Department of Robotics & Automation, Jain (Deemed to be University), Bengaluru, India

## Abstract

This paper presents the design, dynamic model, and control of a two-wheeled self-balancing robot (TWSBR) with three different methods. A TWSBR is an underactuated second-order system in which balancing and motion control is the utmost requirement. Initially, the equations of motion for the TWSBR are derived. The linearization technique is adopted to represent the state space system equations. Some controllers are designed to obtain the desired vertical and horizontal position of the TWSBR. The effectiveness of (proportional–integral–derivative) PID, (Linear–quadratic regulator) LQR, and (artificial neural network) ANN controllers with underactuated systems are studied and their performance is validated in presence of external disturbance as input to the system. Based on rigorous testing with different disturbance signals, ANN models have shown more promising result for controlling two-wheeled robots, under disturbance situations.

*Keywords: TWSBR, Underactuated system, PID controller, LQR controller, ANN controller*

## 1. Introduction

The design and construction of two-wheel self-balancing robots (TWSBR) are one of the attractive fields in control engineering and so far several control methods have been proposed and applied for it. Two-wheel self-balancing robot is additionally a case of cutting-edge advancement in the mechanical technology field like Segway, the human posture systems, the launching of a rocket, and etc. The design of a self-balancing robot is a practical application for the inverted pendulum. A self-balancing robot is dependent on a control framework that is utilized to balance out a flimsy framework utilizing some microcontrollers and sensors. These robots need remarkable strength and capacity because of their little size and force prerequisites and are used for many applications such as surveillance and transportation.

Specifically, the emphasis is on the electro-mechanical systems and control calculations required to empower a robot to act continuously. The comparative ideas can be applied in different control frameworks with complex execution, for example, a humanoid robot, autonomous robots, and so forth.

In this work, a mathematical model of 3 degree-of-freedom (DOF) 2-wheels inverted pendulum is derived and the model will be used for the design of a new robust controller. The dynamic modeling is done directly in terms of variables that are of interest concerning the planning and control of the 2-wheeled inverted pendulum position, inclination, speed, and open for further exploration on heading orientation. The schematic view of the TWSBR is shown in figure 1.

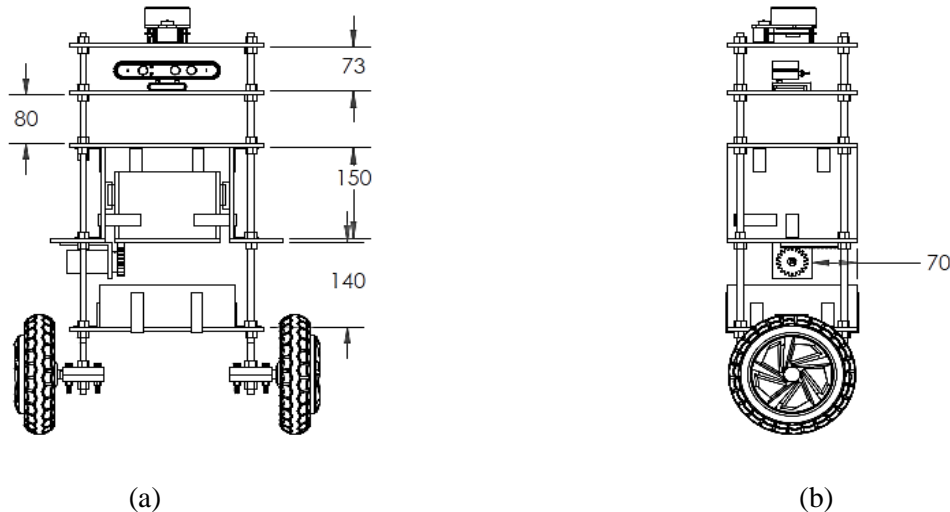


Figure 1. Full assembly schematic diagram; a) Front view, b) Side view

### 2.1 Review of Dynamic Modeling Literature

The TWSBR schemes are applied from apocalyptic navigation vehicles [1-4] to individual transporters [5-9] and even wheelchairs for physically challenged people [10-11]. A dynamic model is an ideal requirement for control designs like adaptive control [5,14], fuzzy logic control [15], and sliding control [16]. The key attributes of TWSBR can be understood through dynamic analysis [3], identification of model [8], and controllability [17]. In addition, pertinent algorithm designs of generation of trajectory [18] and avoidance of obstacle [19] can be helped through relevant dynamic models.

Here, we have three ways to obtain dynamic model of TWSBR. First, the Newtonian method which is more complex process of seeing interactive forces between elements with vector relations [2,12,15,20]. Second, the Lagrangian method which is widely used as motion equations can be derived as Potential energy & Kinetic energy [1,6,8,13,14,16,17,21,22]. Thirdly, Kane's [23] approach is based on operations of vectors similar to mechanics without rigid bodies interactive forces. To verify the system stability, mathematical modeling of SBR with a DC gear motor is realized. Inverted pendulum model is derived and controllers like PID, LQR & LQG is used to control robot. The errors occurred during modeling derivation in previous studies lack knowledge of rotational motion and its comparison. In this paper, dynamic modeling of TWSBR in terms of Lagrangian method is adopted and stability performance is evaluated through different controllers.

### 2.2 Review on Design of Controllers

There are many researches that dedicated their effort to controlling two wheeled robots. Controlling of a self-balancing robot with Arduino in houses with two DC motors, Bluetooth module and drive motor controller is a sample of such efforts. PID, LQR controllers are designed to evaluate system stability. Through simulation results, performance indicates sensitivity to sensor noise causing instability within few seconds. [26,36].

Because of highly unstable two-wheeled robot, control movement gyroscope (CMG) controllers are used to compensate system stability by robot body tilting through torque generated by CMG [27,38]. Similarly, myRio controller which includes on-board accelerometer, Peripheral module gyroscope and PD controller have been used to hold robot upright under disturbance. Simulated Labview results shows effectiveness of controller to hold robot upright due to weight of model and external disturbances a matter of concern [28,36].

Due to stability concerns of two wheeled robot, Kalman filter with PID controller is designed to control the stability issue in robot against disturbance. Their performance against disturbance is studied for PID controller with Kalman filter, (Linear-quadratic regulator) LQR, (Linear-quadratic-Gaussian control) LQG, H<sub>2</sub> & H<sub>∞</sub> so on [29,30,35]. In addition to above controllers, double loop control technique is adopted to robot to regulate speed, design pitch angle of the robot in addition to maintain system stable against disturbance [33,34]. Sliding mode control (SMC) techniques is also adopted for balancing the robot at high-speed rate by adjusting the sliding

mode parameters [40]. Two arms added to the robot for balancing & resting activities with help of remote-controlled operation [41]. Two inputs like angle and angle change are used for proper balancing of a pole-cart model and simulation is carried out in Matlab software. Proper robot movement is achieved through DC motor and H-bridge power circuits [42].

In recent years with improving in the Artificial Intelligence (AI) technology all around the world using of AI model is plausible for controlling the robots. One of the most important structures for prediction of the output is neural network and its branches. Using of presented neural network for prediction of output with desired target is sample of using AI for controlling two wheeled robots [44]. Other AI algorithms such as reinforcement learning has shown good response for solving similar problem to two wheeled robots too [45]. Benefit of using reinforcement learning algorithms can be seen in unsupervised nature of the problem for predicting next actions in order to solve the problem of unsustainability of robots [45]. Based on non-linear nature of the two wheeled robots use of fuzzy logic as controller has shown good performance for such a system. Zhao et al, have focused on comparing performance of the singleton fuzzy controllers and non-singleton fuzzy controllers for control of two wheeled robots. They concluded that performance of non-singleton fuzzy logic controllers is better than that of singleton fuzzy logic controllers for two wheeled robots' controller [46]. As it is clear using different AI methods for controlling such a robots will help to make whole structure of the robots stable while movement. Based on recent achievement on AI algorithms for two wheeled robots' control in this research using of ANN with fuzzy logic as been tested for controlling of two wheeled robots.

The theoretical dynamic model is applied to govern the entire system to construct the control system. The dynamic model has nonlinear structure. It should be linearized in the way to design a linear controller. At zero of tilt angle, the robot system has its quasi-equilibrium state. So, in this case the linearized model is assumed that the variation of the tilt angle is small enough to be neglected. Then we have this linearized model in state space form.

### 3. Mathematical Modeling of TWSBR

The forces and moments acting on the TWSBR model and the representation of the wheels are shown in figure 2(a) and (b) respectively [24]. The system variables are shown in table 1.

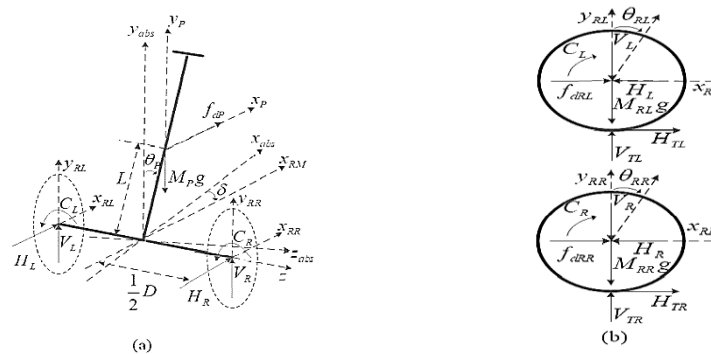


Figure 2. Presentation of forces and moments that are acting; (a) on the TWSBR, (b) robot's wheels [24].

The right wheel mathematical equations are given:

$$J_{P\theta} \ddot{\theta}_P = M_P g L \theta_P + (M_{RR} \ddot{x}_{RR} + M_{RL} \ddot{x}_{RL}) L - (f_{dRR} + f_{dRL}) L + (J_{RR} \ddot{\theta}_{RR} + J_{RL} \ddot{\theta}_{RL}) \frac{L}{R} - (C_R + C_L) \frac{L}{R} \tag{1}$$

$$M_{RR} \ddot{x}_{RR} = f_{dRR} + H_{TR} - H_R - b \dot{x}_{RR} - f_c \text{sgn}(\dot{x}_{RR}) \tag{2}$$

$$M_{RR} \dot{y}_{RR} = V_{TR} - M_{RR} g - V_R \tag{3}$$

$$J_{RR} \ddot{\theta}_{RR} = C_R - (H_{TR} - b \dot{x}_{RR} - f_c \text{sgn}(\dot{x}_{RR})) R \tag{4}$$

Similarly, the left wheel can be modelled as follows

$$M_{RL}\ddot{x}_{RL} = f_{dRL} + H_{TL} - H_L - b\dot{x}_{RL} - f_c \text{sgn}(\dot{x}_{RL}) \quad 5$$

$$M_{RL}\ddot{y}_{RL} = V_{TL} - M_{RL}g - V_L \quad 6$$

$$J_{RL}\ddot{\theta}_{RL} = C_L - (H_{TL} - b\dot{x}_{RL} - f_c \text{sgn}(\dot{x}_{RL}))R \quad 7$$

Table 1. System variable

Parameter	Nomenclature	Parameter	Nomenclature
$x_{RR}$	Displacement of right wheel in horizontal direction	b	Viscous Friction
$x_{RL}$	Displacement of left wheel in horizontal direction	$f_c$	Static friction
$M_{RR}$	right wheel mass	$C_R$	Applied torque on the right wheel
$M_{RL}$	left wheel mass	$C_L$	Applied torque on the left wheel
$y_{RR}$	Displacements of right wheel in vertical direction	$J_{RR}$	Moments of inertia of the right wheel
$y_{RL}$	Displacements of left wheel in vertical direction	$J_{RL}$	Moments of inertia of the left wheel
$\theta_{RR}$	Shaft angle of right wheel	R	Wheel radius
$\theta_{RL}$	Shaft angle of left wheel	$H_R, H_L, V_R$ and $V_L$	Reaction forces between wheels & chassis
$f_{dRR}$	Forces applied to centre of right wheel	$H_{TR}, H_{TL}, V_{TR}$ and $V_{TL}$	Reaction forces between wheels & ground
$f_{dRL}$	Forces applied to centre of left wheel	sgn(.)	Standard signum function
$M_P$	Mass of the chassis	L	Distance between the z axis and the CG of the chassis
$f_{dP}$	Force applied to the centre of gravity (CG)	D	Lateral distance between the contact patches of the wheels
$\theta_P$	Tilt angle of the TWSBR	$x_{RM}$	Position of the TWSBR
$\theta_y$	Yaw angle of the TWSBR	$v_{RM}$	Velocity of the TWSBR
$J_{p\theta}$	Moments of inertia of chassis wrt Z axis	$\omega_P$	TWSBR tilt angular velocity
$J_{py}$	Moments of inertia of chassis wrt y axis	$\omega_Y$	TWSBR yaw angular velocity

Further, the motion of the chassis for translation and rotation is expressed as:

$$M_P\ddot{x}_P = f_{dP} + H_R + H_L \quad 8$$

$$M_P\ddot{y}_P = V_R + V_L - M_Pg \quad 9$$

$$J_{P\theta}\ddot{\theta}_P = (V_R + V_L)L\sin\theta_P - (H_R + H_L)L\cos\theta_P - (C_L + C_R) \quad 10$$

$$\ddot{\theta}_y J_{Py} = (H_L - H_R)D/2 \quad 11$$

From equation 9 we have

$$V_R + V_L = M_P g \quad 12$$

From equation 2 & 5

$$V_{TR} + V_{TL} = M_{RR}\ddot{y}_{RR} + M_{RL}\ddot{y}_{RL} + (M_{RR} + M_{RL})g + M_P g \quad 13$$

From Eq3 and 6:

$$H_{TR} + H_{TL} = -\frac{J_{RR}\ddot{\theta}_{RR} + J_{RL}\ddot{\theta}_{RL}}{R} + \frac{C_R + C_L}{R} + (\dot{x}_{RR} + \dot{x}_{RL})b + (\text{sgn}(\dot{x}_{RR}) + \text{sgn}(\dot{x}_{RL}))f_c \quad 14$$

From Eq1 and 4:

$$H_R + H_L = -M_{RR}\ddot{x}_{RR} - M_{RL}\ddot{x}_{RL} + f_{dRR} + f_{dRL} + H_{TR} + H_{TL} - (\dot{x}_{RR} + \dot{x}_{RL})b - (\text{sgn}(\dot{x}_{RR}) + \text{sgn}(\dot{x}_{RL}))f_c \quad 15$$

From Eq3, 6 and 7:

$$H_R + H_L = -M_{RR}\ddot{x}_{RR} - M_{RL}\ddot{x}_{RL} + f_{dRR} + f_{dRL} - \frac{J_{RR}\ddot{\theta}_{RR} + J_{RL}\ddot{\theta}_{RL}}{R} + \frac{C_R + C_L}{R} = M_P\ddot{x}_P - f_{dP} \quad 16$$

Eq9:

$$J_{P\theta}\ddot{\theta}_P = M_P g L \theta_P + (M_{RR}\ddot{x}_{RR} + M_{RL}\ddot{x}_{RL})L - (f_{dRR} + f_{dRL})L + (J_{RR}\ddot{\theta}_{RR} + J_{RL}\ddot{\theta}_{RL})\frac{L}{R} - (C_R + C_L)\frac{L}{R} \quad 17$$

If we have  $x_P = \frac{x_{RR} + x_{RL}}{2}$ ,  $y_P = \frac{y_{RR} + y_{RL}}{2}$  and  $M_{RR} = M_{RL}$  then

From Eq2 and 5:

$$V_{TR} + V_{TL} = 2M_{RR}\dot{y}_P + 2M_{RR}g + M_P g \quad 18$$

From Eq3 and 6:

$$H_{TR} + H_{TL} = -\frac{J_{RR}\ddot{\theta}_{RR} + J_{RL}\ddot{\theta}_{RL}}{R} + \frac{C_R + C_L}{R} + 2\dot{x}_P b + (\text{sgn}(\dot{x}_{RR}) + \text{sgn}(\dot{x}_{RL}))f_c \quad 19$$

From Eq1 and 4:

$$H_R + H_L = -2M_{RR}\ddot{x}_P + f_{dRR} + f_{dRL} - \frac{J_{RR}\ddot{\theta}_{RR} + J_{RL}\ddot{\theta}_{RL}}{R} + \frac{C_R + C_L}{R} = M_P\ddot{x}_P - f_{dP} \quad 20$$

$$M_{RR}\ddot{x}_{RR} = f_{dRR} + H_{TR} - H_R - b * \dot{x}_{RR} - f_c * \text{sign}(\dot{x}_{RR}) \quad 21$$

$$M_{RR}\ddot{y}_{RR} = V_{TR} - (M_{RR} * g) - V_R \quad 22$$

$$J_{RR}\ddot{\theta}_{RR} = C_R - (H_{TR} - b * \dot{x}_{RR} - f_c * \text{sign}(\dot{x}_{RR})) * R \quad 23$$

$$M_{RL}\ddot{x}_{RL} = f_{dRL} + H_{TL} - H_L - b * \dot{x}_{RL} - f_c * \text{sign}(\dot{x}_{RL}) \quad 24$$

$$M_{RL}\ddot{y}_{RL} = V_{TL} - M_{RL} * g - V_L \quad 25$$

$$J_{RL}\ddot{\theta}_{RL} = C_L - (H_{TL} - b * \dot{x}_{RL} - f_c * \text{sign}(\dot{x}_{RL})) * R \quad 26$$

$$M_P\ddot{x}_P = f_{dP} + H_R + H_L \quad 27$$

$$J_{Pt}\ddot{\theta}_P = (V_R + V_L) * L * \theta_P - (H_R + H_L) * L - (C_L + C_R) \quad 28$$

$$\ddot{\theta}_y * J_{Py} = \frac{(H_L - H_R) * D}{2} \quad 29$$

After spending times for calculations, the linearized model of the TWSBR has been presented in the state-space form. The linearized equation is as follow:

$$\begin{bmatrix} \dot{x}_{RM} \\ \dot{v}_{RM} \\ \dot{\theta}_p \\ \dot{\omega}_p \\ \dot{\theta}_y \\ \dot{\omega}_y \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & A_{22} & A_{23} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & A_{42} & A_{43} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & A_{66} \end{bmatrix} \begin{bmatrix} x_{RM} \\ v_{RM} \\ \theta_p \\ \omega_p \\ \theta_y \\ \omega_y \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ B_{21} & 0 \\ 0 & 0 \\ B_{41} & 0 \\ 0 & 0 \\ 0 & B_{62} \end{bmatrix} \begin{bmatrix} u_p \\ u_y \end{bmatrix} + \begin{bmatrix} 0 \\ f_2 \\ 0 \\ f_4 \\ 0 \\ f_6 \end{bmatrix} \quad 30$$

$$k_1 = \frac{N * k_t}{R_a} \quad 31$$

$$k_2 = \frac{N^2 * k_t * k_e}{R * R_a} \quad 32$$

$$m = 2 * (M_p * J_{RR} * L^2 + M_p * M_{RR} * R^2 * L^2 + J_{RR} * J_{Pt} + J_{Pt} * M_{RR} * R^2) \quad 33$$

$$n = 2 * J_{Py} * R^2 - J_{RR} * D^2 - M_{RR} * R^2 * L^2 \quad 34$$

$$A_{22} = \frac{-2 * k_2 * (M_p * R * L^2 + M_p * R^2 * L + J_{Pt} * R)}{m} \quad 35$$

$$A_{23} = \frac{-(M_p^2 * R^2 * L^2 * g)}{m} \quad 36$$

$$A_{42} = \frac{-2 * k_2}{(J_{Pt} + M_p * L)} \quad 37$$

$$A_{43} = \frac{(M_p * g * L)}{(J_{Pt} + M_p * L)} \quad 38$$

$$A_{66} = \frac{-k_2 * D^2 * R}{n} \quad 39$$

$$B_{21} = \frac{k_1 * (M_p * R * L^2 + M_p * R^2 * L + J_{Pt} * R)}{m} \quad 40$$

$$B_{41} = \frac{k_1}{(J_{Pt} + M_p * L)} \quad 41$$

$$B_{62} = \frac{-k_1 * D * R}{n} \quad 42$$

#### 4. CONTROLLER DESIGN & BEHAVIOR ANALYSIS

Figure 3 shown the structure of the non-linear model based on equations (1) to (10). Based on presented structure on Figure 3 input of the model is voltage of motors and outputs are velocity and acceleration. The system is non-linear so; linearization is next step for this process.

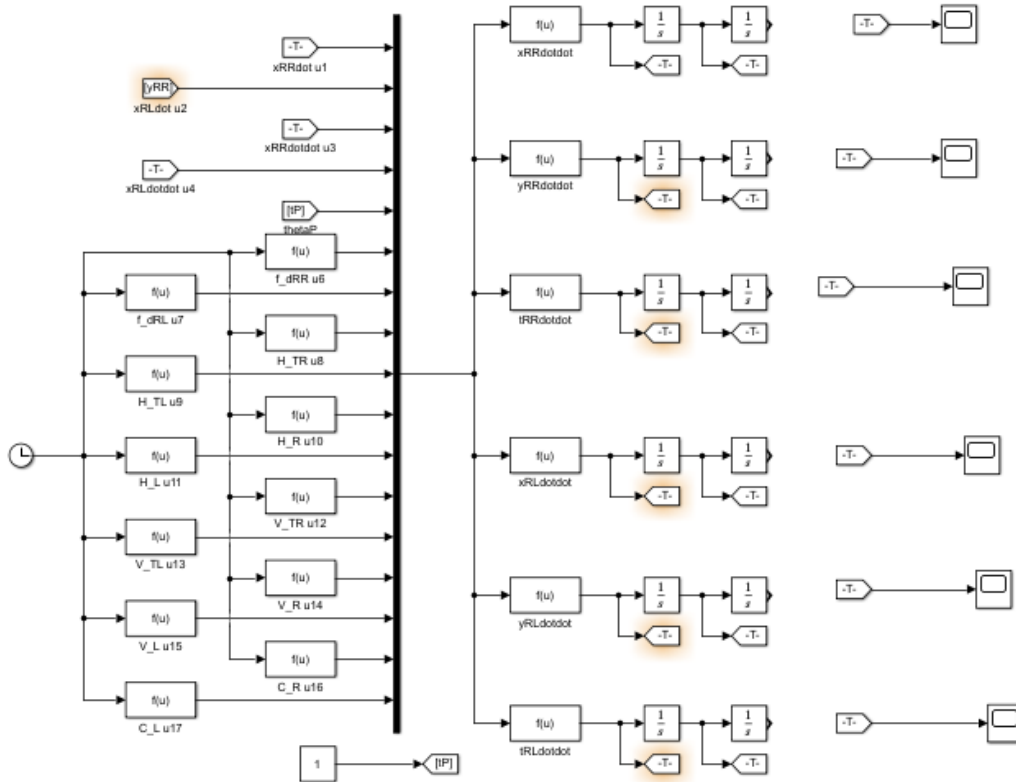


Figure 3. Non-linear model of the TWSBR.

The non-linear model of the TWSBR is linearized with initial conditions and its structure is shown in Figure 4. The initial conditions of  $x$  and  $\theta$  and their derivatives are set to 0.1.

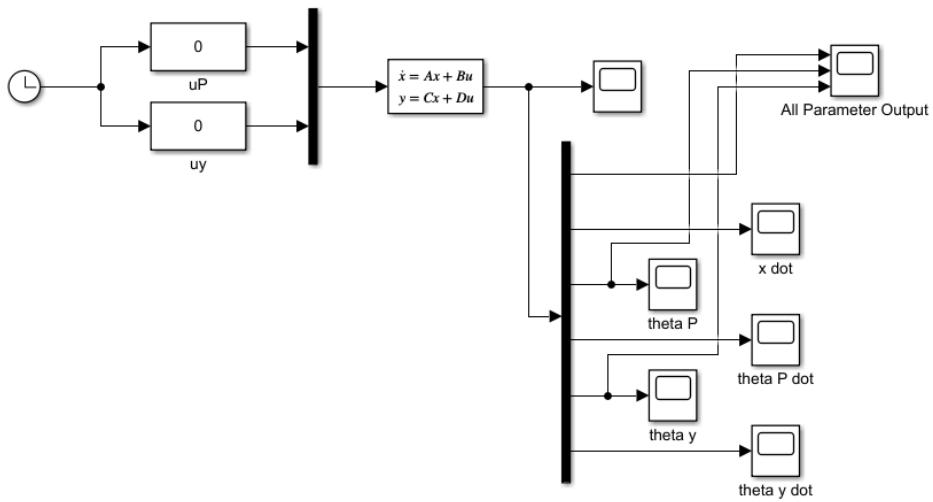


Figure 4. Linear model of the TWSBR

#### 4.1 Design of PID controller

PID controller is developed for controlling positions, yaw and pitch angles of the TWSBR. Here, we have set the desired values, henceforth only the angles can be set because the system is underactuated and three parameters (TWSBR position, vertical angle and horizontal angle) cannot be controlled with just two actuators  $u_P$  and  $u_y$ . The following Figure5 shows PID controller structure for linearized model of the system.

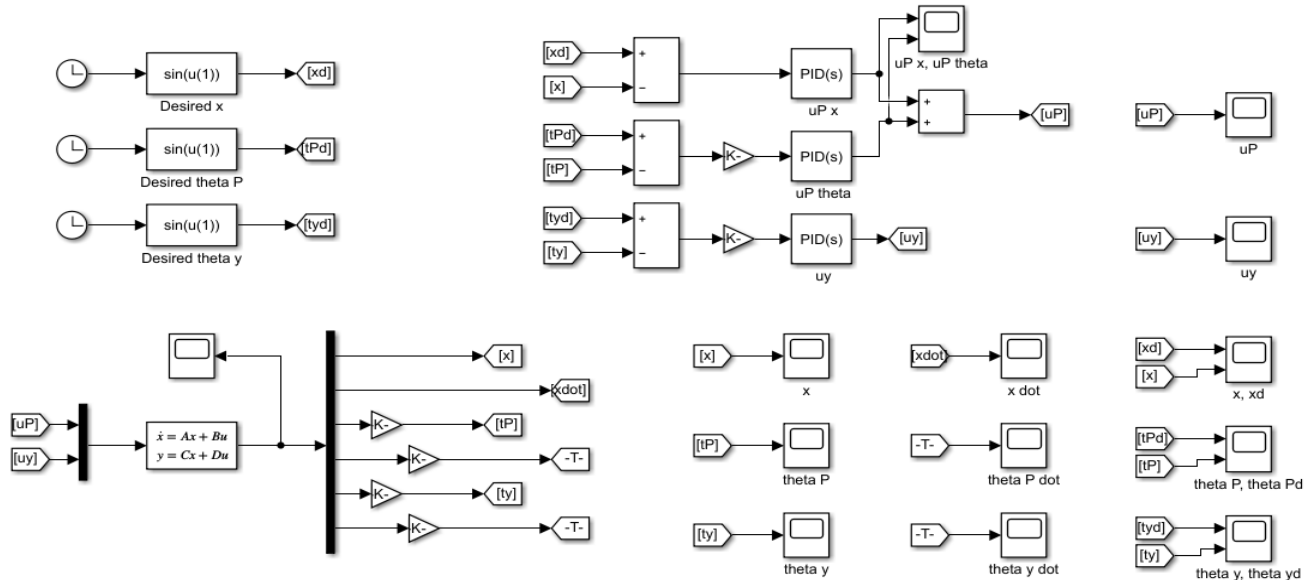


Figure 5. PID controller of the TWSBR

#### 4.2 Design of optimal PID controller to solve underactuated problem

To solve the problem of underactuated system, another PID controller designed based on dividing the TWSBR motion into two subsystems, namely, horizontal motion and vertical angle rotation. In this case, the main purpose is to follow the desired the TWSBR position while moving, if the vertical TWSBR angle differs from the desired angle over 20 degrees, the purpose will change to follow the vertical angle feedback signals. The simplified model of the PID controller with feedback is shown in Figure 7 & Figure 6 which gives switching rule of the controller.

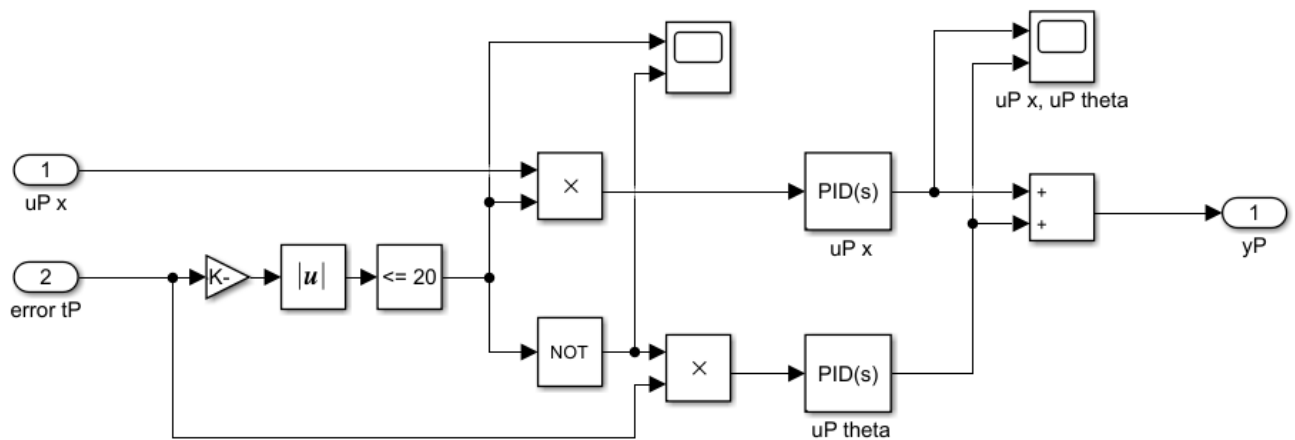


Figure 6. Switching rule of the PID controller.



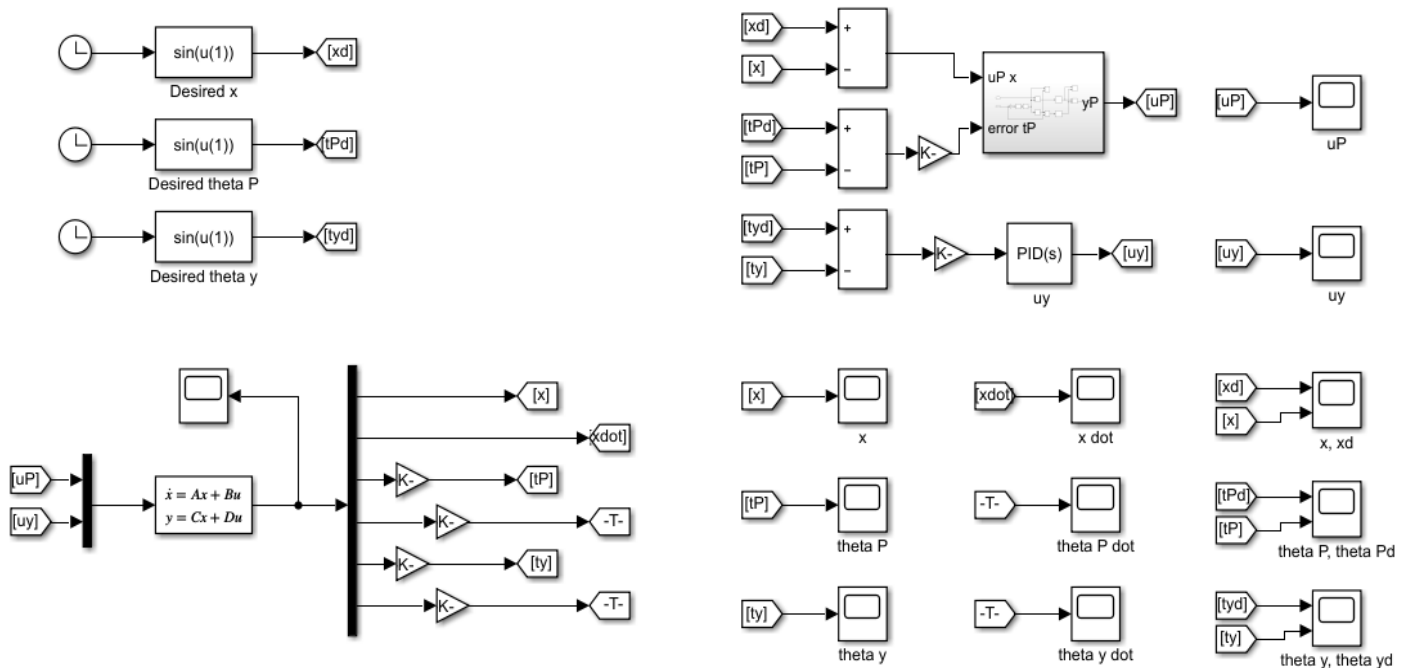


Figure 7. Optimal PID controller.

### 4.3 Design of LQR controller

In addition to PID controller, LQR controller is also designed to control the position and angle of the system. LQR controller is designed to control two outputs  $x$ ,  $y$  position and yaw angles. This controller operates on the basis of a predefined function and unforeseen factors such as unknown disturbances and unmodulated system parameters affect its performance. LQR controller can be defined based on different criteria such as time, energy, error, and etc. Also, LQR controller can satisfy several constraints such as time limit and interval of variables. The simulation results show that the displacement is eventually returning to zero angle and cart moved to new desired position. Figure 8 shows the LQR controller structure of the TWSBR.

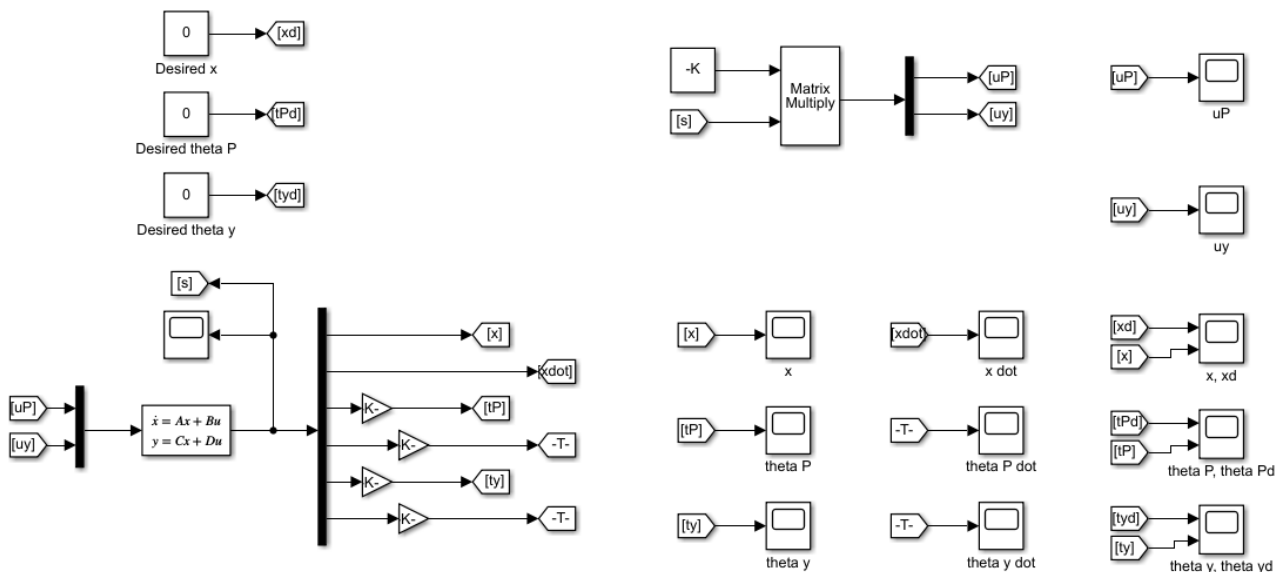


Figure 8. Optimal LQR controller.

## 5. Controller Design of System Model

The controllers is designed to compensate one shape of signals for six disturbances to observe the model performance. The disturbance signal is shown in Figure 9.

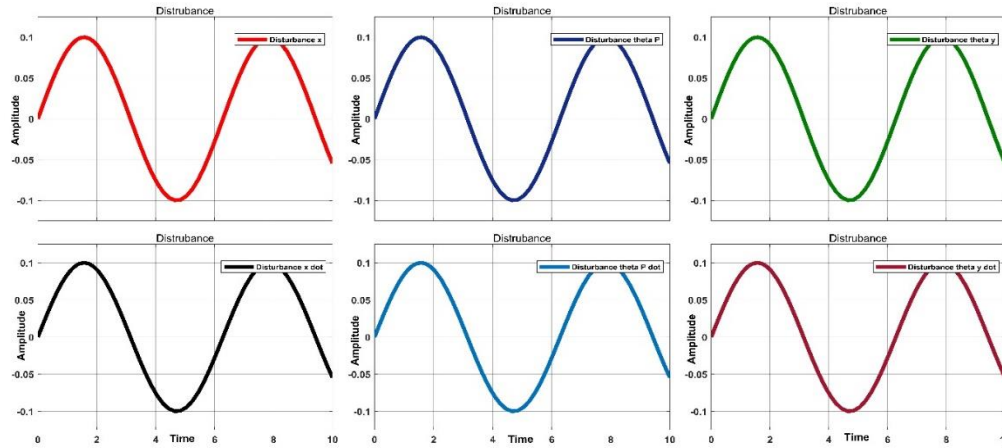


Figure 9. Disturbance signals

Figure 10 shows the PID controller adopted along with disturbance input to validate the system performance.

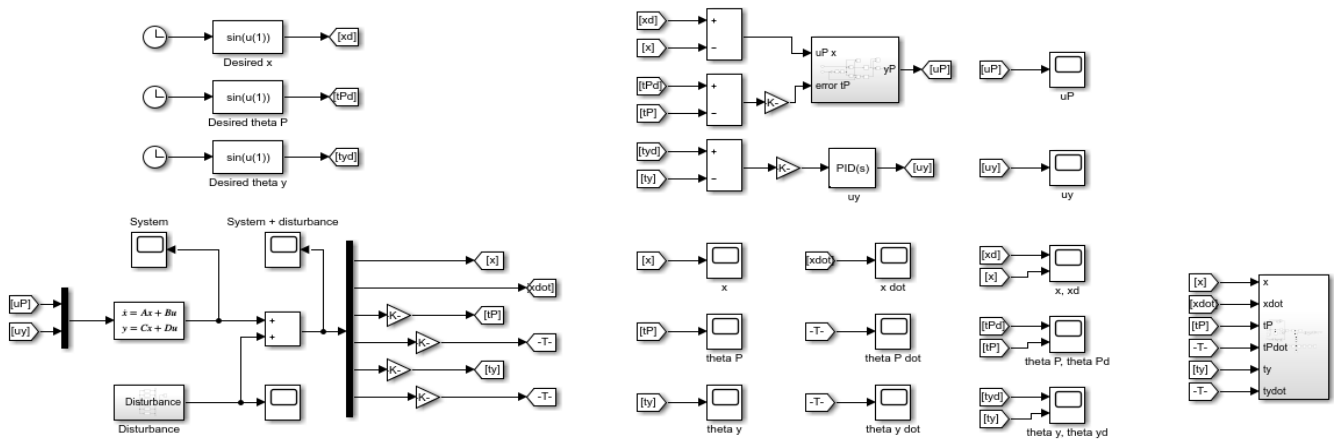


Figure 10. PID controller along with disturbance input.

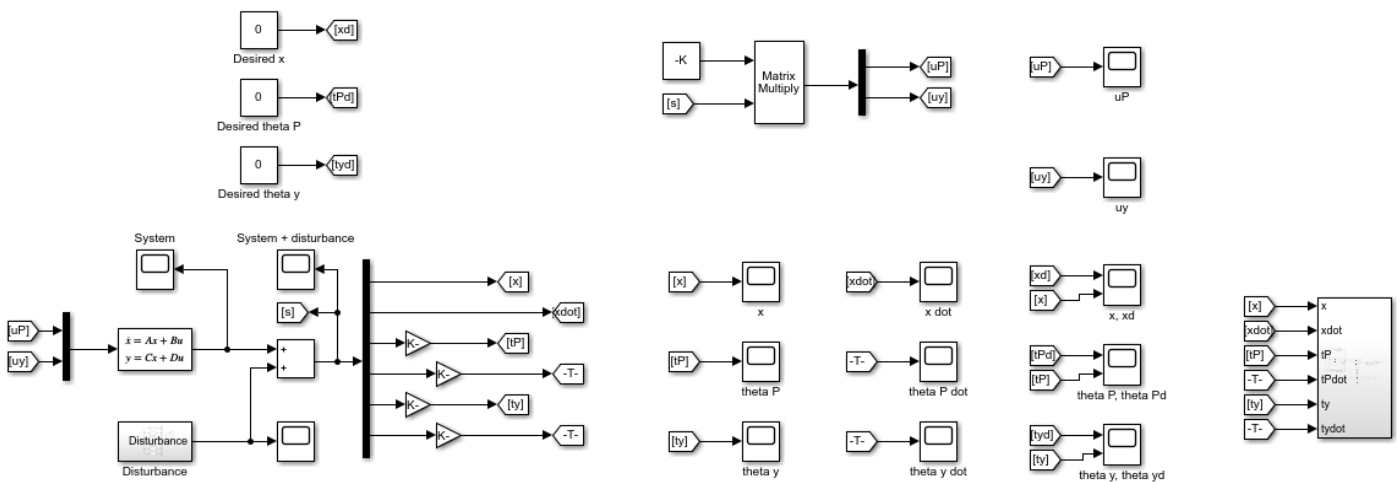


Figure 11 shows the LQR Controller of the TWSBR system.

Figure 11. LQR controller along with disturbance input.

### 5.1 Artificial Neural Network

Artificial neural networks or ANN is a computing algorithm inspired by the biological neural networks that constitute brains. An ANN is based on a collection of connected nodes called artificial neurons. Each connection, like the synapses in a brain, can transmit a signal to other neurons. An artificial neuron receives a signal and processes it then ANN can decide to activate the corresponding neurons or not. Excitement of these neurons will decide the output of the model. Its inference system corresponds to a set of fuzzy IF–THEN rules that have learning capability to approximate nonlinear functions. Figure 12 shows the ANN controller structure of the TWSBR.

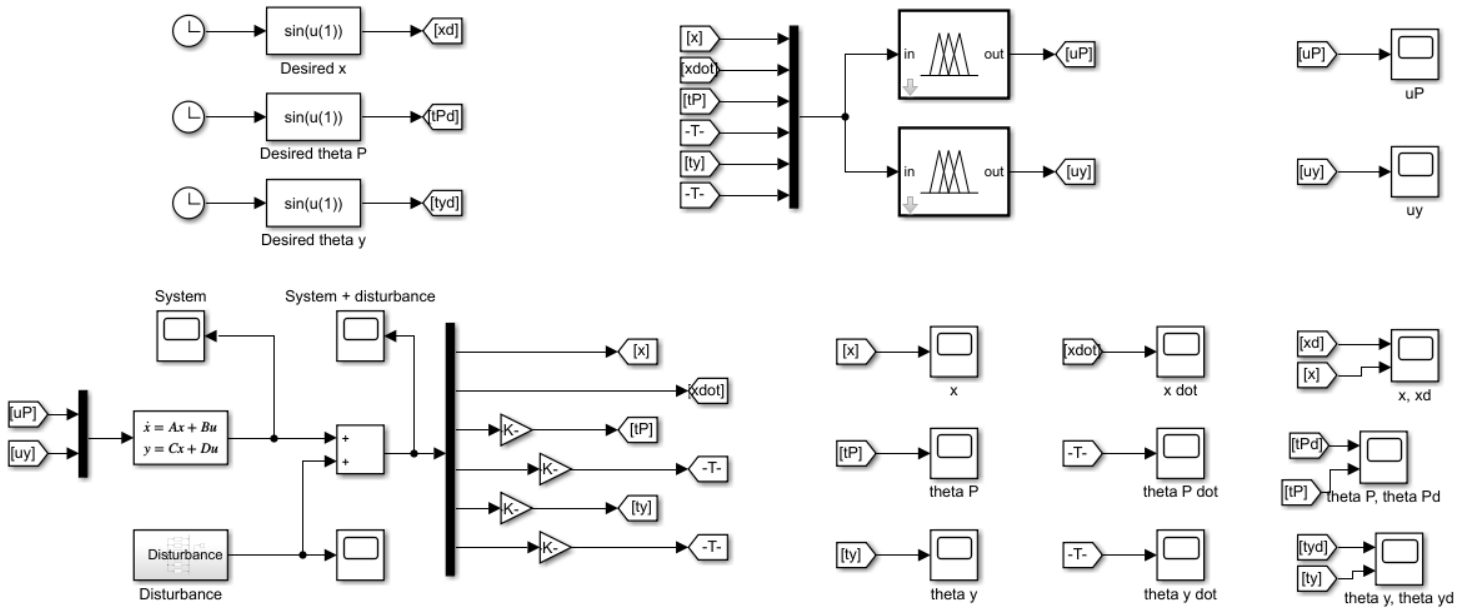


Figure 12. ANN controller structure along with disturbance input

For controlling of the output signals Adaptive Neuro-Fuzzy Inference System or ANFIS based on Sugeno representation strategy has been used. Sugeno ANFIS system has 5 different layers. First layer is used for calculating of membership function, second layer is used for generating the output for them by finding the product of membership function. Third layer is used for normalization. Forth layer is Consequent Layer and last layer is used for Aggregations. The last layer is responsible for firing signals of the output [43]. Proposed Sugeno ANFIS is used for calculating of membership function in order to predict control signals with better precision.

### 6. Results & Discussion

In this section result of propose model is discussed. This result has been achieved with testing models based on disturbance input signals. Table 2 shows the initial values of the system variable.

Table 2. Initial values of the system variable

Parameter	Values	Parameter	Values	Parameter	Values
$M_P$	10	$M_{RR}$	2	R	0.3
$J_{Pt}$	6	$J_{RR}$	3	G	9.81
$J_{Py}$	5	N	8	$R_a$	2
L	1	$k_t$	5	$k_e$	3
D	0.4				

Based on the adopted system model, the results of simulations of the non-linear model of the TWSBR are shown in figures 13a, 13b, 13c, 13d, 13e. The displacement and angle of the right and left wheels are also obtained.

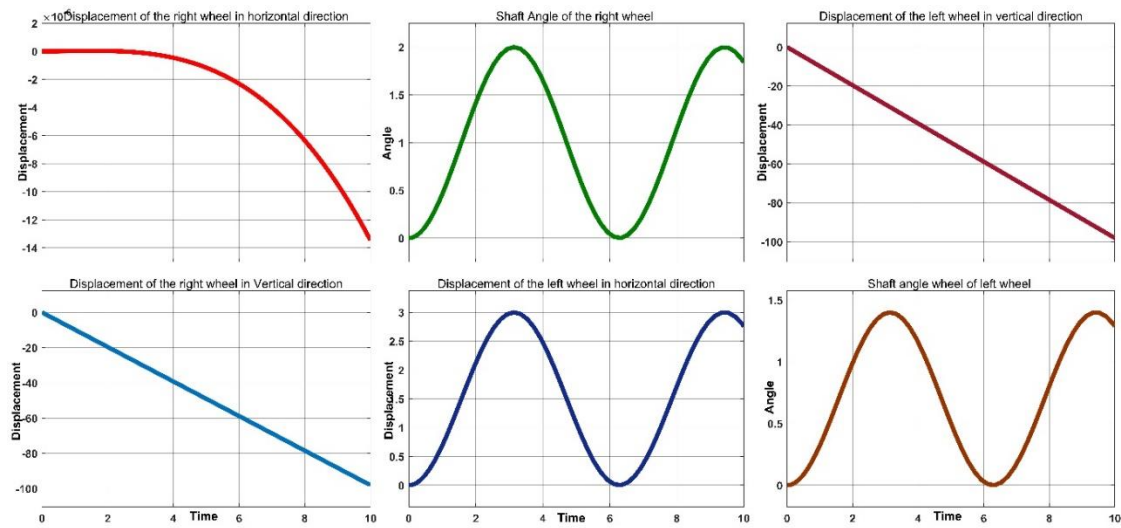


Figure 13. (a) Displacements of; (a) right wheel in vertical direction, (b) right wheel in horizontal direction, (c) right wheel in Angle of the right wheel, (d) left wheel in horizontal direction, (e) of the left wheel in vertical direction, (f) of the left wheel in angle of the left wheel.

After linearizing the system model, results are obtained as shown in figure 14a, 14b & 14c which plot position, tilt angle and yaw angle of the TWSBR.

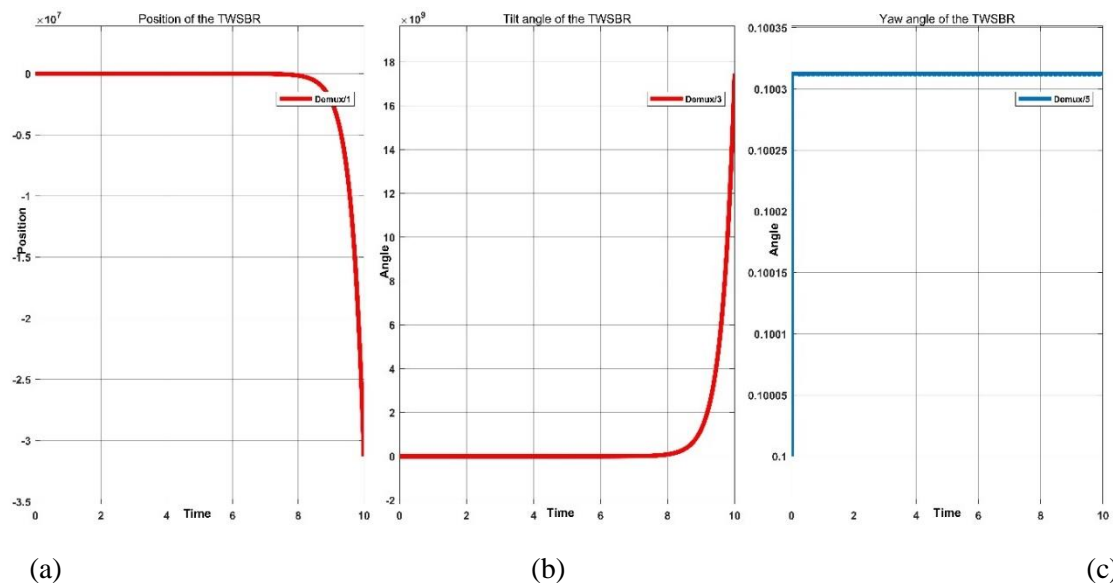


Figure 14. (a) Position of the TWSBR, (b) Tilt angle of the TWSBR, (c) Yaw angle of the TWSBR.

For controlling the position, vertical and horizontal angles of the TWSBR a PID controller is used. The result of controlling robot in mentioned directions has shown in figure 15a, 15b & 15c respectively.

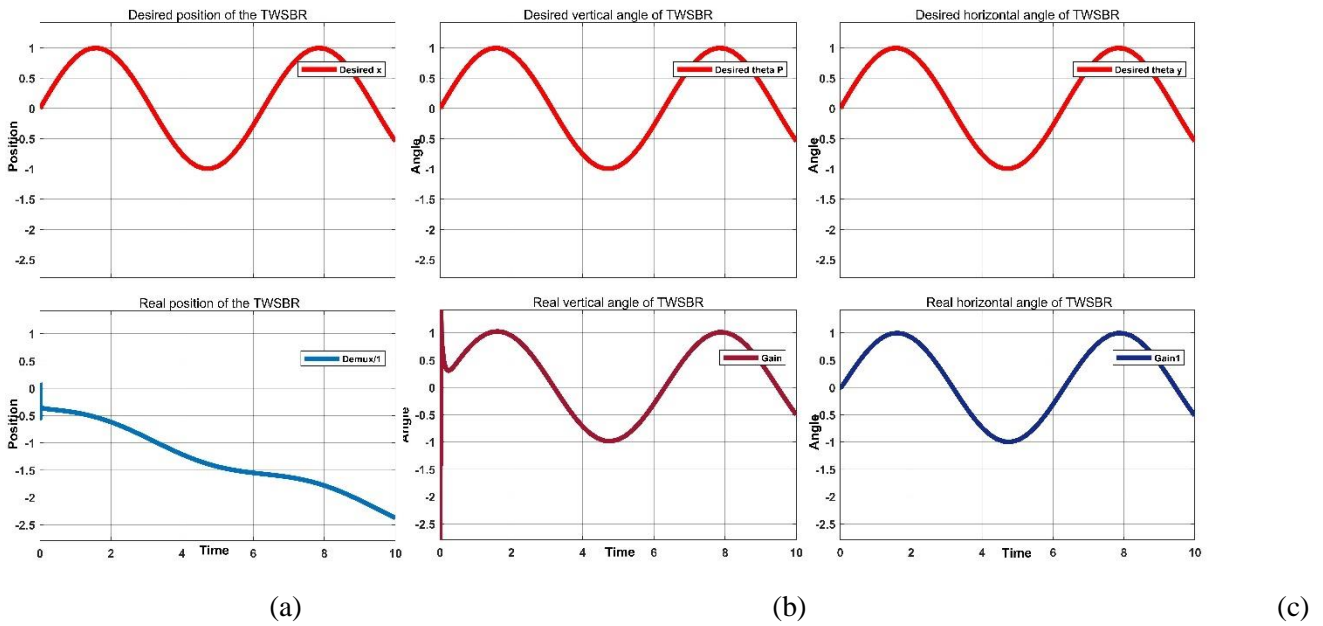


Figure 15. (a) Desired and real positions, (b) Desired and real vertical angles, (c) Desired and real horizontal angles.

As it has shown the on Figure 15 the desired angles on horizontal and vertical angle have been perused precisely but result on the desired position in what positions has been perused are different. This fact shows the priority of controlling the robot. The roll, yaw angles and position of the TWSBR are obtained for the underactuated system model with advanced PID controller and shown in figure 16a, 16b & 16c respectively.

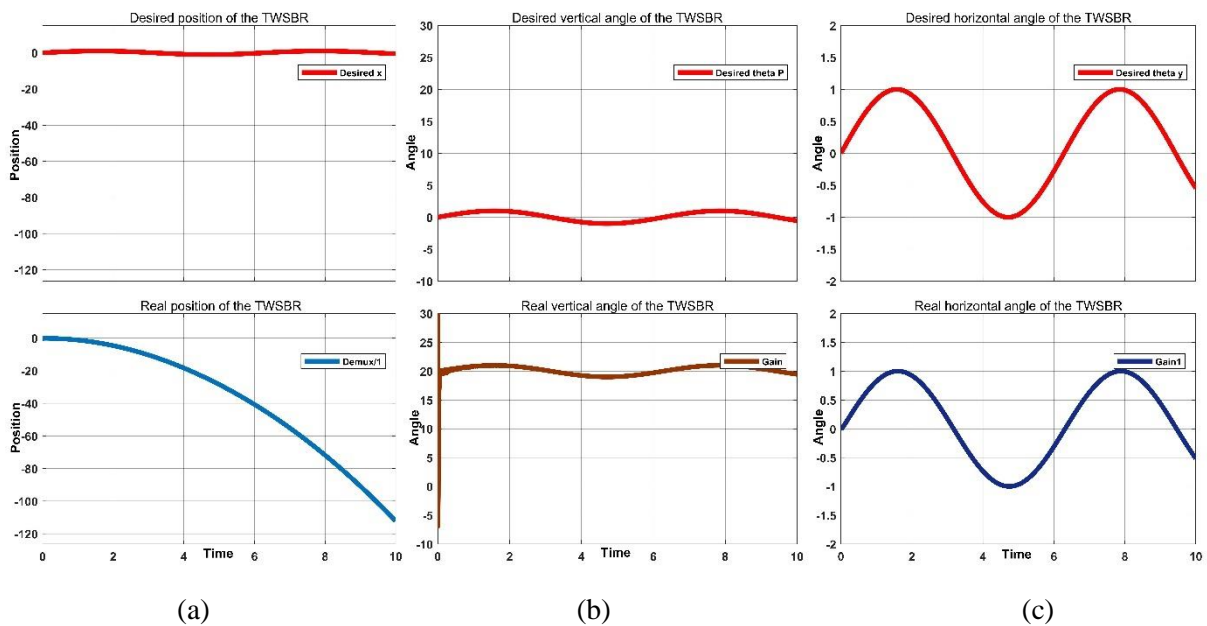


Figure 16. (a) Desired and real positions, (b) Desired and real vertical angles, (c) Desired and real horizontal angles.

On simulating system model of LQR controller the position, vertical position and horizontal position of TWSBR are plotted as shown in figure 17a, 17b & 17c respectively.

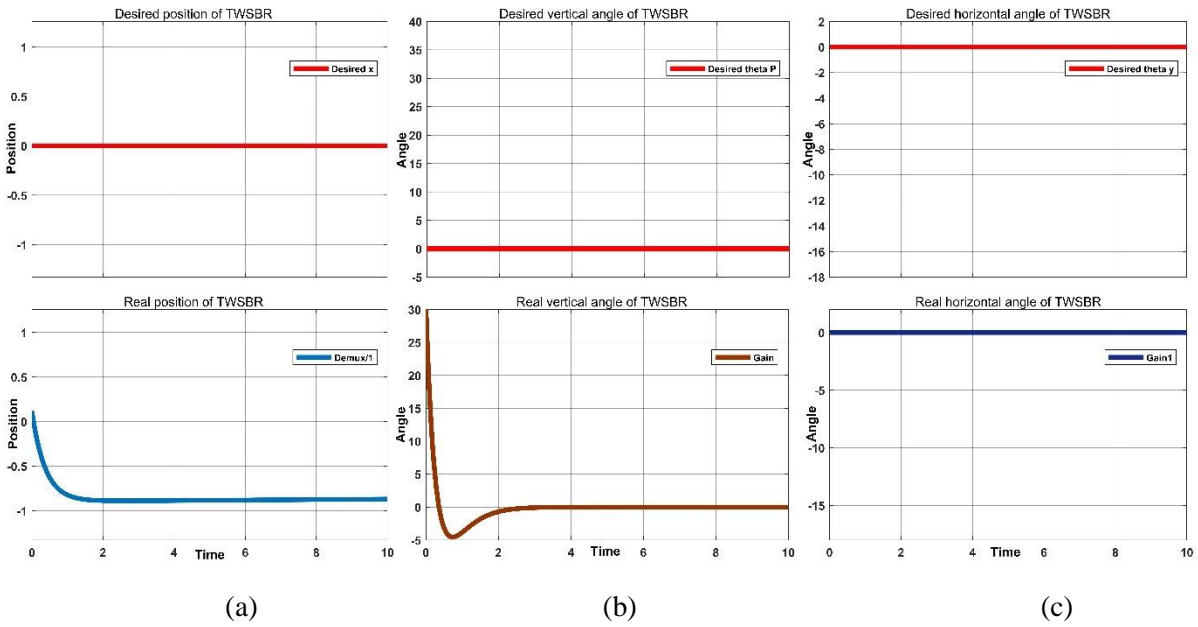


Figure 17. (a) Desired and real positions, (b) Desired and real vertical angles, (c) Desired and real horizontal angles.

The position, vertical position and horizontal position of the TWSBR are obtained for system model with PID controller along with disturbance input as shown in figure 18a, 18b & 18c respectively.

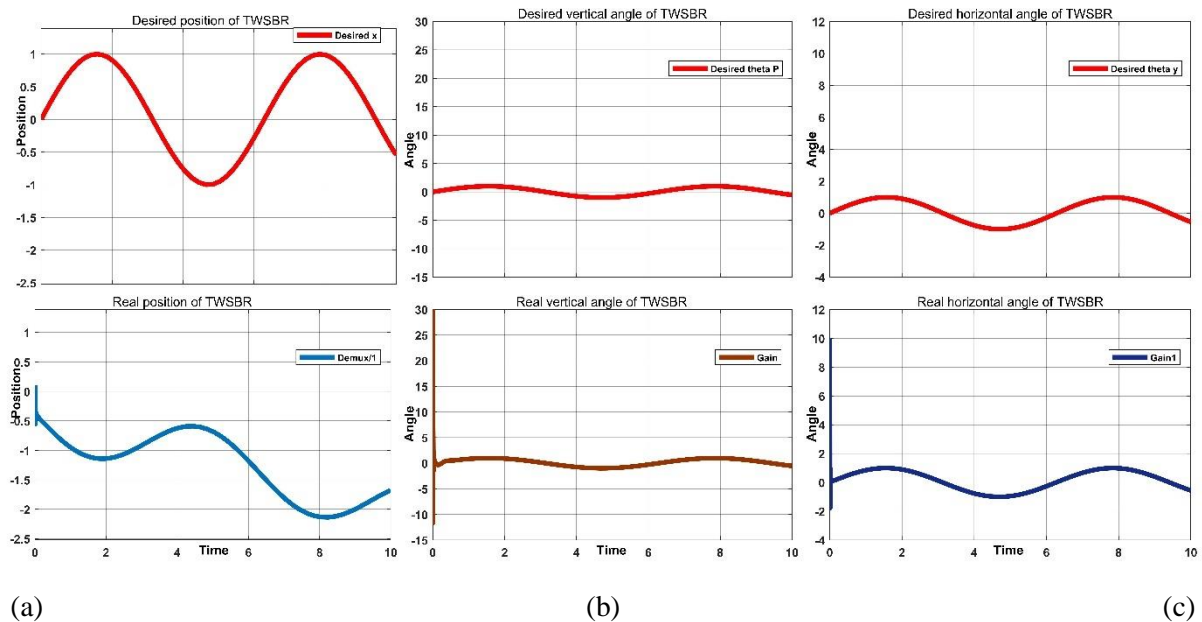


Figure 18. (a) Desired and real position, (b) Desired and real vertical angle, (c) Desired and real horizontal angle.

The position, vertical position and horizontal position of the TWSBR is obtained for system model with another optimal PID controller along with disturbance input as shown in figure 19a, 19b & 19c respectively.

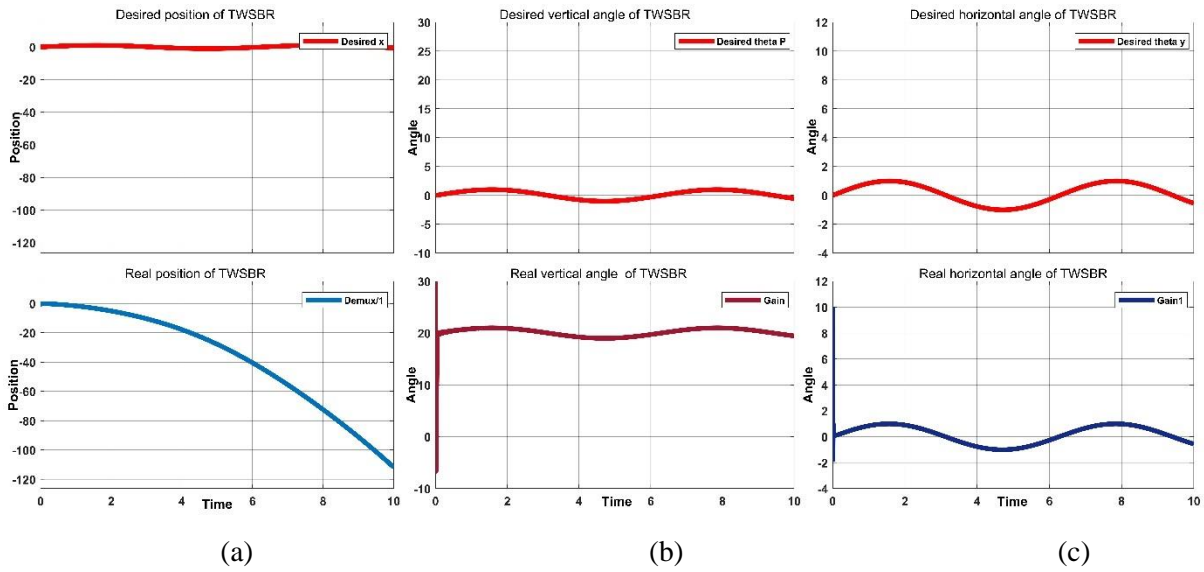


Figure 19. (a) Desired and real positions, (b) Desired and real vertical angles, (c) Desired and real horizontal angle.

The position, vertical position and horizontal position of the TWSBR is obtained for system model with LQR controller along with disturbance input as shown in figure 20a, 20b & 20c respectively.

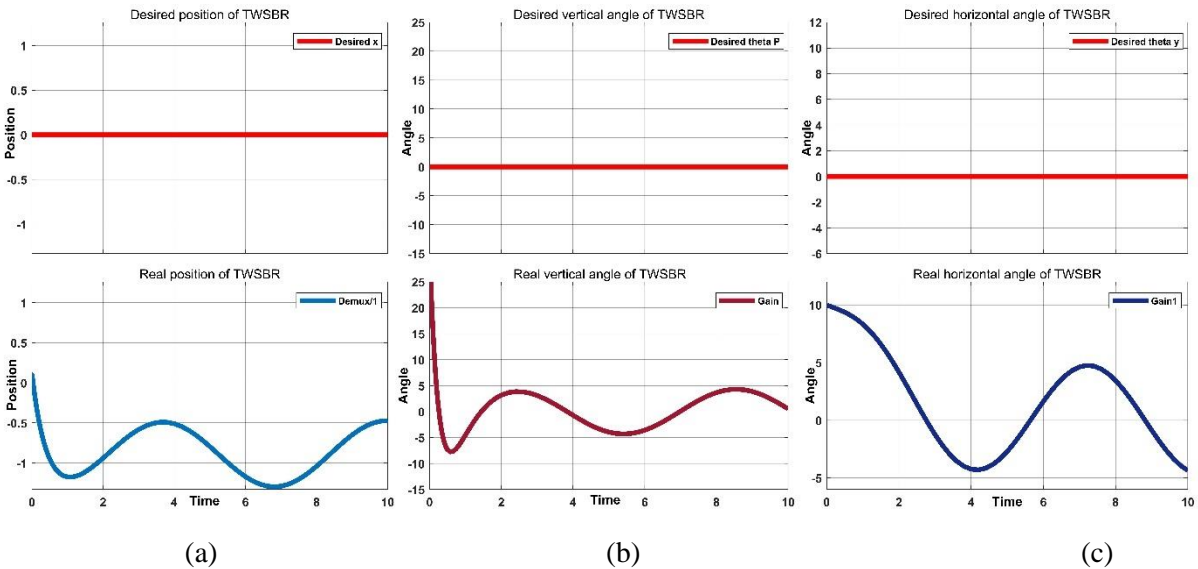


Figure 20. (a) Desired and real positions, (b) Desired and real vertical angles, (c) Desired and real horizontal angles.

One can see that the results are affected and change by disturbance, because LQR is actually a way to reduce a cost function before running the system and is asynchronously unlike other controllers like PID, determines the control inputs of the system and is not resistant to disturbances. The position, vertical position and horizontal position of the TWSBR are obtained for system model with ANN controller along with disturbance input as shown in figure 21a, 21b & 21c respectively.

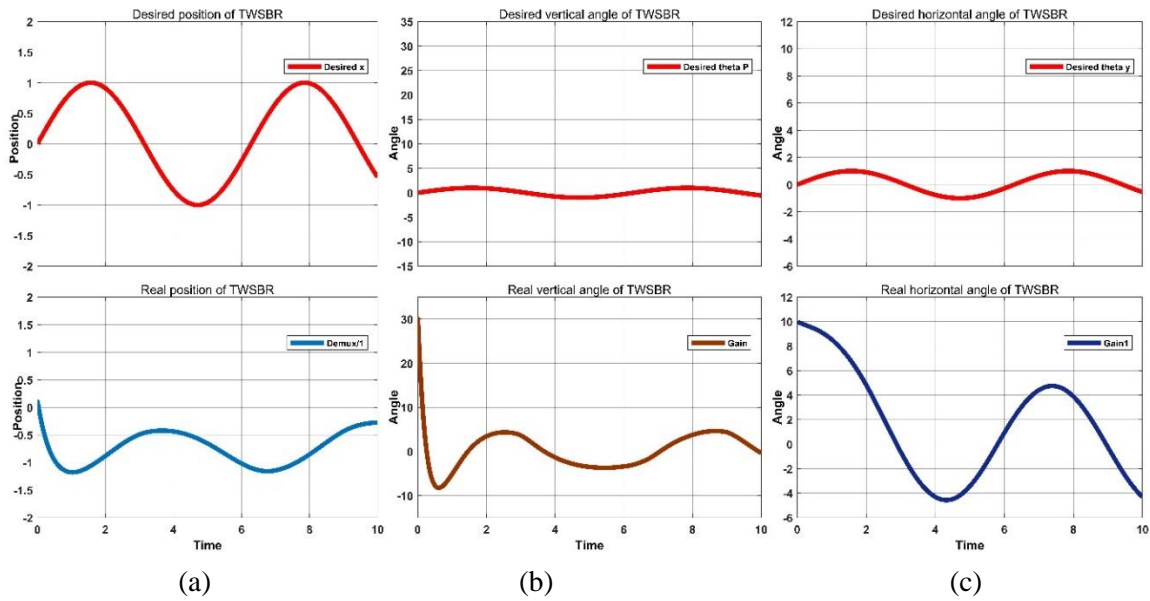


Figure 21. (a) Desired and real positions, (b) Desired and real vertical angles, (c) Desired and real horizontal angles.

The simulation time is 10 seconds and is enough for the system model to move and achieve the desired position and balance. The position, vertical and horizontal angle responses of the TWSBR with PID, optimal PID, LQR and ANN controller are simulated and results show satisfactory performance. To analyze the effectiveness of the controllers, the system is subjected to different disturbance signals, then the simulation results with controllers are obtained and studied. The conventional controllers PID performance is satisfactory for actuated systems. The artificial neural networks controller performance is more suited for balancing motion of the robot. Even LQR controller regains system balance but on account of disturbance, system loses its resistance and balance. Comparison between result on the Figure 21 (a), Figure 20 (a), Figure 17 (a) shows that for controlling position of the robots in the movement to the desired position use of ANFIS model is going to cause lower average loss. But result on the Figure 17 (b) and Figure 17 (c) are showing that with use of advanced PID model vertical and horizontal angles can get controlled better. But in total based on Figure 21 (a), Figure 21 (b), and Figure 21 (c) response time of ANFIS model is higher than LQR and Advanced PID. Also Based on result on the Figure 21 (a) best robust signals for controlling the position of motion comes from using ANN controller. Achieved result is showing fuzzy nature of the proposed model for controlling position is reached to better result compare to ordinal PID, and LQR.

## 7. Conclusion

This paper presented a robust optimal control method for a TWSBR using PID, LQR & ANN controllers. The mathematical modelling of the TWSBR has been well established in state space form. Then the system is decomposed into subsystems to achieve self-balancing and desired motion. Through simulations results, the proposed controllers have shown significant controlling actions with disturbance input added to the system model. PID, LQR and ANN controllers presented desired results for normal and disturbance model. Based on testing presented models for control with input and disturbance signals, for controlling position of the robots ANFIS (ANN) model is better tuned and its response time is higher than other presented models. Presented structure is valid for controlling position of the robots, but for controlling horizontal and vertical angles of the robots it is better to use advanced PID controller. An important topic for further research is to develop a new controller method for a TWSBR to running at different speeds with fluctuating loads.



## References

- [1] Y. Ha and S. Yuta, "Trajectory tracking control for navigation of the inverse pendulum type selfcontained mobile robot," *Robotics and Autonomous Systems*, vol. 17, pp. 65-80, 1996.
- [2] F. Grasser, A. D'Arrigo, S. Colombi, and A. C. Rufer, "Joe: a mobile, inverted pendulum," *IEEE Trans. on Industrial Electronics*, vol. 49, no. 1, pp. 107-114, February 2002.
- [3] Y. Kim, S. Kim, and Y. Kwak, "Dynamic analysis of a nonholonomic two-wheeled inverted pendulum robot," *Journal of Intelligent and Robotic Systems*, vol. 44, no. 1, pp. 25-46, 2005.
- [4] T. Takei, R. Imamura, and S. Yuta, "Baggage transportation and navigation by a wheeled inverted pendulum mobile robot," *IEEE Trans. on Industrial Electronics*, vol. 56, no. 10, pp. 3985-3994, 2009.
- [5] S. C. Lin, P. S. Tsai, and H. C. Huang, "Adaptive robust self-balancing and steering of a two-wheeled human transportation vehicle," *Journal of Intelligent and Robotic Systems*, vol. 62, no. 1, pp. 103- 123, April 2011.
- [6] P. Petrov and M. Parent, "Dynamic modeling and adaptive motion control of a two-wheeled selfbalancing vehicle for personal transport," *Proc. Of 13th Int. IEEE Conf. on Intelligent Transportation Systems*, pp. 1013-1018, 2010.
- [7] H. Azizan, M. Jafarinasab, S. Behbahani, and M. Danesh, "Fuzzy control based on LMI approach and fuzzy interpretation of the rider input for two wheeled balancing human transporter," *Proc. of 8th IEEE Int. Conf. on Control and Automation*, pp. 192-197, 2010.
- [8] M. Baloh and M. Parent, "Modeling and model verification of an intelligent self-balancing twowheeled vehicle for an autonomous urban transportation system," *Proc. of the Conf. on Computational Intelligence, Robotics, and Autonomous Systems*, pp. 1-7, 2003.
- [9] General Motors, [Online video]. Available: [http:// en.wikipedia.org/wiki/General\\_Motors\\_EN-V](http://en.wikipedia.org/wiki/General_Motors_EN-V), <http://youtu.be/zoKxx0GEEFE>
- [10] H. Ustal and J. L. Minkel, "Study of the independence IBOT 3000 mobility system: an innovative power mobility device, during use in community environments," *Archives of Physical Medicine and Rehabilitation*, vol. 85, no. 12, pp. 2002-2010, December 2004.
- [11] Genny Mobility, [Online video]. Available: <http://www.gennymobility.com/Genny/Concept.aspx>, <http://youtu.be/7DfcjRcoef0>
- [12] S. Miao and Q. Cao, "Modeling of self-tilt-up motion for a two-wheeled inverted pendulum," *Industrial Robot: An International Journal*, vol. 38, no. 1, pp. 76-85, January 2011.
- [13] S. Jeong and T. Takayuki, "Wheeled inverted pendulum type assistant robot: design concept and mobile control," *Intelligent Service Robotics*, pp. 313- 320, 2008.
- [14] Z. Li and J. Luo, "Adaptive robust dynamic balance and motion controls of mobile wheeled inverted pendulums," *IEEE Trans. on Control Systems Technology*, vol. 17, no. 1, pp. 233-241, January 2009.
- [15] C. H. Huang, W. J. Wang, and C. H. Chiu, "Design and implementation of fuzzy control on a twowheel inverted pendulum," *IEEE Trans. on Industrial Electronics*, vol. 58, no. 7, pp. 2988-3001, July 2011.
- [16] J. Huang, Z.-H. Guan, T. Matsuno, T. Fukuda, and K. Sekiyama, "Sliding mode velocity control of mobile-wheeled inverted-pendulum systems," *IEEE Trans. on Robotics*, vol. 26, no. 4, pp. 750-758, August 2010.
- [17] A. Salerno and J. Angeles, "A new family of two wheeled mobile robot: modeling and controllability," *IEEE Trans. on Robotics*, vol. 23, no. 1, pp. 169-173, February 2007.
- [18] K. Pathak and S. Agrawal, "Band-limited trajectory planning and tracking for certain dynamically stabilized mobile systems," *Journal of Dynamic Systems, Measurement, and Control*, vol. 128, no. 1, pp. 104-111, 2006.
- [19] K. Teeyapan, J. Wang, T. Kunz, and M. Stilman. "Robot limbo: optimized planning and control for dynamically stable robots under vertical obstacles," *IEEE Int. Conf. on Robotics and Automation*, pp. 4519-4524, 2010.

- [20] D. Choi and J. Oh, "Human-friendly motion control of a wheeled inverted pendulum by reduced-order disturbance observer," Proc. of IEEE Int. Conf. on Robotics and Automation, pp. 2521-2526, May 2008.
- [21] K. Pathak, J. Franch, and S. Agrawal, "Velocity and position control of a wheeled inverted pendulum by partial feedback linearization," IEEE Trans. on Robotics, vol. 21, no. 3, pp. 505-513, June 2005.
- [22] M. C. Tsai and J. S. Hu, "Pilot control of an auto balancing two wheeled cart," Advanced Robotics, vol. 21, no. 7, pp. 817-827, 2007.
- [23] T. R. Kane and D. A. Levinson, Dynamics: Theory and Applications, McGraw-Hill Book Company, 1985.
- [24] Long Chen , Hai Wang , Yunzhi Huang , Zhaowu Ping , Ming Yu , Xuefeng Zheng , Mao Ye , Youhao Hu" Robust hierarchical sliding mode control of a two-wheeled self-balancing vehicle using perturbation estimation", 2019 Mechanical Systems and Signal Processing Volume 139, May 2020, 106584, Elsevier.
- [25] Ines Jmel, Habib Dimassi, Salim Hadj-Said, Faouzi Msahli , "An adaptive observer for two wheeled self-balancing robot with a varying center of mass", 2019 19th international conference on Sciences and Techniques of Automatic control & computer engineering (STA), Sousse, Tunisia, March 24-26, 2019.
- [26] Vincent Y. Philippart, Kristian O. Snel, Antoine M. de Waal, Jeedella S.Y. Jeedella, Esmaeil Najafi, "Model-based Design for a Self-balancing Robot using the Arduino Micro-controller Board", 2019 IEEE.
- [27] Ji-Hyun Park and Baek-Kyu Cho, "Development of a self-balancing robot with a control moment gyroscope", 2018 International Journal of Advanced Robotic Systems.
- [28] Shyamala Sarathy, Mariyam Hibah M M, Anusooya S, S.Kalaivani "Implementation of Efficient Self Balancing Robot" 2018 IEEE.
- [29] Md. Iman Ali and Md. Modasser Hossen "A Two-Wheeled Self-Balancing Robot with Dynamics Model" Proceedings of the 2017 4th International Conference on Advances in Electrical Engineering (ICAEE), 28-30 September, Dhaka, Bangladesh.
- [30] Magdi S. Mahmoud and Mohammad T. Nasir "Robust Control Design of Wheeled Inverted Pendulum Assistant Robot" IEEE/CAA JOURNAL OF AUTOMATICA SINICA, VOL. 4, NO. 4, OCTOBER 2017.
- [31] P. FRANKOVSKÝ, L. DOMINIK, A. GMITERKO, I. VIRGALA P. KURYLO O. PERMINOVA, "Modeling of Two-Wheeled Self-Balancing Robot Driven by DC Gearmotors" Int. J. of Applied Mechanics and Engineering, 2017, vol.22, No.3, pp.739-747.
- [32] Keerthi Prakash, Koshy Thomas "Study of Controllers for a Two Wheeled Self-balancing Robot" 2016 International Conference on Next Generation Intelligent Systems (ICNGIS).
- [33] WANG Xin, CHEN Songlin\*, CHEN Ting and YANG Baoqing "Study on control design of a two-wheeled self-balancing robot based on ADRC" Proceedings of the 35th Chinese Control Conference July 27-29, 2016, Chengdu, China.
- [34] Congying Qiu and Yibin Huang "The Design of Fuzzy Adaptive PID Controller of Two-Wheeled Self-Balancing Robot" International Journal of Information and Electronics Engineering, Vol. 5, No. 3, May 2015.
- [35] Fengxin Sun , Zhen Yu, Haijiao Yang "A Design for Two-Wheeled Self-Balancing Robot Based on Kalman Filter and LQR" 2014 International Conference on Mechatronics and Control (ICMC) July 3 - 5, 2014, Jinzhou, China.
- [36] Liangliang Cui, Yongsheng Ou, Junbo Xin, Dawei Dai, Xiang Gao "Control of a Two-Wheeled Self-Balancing Robot with Support Vector Regression Method" 2014 IEEE.
- [37] Osama Jamil, Mohsin Jamil, Yasar Ayaz, Khubab Ahmad, "Modeling, Control of a Two-Wheeled Self-Balancing Robot" 2014 International Conference on Robotics and Emerging Allied Technologies in Engineering (iCREATE) Islamabad, Pakistan, April 22-24, 2014.
- [38] Hau-Shiue Juang and Kai-Yew Lum "Design and Control of a Two-Wheel Self-Balancing Robot using the Arduino Microcontroller Board", 2013 10th IEEE International Conference on Control and Automation (ICCA) Hangzhou, China, June 12-14, 2013.

- [39] Wei An<sup>1</sup> and Yangmin Li<sup>1,2\*</sup>, Senior Member, IEEE, "Simulation and Control of a Two-wheeled Self-balancing Robot", Proceeding of the IEEE International Conference on Robotics and Biomimetics (ROBIO) Shenzhen, China, December 2013.
- [40] Junfeng Wu, Yuxin Liang, Zhe Wang, "A Robust Control Method of Two-Wheeled Self-Balancing Robot" 2011 The 6th International Forum on Strategic Technology.
- [41] Tao Feng, Tao Liu, Xu Wang, , Zhao Xu, Meng Zhang, Sheng-chao Han, "Modeling and Implementation of Two-wheel Selfbalancing Robot Equipped With Supporting Arms" 2011 IEEE.
- [42] Samer Miasa, Mohammad Al-Mjali, Anas Al-Haj Ibrahim, and Tarek Tutunji "Fuzzy control of a two-wheel balancing robot using DSPIC" 2010 IEEE.
- [43]. Cortés-Antonio, Prometeo, et al. "Learning rules for Sugeno ANFIS with parametric conjunction operations." *Applied Soft Computing* 89 (2020): 106095.
- [44]. Nguyen, Duc-Minh, Nguyen Van-Tiem, and Trong-Thang Nguyen. "A neural network combined with sliding mode controller for the two-wheel self-balancing robot." *IAES International Journal of Artificial Intelligence* 10.3 (2021): 592.
- [45]. Raudys, Aistis, and Aušra Šubonienė. "A Review of Self-balancing Robot Reinforcement Learning Algorithms." *International Conference on Information and Software Technologies*. Springer, Cham, 2020.
- [46]. Zhao, Tao, et al. "Non-singleton general type-2 fuzzy control for a two-wheeled self-balancing robot." *International Journal of Fuzzy Systems* 21.6 (2019): 1724-1737.