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γ_h- graphs of graph

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Abstract **-** Consider the family of all γ-sets of a graph G and define the γ-graph $G(\gamma) = (V(\gamma), E(\gamma))$ of G to be the graph whose vertices $V(\gamma)$ correspond one to one with the γ - sets of G and two γ - sets, say S_1 and S_2, form an edge in $E(\gamma)$, if there exists a vertex $v \in S_1$ and a vertex $w \in S_2$ such that (i) v is adjacent to w and (ii) $S_1 = S_2 - \{w\} \cup \{v\}$and $S_1 = S_2 - \{v\} \cup \{w\}$. (i.e) two γ - sets are said to be adjacent if they differ by one vertex and two vertices defining this difference are adjacent in G. Plenty of work has been carried out in this topic. This paper falls in the same line taking a little diversion. The concept of γ_h graph is introduced as follows, Consider the family of all γ_h - sets of a graph G and define the γ_h-graph $G(\gamma_h) = (V(\gamma_h), E(\gamma_h))$ of G to be the graph whose vertices $V(G(\gamma_h))$ correspond one to one with the γ_h-sets of G and two γ_h - sets, say S_1 and S_2, form an edge in $E(G(\gamma_h))$, if there exists a vertex $v \epsilon S_1$ and a vertex $w \epsilon S_2$ such that (i) v is at a distance of 2 to w and (ii) $S_1 = S_2 - \{w\} \cup \{v\}$and $S_2 = S_1 - \{v\} \cup \{w\}$. (i.e) two γ_h- sets are adjacent in $G(\gamma_h)$ if they differ by one vertex and two vertices defining this difference are at a distance 2 in G. γ_h graphs of basic families such as paths, cycles, and wheels have been identified.

Keywords: dominating sets, γ-graphs, Hop dominating set.

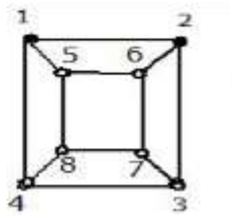
INTRODUCTION

Constructing new families of graphs from a given family of graphs based on some graph parameters or concepts has been an attractive research line in Graph Theory. γ-graphs is one such family introduced by GRED H . FRICKE in 2011. Motivated by this idea, taking a little diversion, yet another family of graphs is introduced and dealt with in this paper. The necessary concepts and results due to our pioneers are stated below.

PRELIMINARIES

Definition 2.1. [3] The hop graph H(G) of a graph G is the graph obtained from G by taking $V(H(G)) = V(G)$ and joining two vertices u, v in H(G) if and only if they are at a distance 2 in G.

Example 2.2.



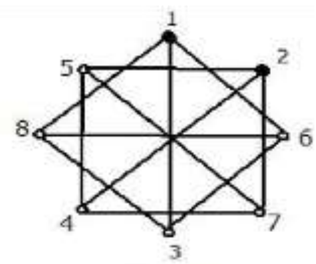


Figure 1: Q_3 **Figure 2 : $H(Q_3)$**

Theorem 2.3. [3] $\gamma_h(G) = \gamma(H(G))$.

Theorem 2.4[3]
$$H(P_n) = \begin{cases} P_{\frac{n}{2}} \cup P_{\frac{n}{2}} & if\ n\ is\ even \\ P_{\lceil \frac{n}{2} \rceil} \cup P_{\lceil \frac{n}{2} \rceil} & if\ n\ is\ odd \end{cases}$$

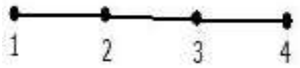
Theorem 2.5. [3]
$$H(C_n) = \begin{cases} C_{\frac{n}{2}} \cup C_{\frac{n}{2}} & if\ n\ is\ even \\ C_n & if\ n\ is\ odd \end{cases}$$

Theorem 2.6. [3] $H(W_n) = K_1 \cup G_n$, where G_n is a n-2 regular graph on n vertices.

Definition 2.7. [2] **γ- Graphs**

Consider the family of all γ-sets of a graph G and define the γ-graph $G(\gamma) = (V(\gamma), E(\gamma))$ of G to be the graph whose vertices $V(\gamma)$ correspond one to one with the γ-sets of G and two γ-sets, say S_1 and S_2, form an edge in $E(\gamma)$, if there exists a vertex $v \in S_1$ and a vertex $w \in S_2$ such that (i) v is adjacent to w and (ii) $S_1 = S_2 - \{w\} \cup \{v\}$ and $S_1 = S_2 - \{v\} \cup \{w\}$. With this definition, two γ– sets are said to be adjacent if they differ by one vertex and two vertices defining this difference are adjacent in G.

Illustration



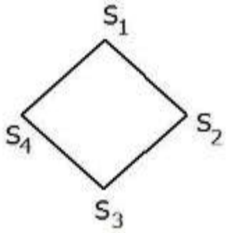


Figure 3: P_4 **Figure 4: $\gamma(P_4)$**

The γ-sets are $S_1 = \{1,3\}$, $S_2 = \{1,4\}$, $S_3 = \{2,4\}$,

$S_4 = \{2,3\}$.

Theorem 2.8. [2] $C_{3k+2}(\gamma) \square C_{3k+2}$.

Theorem 2.9. [2] For $k \geq 2, C_{3k}(\gamma) \square K_3$.

Theorem 2.10. [2] $P_{3k}(\gamma) \square K_1$.

Theorem 2.11. [2] $P_{3k+2}(\gamma) \square P_{k+2}$.

Theorem 2.12. [2] For $k \geq 2, P_2\square(P_{2k+1})(\gamma) \simeq \overline{K_2}$.

Theorem 2.13. [2] $P_{3k+1}(\gamma) \square SG(k + 1)$.

Theorem 2.14. [2] If $G \cup H$ denotes the disjoint union of two graphs G and H, then $(G \cup H)(\gamma) \simeq G(\gamma)\square H(\gamma)$

SWING GRIDS

GERD H.FRICKE et.al have concluded in [2] that the γ-graphs of cycles of orders 3k +1 are much more complex. It has been proved that the γ-graph $C_{3k+1}(\gamma)$ is connected and has some of the same structural properties as $P_{3k+1}(\gamma)$. In this section, we make an attempt to identify the γ-graphs of C_{3k+1}.

Definition 3.1.

Let $S_wG(n : 1)$ be the graph with $V(S_wG(n : 1)) =$

$$\{u_{ij}/1 \leq i \leq \lceil \tfrac{n+2}{2} \rceil, 1 \leq j \leq \lfloor \tfrac{3n+1}{2} \rfloor\} \cup \{v_{ij}/1 \leq i \leq \lceil \tfrac{n+2}{2} \rceil - 1, 1 \leq j \leq \lfloor \tfrac{3n+1}{2} \rfloor\}$$
$$E(S_wG(n:1)) = \{v_{i1}u_{i1}, v_{i1}u_{i+1,1}, /1 \leq i \leq \lceil \tfrac{n+2}{2} \rceil - 1\} \cup \{v_{ij}u_{i,j-1}, v_{ij}u_{ij}, v_{ij}u_{i+1,j-1}, v_{ij}u_{i+1,j}$$
$$1 \leq i \leq \lceil \tfrac{n+2}{2} \rceil - 1, 2 \leq j \leq \lfloor \tfrac{3n+1}{2} \rfloor\} \cup \{v_{i1}u_{i+1,\lfloor \frac{3n+1}{2} \rfloor}, v_{i1}u_{i+2,\lfloor \frac{3n+1}{2} \rfloor}/1 \leq i \leq \lceil \tfrac{n+2}{2} \rceil - 2\} \cup$$
$$\{v_{\lceil \frac{n+2}{2} \rceil - 1,1}u_{\lceil \frac{n+2}{2} \rceil,\lfloor \frac{3n+1}{2} \rfloor}, v_{\lceil \frac{n+2}{2} \rceil - 1,1}u_{1,\lfloor \frac{3n+1}{2} \rfloor}\}$$

Definition 3.2.

Let $S_wG(n : 2)$ be the graph with $V(S_wG(n : 2)) =$

$$\{u_{ij}/1 \leq i \leq \lfloor \tfrac{n}{2} \rfloor + 1, 1 \leq j \leq \lfloor \tfrac{3n}{2} \rfloor\} \cup \{v_{ij}/1 \leq i \leq \lfloor \tfrac{n}{2} \rfloor + 1, 1 \leq j \leq \lfloor \tfrac{3n}{2} \rfloor + 1\}$$
$$E(S_wG(n:2)) = \{u_{1j}v_{1j}, u_{1j}v_{1}, j+1, /1 \leq j \leq \lfloor \tfrac{3n}{2} \rfloor\} \cup \{u_{ij}v_{i-1,j}, u_{ij}v_{i-1,j+1}, u_{ij}v_{ij}, u_{ij}v_{i,j+1}/2 \leq$$
$$i \leq \lfloor \tfrac{n}{2} \rfloor + 1, 1 \leq j \leq \lfloor \tfrac{3n}{2} \rfloor\} \cup \{v_{i1}v_{i,\lfloor \frac{3n}{2} \rfloor + 1}, v_{i1}v_{i+1,\lfloor \frac{3n}{2} \rfloor + 1}/1 \leq i \leq \lfloor \tfrac{n}{2} \rfloor\} \cup \{v_{\lfloor \frac{n}{2} \rfloor + 1,1}v_{1,\lfloor \frac{3n}{2} \rfloor + 1}\}$$

Let us call $S_wG(n : 1)$ as 'Swing Grids of Type 1' and $S_wG(n : 2)$ as 'Swing Grids of Type 2 '

Example 3.3.

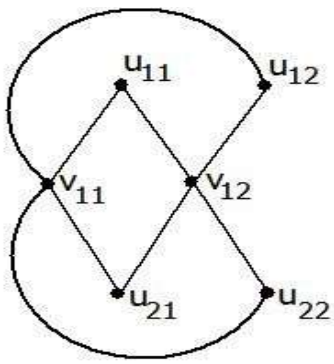


Figure 5 : Swing Grids graph $S_W G(1 : 1)$

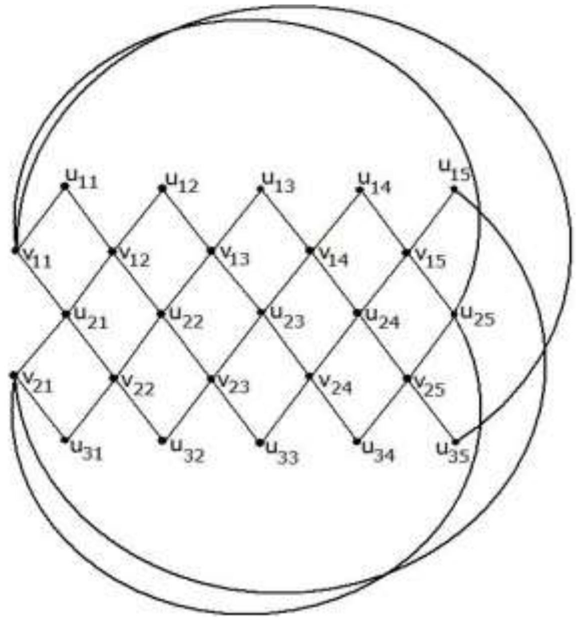


Figure 6 : Swing Grids graph S_W G(3 : 1)

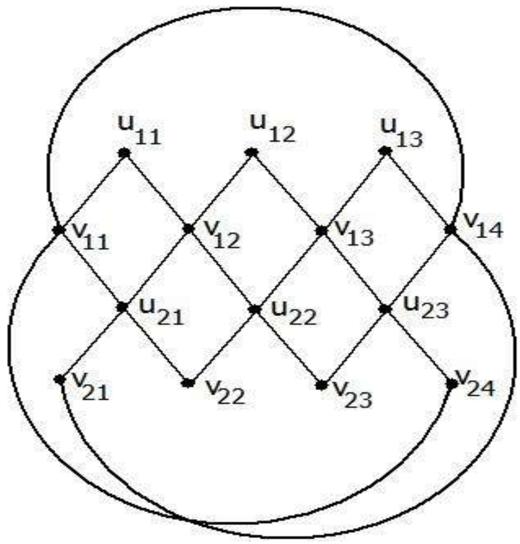


Figure 7 : Swing Grids graph S_W G(2 : 2)

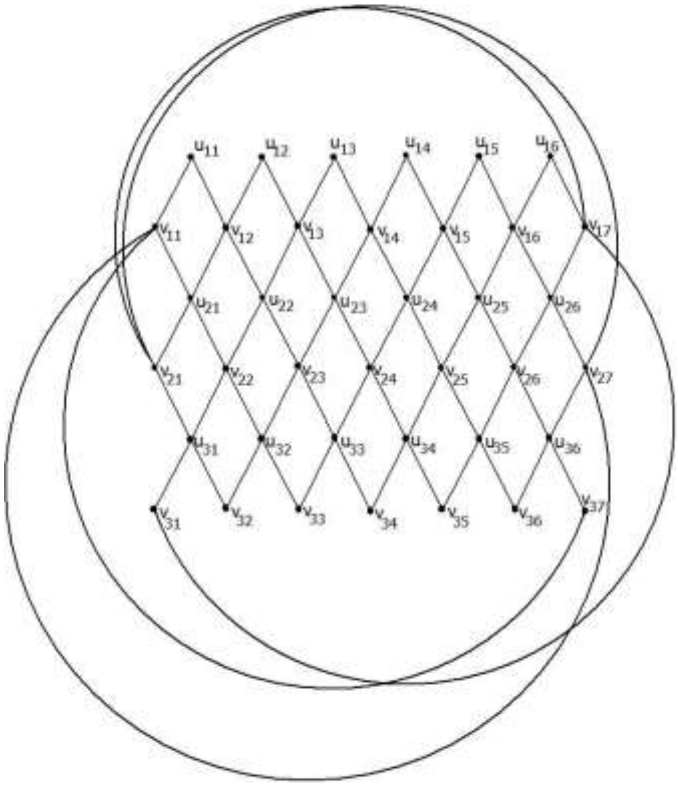


Figure 8 : Swing Grids graph S_W G(4: 4)

Theorem 3.4. $(i)\ \gamma(S_wG(n:1)) = \frac{(3n+1)(n+2)}{2} = \gamma(S_wG(n:2))$.

(ii) The number of vertices of degree 2 in

$$S_wG(n:1) = \begin{cases} 3n+1 & if\ n\ is\ odd \\ 3n & if\ n\ is\ even \end{cases}$$

(iii)The number of vertices of degree 2 in

$$S_wG(n:2) = \begin{cases} 3n & if\ n\ is\ odd \\ 3n+1 & if\ n\ is\ even \end{cases}$$

(iv)The number of edges in $S_wG(n:1)$

$= \{(3n+1)(n+1) \quad if\ n\ is\ odd\ 3n^2+4n+2 \quad if\ n\ is\ even$

(v) The number of edges in $S_wG(n:2)$

$= \{3n^2+4n+2 \quad if\ n\ is\ odd\ (3n+1)(n+1) \quad if\ n\ is\ even$

Proof: By construction of G(n : 1)

$u_{1j}\ , 1 \le j \le \lfloor \frac{3n+1}{2} \rfloor\ and\ u_{\lceil \frac{n+2}{2} \rceil, j}\ , 1 \le j \le \lfloor \frac{3n+1}{2} \rfloor$ have degree 2.

Therefore, Number of vertices of degree 2 =$\lfloor \frac{3n+1}{2} \rfloor - \lfloor \frac{3n+1}{2} \rfloor$

$$= \begin{cases} 2(\frac{3n+1}{2}) & if\ n\ is\ odd \\ 2(\frac{3n}{2}) & if\ n\ is\ even \end{cases}$$

$$= \begin{cases} 3n+1 & if\ n\ is\ odd \\ 3n & if\ n\ is\ even \end{cases}$$

The remaining vertices have degree 4.

(ie)The number of vertices of degree 4

$$= \begin{cases} \frac{n(3n+1)}{2} & if\ n\ is\ odd \\ \frac{n(3n+1)}{2}+1 & if\ n\ is\ even \end{cases}$$

(iii)The number of vertices of degree 2 in

$$S_wG(n:2) = \begin{cases} 3n & if\ n\ is\ odd \\ 3n+1 & if\ n\ is\ even \end{cases}$$

Proof:

By construction of $S_wG(n:2)$ $u_{1j}, 1 \leq j \leq \lfloor \frac{3n}{2} \rfloor$

and $v_{\lfloor \frac{n}{2} \rfloor+1,j}, 1 \leq j \leq \lfloor \frac{3n}{2} \rfloor + 1$.

Therefore, number of vertices of degree 2

$= \{3n \quad if\ n\ is\ odd\ 3n+1 \quad if\ n\ is\ even$

The remaining vertices have degree 4.

(ie)The number of vertices of degree 4

$$= \begin{cases} \frac{n(3n+1)}{2}+1 & if\ n\ is\ odd \\ \frac{n(3n+1)}{2} & if\ n\ is\ even \end{cases}$$

(vi)The number of edges in $S_wG(n:1)$

Case(i): When n is odd

$$2\epsilon(S_wG(n:1)) = 2 \times \left[2\left(\frac{3n+1}{2}\right)\right] + \left[4 \times \frac{n(3n+1)}{2}\right] = 2(3n+1) + 2n(3n+1) = 2(3n+1)(n+1)$$
$$2\epsilon = 2(3n+1)(n+1)$$
$$\epsilon = (3n+1)(n+1).$$

Case(ii): When n is even

$$2\epsilon(S_wG(n:1)) = (2 \times 3n) + 4\left(\frac{n(3n+1)}{2}+1\right) = 2(3n) + 4\left(\frac{n(3n+1)+2}{2}\right)$$
$$2(3n) + 2(n(3n+1)+2) = 2(3n + n(3n+1)+2)$$
$$2\epsilon = 2[3n + 3n^2 + n + 2]$$
$$\epsilon = 3n^2 + 4n + 2.$$

(v)The number of edges in $S_wG(n:2)$

Case(i): When n is odd

$$2\epsilon(S_wG(n:2)) = 2((3n) + 4\left(\frac{n(3n+1)}{2}+1\right)$$
$$2\epsilon = 2(3n + n(3n+1) + 2)$$
$$\epsilon = 3n^2 + 4n + 2.$$

Case(ii): When n is even

$$2\epsilon S_wG(n:2) = 2(3n+1) + 4\left(\frac{n(3n+1)}{2}\right)$$
$$2(3n+1) + 2(n(3n+1)) = 2((3n+1) + n(3n+1))$$
$$2\epsilon = 2((3n+1)(n+1))$$
$$\epsilon = (3n+1)(n+1).$$

Theorem 3.5.

i) $C_{3k+1}(\gamma) \cong S_wG(k:1)$ if k is odd

ii) $C_{3k+1}(\gamma) \cong S_wG(k:2)$ if k is even

Proof:

(i) If $(a_1, a_2, \ldots a_{k+1})$, $a_1 < a_2 < \ldots < a_{k+1}$ is a γ-set of C_{3k+1} then (i)

$$|a_i - a_{i+1}| \leq 3$$
$$(ii) \Sigma_{i=1}^{k} |a_i - a_{i+1}| + |a_1 - a_k| = 3k+1$$
$$\{|a_i - a_{i+1}| / 1 \leq i \leq k\} \cup \{|a_1 - a_k|\} = \{1, \overbrace{3,3,\ldots 3}\ k\ times\}\ \ or\ \{2,2,\overbrace{3,3,\ldots}(k-1)\ times\}$$

Therefore, The number of possibilities of γ-sets are tabulated below

When k is odd

Type	Difference sets	Number of γ- sets
1	1 followed by k 3’ s	3 k +1
2	Two 2’ s followed by (k-1) 3’ s	3 k +1
3	2,3,2 followed by (k-2) 3’ s	3 k +1
4	2,3,3,2 followed by (k-3) 3’ s	3 k +1
.	.	.
.	.	.
.	.	.
k+1 2	$2,3,3.\dots\dots\dots3\left(\frac{k-1}{2}-1\right)$times, 2 followed by $\left(\left(\frac{k-1}{2}\right)+1\right)3's$	3k + 1
k+3 2	$2,3,3.\dots\dots\dots3\left(\frac{k-1}{2}\right)$times, 3 followed by $\left(\frac{k-1}{2}\right)$ 3’s	$\left(\frac{3k+1}{2}\right)$

Table 1: The number of possibilities of γ-sets, when k is odd

Therefore, the total number of vertices =

$$(3k + 1)\left(\frac{k+1}{2}\right) + \left(\frac{3k+1}{2}\right) = \frac{(3k+1)(k+2)}{2}.$$

When k is even,

Type	Difference sets	Number of γ- sets
1	1 followed by k 3’ s	3 k +1
2	Two,2’ s followed by(k-1) 3’ s	3 k +1
3	2,3,2 followed by (k-2) 3’ s	3 k +1
.	.	.
.	.	.
.	.	.
k+2 2	2 followed $by\ \left(\frac{k-2}{2}\right)3's$, 2 followed $by\ \left(\frac{k+2}{2}\right)\ 3's$	3 k +1

Table 2: The number of possibilities of γ-sets, when k is even

Therefore, the total number of vertices = $\frac{(3k+1)(k+2)}{2}$.

Since we are dealing with cycles it is enough to consider γ- sets starting with 1.

Consider the type 1 γ-sets (1,2,5,8,11,...3k-4,3k-1). It is adjacent to (1,3,5,8,...3k-1), (2,5,8,...3k4 ,3k-1, 3k+1). So type1 vertices have degree 2.

Consider the type2 γ-set (1 ,3,5, 8,11,...3k-1).

It is adjacent to

(1,4,5,8,11,...3k-1), (3,5,8,...3k+1), (1,3,6,...3k-1).

This type 2 vertices have degree 4.

Proceeding as before it can be noted that the remaining vertices have degree 4 for all types. Arrange the vertices as follows. Start with the γ–set (1, 2, 5, 8 , ..., 3k-1). Place it as the first point in the first row and call it u_{11}. Look for the 4- degree vertices adjacent to u_{11}.

Take them as the first two vertices in the second row. Call them as v_{11},v_{12}. The 2-degree vertex adjacent to v_{12} will be taken as u_{12}. v_{22} will be the next vertex adjacent to u_{12}. Proceed like this until $u_{1,\lfloor\frac{3k+1}{2}\rfloor}$, $1 \leq j \leq \lfloor\frac{3k+1}{2}\rfloor$ if k is odd and $u_{1,\lfloor\frac{3k}{2}\rfloor}$ if k is even.

The third row is formed by taking u_{2j} as the vertex adjacent to both v_{1j} and $v_{1,j+1}$. Follow the same procedure for choosing the rest of the rows and naming them alternatively as u_{ij} and v_{ij} until we arrive at the (k+2)nd row. Add the remaining edges. It can be easily checked that C_{3k+1} are swing grids.

Illustration:

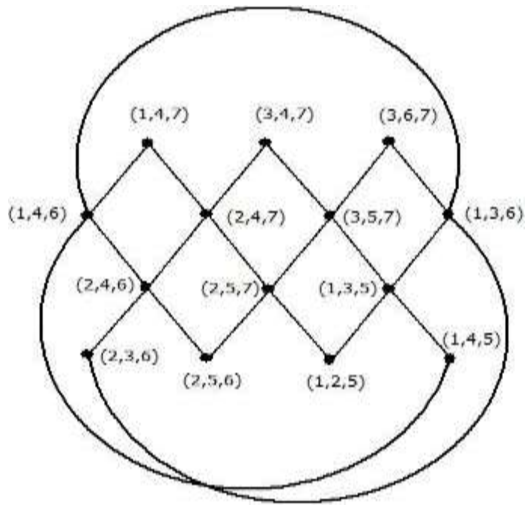


Figure 9 : $C_7(\gamma)$

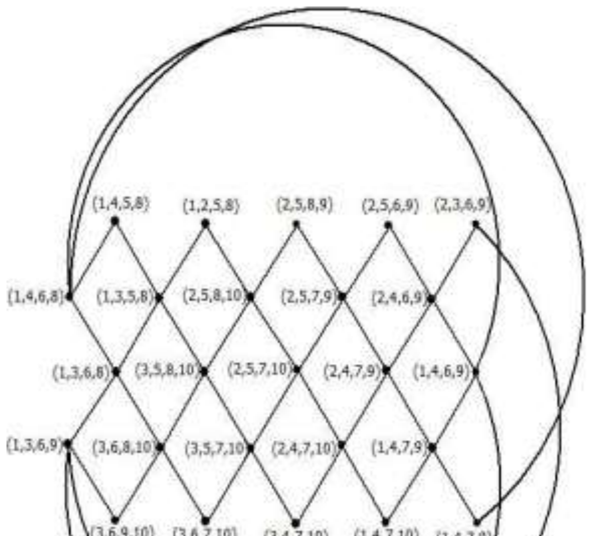


Figure 9 : $C_{10}(\gamma)$

γ_h- Graph

γ-graphs of standard families have already been identified. Instead of identifying γ_h- graphs directly, hop graphs can be determined and their γ-graphs can be identified. This approach results in the following theorems.

Definition 4.1. Consider the family of all γ_h- sets of a graph G and define the γ_h- graph $G(\gamma_h) = (V(\gamma_h), E(\gamma_h))$ of G to be the graph whose vertices $V(G(\gamma_h))$ correspond one to one with the γ_h-sets of G and two γ_h-sets, say S_1 and S_2, form an edge in $E(G(\gamma_h))$, if there exists a vertex $v \in S_1$ and a vertex $w \in S_2$ such that (i) v is at a distance of 2 to w and (ii) $S_1 = S_2 - \{w\} \cup \{v\}$ and $S_2 = S_1 - \{v\} \cup \{w\}$.

(i.e) two γ_h- sets are adjacent in $G(\gamma_h)$ if they differ by one vertex and two vertices defining this difference are at a distance 2 in G.

Example 4.2. Consider C_5,

The γ_h- sets of C_5 are $S_1 = \{v_1,v_2\}$, $S_2 = \{v_2,v_3\}$, $S_3 = \{v_3,v_4\}$, $S_4 = \{v_4,v_5\}$, $S_5 = \{v_5,v_1\}$.

C_5 of γ_h as follows,

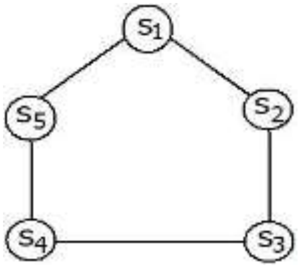


Figure 10 : γ_h – graph of C_5

Theorem 4.3. γ_h- graph of any graph G will be the same as γ- graph of H(G) where H(G) denotes the hop graph of G .

i.e. $G(\gamma_h) \cong (H(G))(\gamma)$

Proof.

We know that $\gamma_h(G) = \gamma(H(G))$.

Any γ_h- set of G will be a γ - set of H(G) and vice versa.

γ_h-graph $G(\gamma_h) = (V(\gamma_h), E(\gamma_h))$ of G is the graph whose vertices $V(G(\gamma_h))$ correspond one to one with the γ_h - sets of G and two γ_h-sets, say S_1 and S_2, form an edge if they differ by one vertex and two vertices defining this difference are at a distance 2 in G.

i.e. Vertex set of $G(\gamma_h)$ will be the set of all γ - sets of H(G) and edges of $G(\gamma_h)$ will be nothing but edges of $(H(G))(\gamma)$.

Theorem 4.4 .

$$P_n(\gamma_h) \cong \begin{cases} K_1 \square K_1 & if\ n = 6k \\ S_G(k+1) \square K_1 & if\ n = 6k+1 \\ S_G(k+1) \square S_G(k+1) & if\ n = 6k+2 \\ P_{k+2} \square S_G(k+1) & if\ n = 6k+3 \\ P_{k+2} \square P_{k+2} & if\ n = 6k+4 \\ K_1 \square P_{k+2} & if\ n = 6k+5 \end{cases}$$

Proof:

$P_n(\gamma_h)$ is the disjoint union of 2 paths, one of length $\lceil \frac{n}{2} \rceil$ and other of length $\lfloor \frac{n}{2} \rfloor$

We know that, $$H(P_n) = \begin{cases} P_{\frac{n}{2}} \cup P_{\frac{n}{2}} & if\ n\ is\ even \\ P_{\lceil \frac{n}{2} \rceil} \cup P_{\lceil \frac{n}{2} \rceil} & if\ n\ is\ odd \end{cases}$$

Case(i): n = 6k

$\lceil \frac{n}{2} \rceil = 3k = \lfloor \frac{n}{2} \rfloor$
$H(P_{6k}) \cong P_{3k} \cup P_{3k}$
$(P_{3k} \cup P_{3k})(\gamma) \cong P_{3k}(\gamma) \Box P_{3k}(\gamma)$
Therefore $P_{6k}(\gamma_h) \cong K_1 \Box K_1$.

Case (ii): n = 6k+1

$\lceil \frac{n}{2} \rceil = 3k + 1 and \lfloor \frac{n}{2} \rfloor = 3k$

n is odd and hence $H(P_{6k+1}) = P_{3k+1} \cup P_{3k}$.

$(P_{3k+1} \cup \ P_{3k})(\gamma) \cong P_{3k+1}(\gamma) \Box P_{3k}(\gamma)$

Therefore, $P_{6k+1}(\gamma_h) \cong S_G(k+1) \Box K_1$

Case(iii): n = 6k+2

$H(P_{6k+2}) \cong P_{3k+1} \cup \ P_{3k+1}$
$(P_{3k+1} \cup \ P_{3k+1})(\gamma) \cong P_{3k+1}(\gamma) \Box P_{3k+1}(\gamma)$
Therefore, $P_{6k+2}(\gamma_h) \cong S_G(k+1) \Box S_G(k+1)$

Case(iv): n = 6k+3

$\lceil \frac{n}{2} \rceil = 3k + 2 and \lfloor \frac{n}{2} \rfloor = k+1$

Here n is odd and hence $H(P_{6k+3}) \cong P_{3k+2} \cup P_{3k+1}$

$(P_{3k+2} \cup P_{3k+1})(\gamma) \cong P_{3k+2}(\gamma) \Box P_{3k+1}(\gamma)$

Therefore, $P_{6k+3}(\gamma_h) \cong P_{k+2} \Box S_G(k+1)$.

Case(v): n = 6k+4

$H(P_{6k+4}) \cong P_{3k+2} \cup P_{3k+2}$
$(P_{3k+2} \cup P_{3k+2})(\gamma) \cong P_{3k+2}(\gamma) \Box P_{3k+2}(\gamma)$
Therefore, $P_{6k+4}(\gamma_h) \cong P_{k+2} \Box P_{k+2}$

Case(vi): n = 6k +5

$\lceil \frac{n}{2} \rceil = 3k + 3 and \lfloor \frac{n}{2} \rfloor = 3k+2$

Here n is odd and hence $H(P_{6k+5}) \cong P_{3k+3} \cup P_{3k+2}$

$(P_{3k+3} \cup \ P_{3k+2})(\gamma) \cong P_{3k+3}(\gamma) \Box P_{3k+2}(\gamma)$

where 3k + 3 = 3(k + 1)

For k+1 ≥ 2,

$$(P_{3k+3} \cup \ P_{3k+2})(\gamma) \cong P_{3(k+1)}(\gamma) \Box P_{3k+2}(\gamma)$$

Therefore, $P_{6k+5}(\gamma_h) \cong K_1 \Box P_{k+2}$.

Hence

$$P_n(\gamma_h) \cong \begin{cases} K_1 \Box K_1 & if \ n = 6k \\ S_G(k+1) \Box K_1 & if \ n = 6k+1 \\ S_G(k+1) \Box S_G(k+1) & if \ n = 6k+2 \\ P_{k+2} \Box S_G(k+1) & if \ n = 6k+3 \\ P_{k+2} \Box P_{k+2} & if \ n = 6k+4 \\ K_1 \Box P_{k+2} & if \ n = 6k+5 \end{cases}$$

Theorem 4.5

$$C_n(\gamma_h) \cong \begin{cases} \overline{K_3} \,\square\, \overline{K_3} & if\ n = 6k \\ S_wG(2k:2) & if\ n = 6k+1 \\ S_wG(k:1) \,\square\, S_wG(k:1) & if\ n = 6k+2\ is\ odd \\ S_wG(k:2) \,\square\, S_wG(k:2) & if\ n = 6k+2\ is\ even \\ \overline{K_3} & if\ n = 6k+3 \\ C_{3k+2} \,\square\, C_{3k+2} & if\ n = 6k+4 \\ C_{6k+5} & if\ n = 6k+5 \end{cases}.$$

Proof:

We know that,

$$H(C_n) = \begin{cases} C_{\frac{n}{2}} \cup C_{\frac{n}{2}} & if\ n\ is\ even \\ C_n & if\ n\ is\ odd \end{cases}$$

Case(i): n = 6k and $\frac{n}{2} = 3k$

$H(C_{6k}) \cong C_{3k} \cup\ C_{3k}$

$(C_{3k} \cup\ C_{3k})(\gamma) \cong \underline{C_{3k}(\gamma)} \square \underline{C_{3k}}(\gamma) \cong \overline{K_3} \square \overline{K_3}$

Therefore, $C_{6k}(\gamma_h) \cong K_3 \square K_3$.

Case(ii): n = 6k+1

n is odd and hence $H(C_{6k+1}) \cong C_{6k+1} \cong C_{3k+1}$ Therefore, $C_{6k+1}(\gamma_h) \cong S_wG(2k : 2)$

Case(iii): n = 6k+2

$H(C_{6k+2}) \cong C_{3k+1} \cup\ C_{3k+1}$

$(C_{3k+1} \cup\ C_{3k+1})(\gamma) \cong C_{3k+1}(\gamma) \square C_{3k+1}(\gamma)$

Therefore, $C_{6k+2}(\gamma_h) \cong \begin{cases} S_wG(k:1) \square S_wG(k:1) & if\ k\ is\ odd \\ S_wG(k:2) \square S_wG(k:2) & if\ k\ is\ even \end{cases}$

Case (iv): n = 6k + 3.

n is odd and hence $H(C_{6k+3}) = C_{6k+3}$

6k + 3 = 3(2k + 1)

For 2k + 1 ≥ 2, $C_{6k+3}(\gamma) \cong K_3$

Therefore, $C_{6k+3}(\gamma_h) \cong K_3$.

Case (v): n = 6k + 4.

$H(C_{6k+4}) \cong C_{3k+2} \cup C_{3k+2}$

$(C_{3k+2} \cup\ C_{3k+2})(\gamma) \cong C_{3k+2}(\gamma) \square C_{3k+2}(\gamma)$

Therefore, $C_{6k+4}(\gamma_h) \cong C_{3k+2} \square C_{3k+2}$.

Case(vi): n = 6k+5

n is odd and hence $H(C_{6k+5}) = C_{6k+5}$

6k+5 = 3(2k+1) + 2

Therefore, $C_{6k+5}(\gamma) = C_{3(2k+1)+2}(\gamma) \cong C_{6k+5}$

$C_{6k+5}(\gamma_h) \cong C_{6k+5}$

Hence

$$C_n(\gamma_h) \cong \begin{cases} \overline{K_3} \square \overline{K_3} & if\ n = 6k \\ S_wG(2k:2) & if\ n = 6k+1 \\ S_wG(k:1) \square S_wG(k:1) & if\ n = 6k+2 \text{ is odd} \\ S_wG(k:2) \square S_wG(k:2) & if\ n = 6k+2 \text{ is even} \\ \overline{K_3} & if\ n = 6k+3 \\ C_{3k+2} \square C_{3k+2} & if\ n = 6k+4 \\ C_{6k+5} & if\ n = 6k+5 \end{cases}$$

CONCLUSION

N. Sridharan and K. Subramanian have given a different definition for γ graph. According to them two vertices are adjacent in γ graph if they are interchangeable. There is no condition on the two vertices defining the difference. But according to GERD H.FRICKE et.al., while moving to (Hop domination) γ_h graph from (Domination) γ graph, it seems to be convenient to stick on to definition given by GERD H.FRICKE et.al., .Properties of γ_h graphs will be an intersting topic for future research.

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