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Cylindrically Symmetric Cosmological Model for Barotropic Fluid Distribution with Varying Λ

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Abstract

Cylindrically symmetric cosmological model for barotropic fluid distribution with varying Λ is investigated. To get the deterministic solution we have assumed barotropic fluid distribution condition p = γρ, $0\leq \gamma \leq 1$, p being pressure, ρ the matter density and a supplementary condition between metric potential A, B, C, as A = BC. The cosmological constant Λ is varying having value, Λ= $\alpha/_{R^2}$, where R is scalar factor. The physical and geometrical properties of the model are also discussed and the reality condition is satisfied.

1. Introduction:

The problem of cosmological constant is one of the most salient and unsettled problems in cosmology. The smallness of the effective cosmological constant recently observed constitutes the most difficult problems involving cosmology and elementary particle physics theory. (Pradhan [14]).

Abers& Lee [1] predicted that during an early exponential phase, the vaccum energy is treated as large cosmological constant which is expected by Grand Unified Theory (GUT) (Sakharov [16]). Several cosmological model in which Λ is varying or the behavior of universe is given by Λ and universe filled with barotropic fluid distribution have been investigated by numbers of authors, viz. Abdussattar and Vishwakarma [2], Bali et al. [3][4][5][6][8][9][10]. Bali & Saraf [7] and Tiwari [19] studies cosmological model by using law of variation of scalar factor with a variable constant term. Singh et al [17] and Pradhan et al [15] investigated cosmological model with variable G and Λ.

In this paper, we have investigated cosmological model for barotropic fluid with varying Λ taking Λ= $\alpha/_{R^2}$, where R is scalar factor. To get the deterministic solution we have assumed barotropic fluid distribution condition p = γρ, $0\leq \gamma \leq 1$, p being pressure, ρ the matter density and a supplementary condition between metric potential A, B, C, as A = BC. The model also has point type singularity at T = 0 (MacCallum[13]).

2. Metric and Field Equation

We consider the cylindrically symmetric metric given by Marder (1958) in the form as

$$ds^2 = A^2 (dx^2 - dt^2) + B^2 dy^2 + C^2 dz^2 \qquad \text{--------(1)}$$

Where A, B, C are metric potentials and are functions of t-alone.

The energy momentum tensor T_i^j for perfect fluid is given by

$$T_i^j = (\rho + p)v_i v^j + p g_i^j$$

Where ρ is the matter density, p the isotropic pressure, v^i the flow vector satisfying $v_i v^j$ = -1

Einstein field equation is given by

$R_{ij} - \frac{1}{2} R g_{ij} - \Lambda g_{ij} = -8\pi T_i^j$ ……..(2)

Where Λ is cosmological constant and depends on time.

The Einstein field equation (2) for the metric (1) leads to

$\frac{1}{A^2}\left[-\frac{B_{44}}{B} - \frac{C_{44}}{C} - \frac{B_4 C_4}{BC} + \frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} \right] + \Lambda = 8\pi p$……..(3)

$\frac{1}{A^2}\left[-\frac{C_{44}}{C} - \frac{A_{44}}{A} + \frac{A_4^2}{A^2} \right] + \Lambda = 8\pi p$ ……….(4)

$\frac{1}{A^2}\left[-\frac{B_{44}}{B} - \frac{A_{44}}{A} + \frac{A_4^2}{A^2} \right] + \Lambda = 8\pi p$ ………(5)

$\frac{1}{A^2}\left[\frac{B_4 C_4}{BC} + \frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} \right] - \Lambda = 8\pi p$ ………..(6)

We assume the coordinates so that

$v^1 = 0 = v^2 = v^3$, $v_i v^j = -1$

Thus$v^4 = \frac{1}{A}$, $v_4 = -A$

From equation (4) and (5), we have

$\frac{B_{44}}{B} = \frac{C_{44}}{C}$ ………(7)

which leads to

$(CB_4 - BC_4)_4 = 0$

$(CB_4 - BC_4) = k$

which gives

$(\frac{B}{C})_4 = \frac{k}{c^2}$ ……….(8)

To get the result in terms of cosmic time t , we assume

A = BC ………..(9)

By equation (3) and (6) after using p = γρ,A = BC and Λ = $\alpha/_{R^2}$ (where R = BC) leads to

$\frac{B_{44}}{C} + \frac{C_{44}}{C} + (1+\gamma)\frac{B_4 C_4}{BC} = (1-\gamma)\left(\frac{B_4}{B} + \frac{C_4}{C}\right)^2 + (1+\gamma)\alpha$ ……...(10)

Which leads to,

$\frac{\mu_{44}}{\mu} + \frac{5}{4}(\gamma - 1)\frac{\mu_4^2}{\mu^2} = \left(\frac{\gamma-1}{4}\right)\frac{k^2}{\mu^2} + (1+\gamma)\alpha$ ………(11)

To find solution, we put

$\mu_4 = f(\mu)$

Thus $\mu_{44} = ff'$

Now equation (11) leads to

$$2ff' + \frac{5}{4}(\gamma - 1)\frac{f^2}{\mu} = \left(\frac{\gamma - 1}{4}\right)\frac{k^2}{\mu} + (1+\gamma)\alpha\mu$$

$f^2 = \frac{k^2}{5} + \frac{4\alpha(1+\gamma)\mu^2}{(5\gamma-1)}$ ………(12)

which leads to,

$$\frac{d\mu}{\sqrt{\frac{k^2}{5} + \beta^2\mu^2}} = dt$$

$\frac{d\mu}{\sqrt{b^2+\mu^2}} = \beta dt$ ………..(13)

Where $\beta^2 = \frac{4\alpha(1+\gamma)}{5\gamma-1}, b = \frac{k^2}{5\beta^2}$

Thus we have

$\sinh^{-1}\frac{\mu}{b} = \beta(t + t')$

Thus

$\mu = b\ sinh\beta T$ ………..(14)

Where T = t + t', and t' being constant of integration

We consider BC = μ and $\frac{B}{C} = \nu$

Therefore,

$\frac{\nu_4}{\nu} = \frac{k}{\mu}$,

Where k is constant

From equation (14), we have

$$\nu = l\left(tanh\frac{\beta T}{2}\right)^{k/b\beta}$$

$\nu = l tanh^N\left(\frac{\beta T}{2}\right)$ ……….(15)

Where l being constant of integration and N = $\frac{k}{b\beta}$

Now

$B^2 = \mu\nu$

Which gives,

$B^2 = 2bl\ sinh^{1+N}\left(\frac{\beta T}{2}\right)cosh^{1-N}\left(\frac{\beta T}{2}\right)$ ………(16)

And,

$C^2 = \frac{\mu}{\nu}$

Which gives

$C^2 = \frac{2b}{l}sinh^{1-N}\left(\frac{\beta T}{2}\right)cosh^{1+N}\left(\frac{\beta T}{2}\right)$ …………(17)

And

A = BC

Which gives

A = b sinhβT …………..(18)

Thus the metric (1) leads to the form

$ds^2 = b^2 sinh^2(\beta T)(dx^2 - dt^2) + 2bl\ sinh^{1+N}\left(\frac{\beta T}{2}\right)cosh^{1-N}\left(\frac{\beta T}{2}\right)dy^2 +$
$\frac{2b}{l}sinh^{1-N}\left(\frac{\beta T}{2}\right)cosh^{1+N}\left(\frac{\beta T}{2}\right)dz^2$ …………..(19)

4. Some Geometrical and physical properties

The matter density (ρ) for the model (19) is given by

$\rho = \frac{1}{8\pi b^2 sinh^4(\beta T)}\left[\frac{5}{4}cosh^2(\beta T) - N^2\beta^2\right]$ ……….(20)

The reality condition $\rho > 0$ Ellis (1971) leads to ,

$\rho > 0 \Longrightarrow 5cosh^2(\beta T) > 4N^2\beta^2$ ………(21)

The expansion (θ) is given by

$$\theta = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^i}\left(v^2\sqrt{-g}\right)$$

$= \frac{1}{A}\left(\frac{A_4}{A} + \frac{B_4}{B}\frac{C_4}{C}\right)$

$= \frac{2\beta cosh\beta T}{sinh^2(\beta T)}$ ……… (22)

Shear tensor is given by

$$\sigma_{ij} = \frac{1}{2}\left(v_{i;j} + v_{j;i}\right) + \frac{1}{2}\left(\dot{v_i}v_j + \dot{v_j}v_i\right) - \frac{1}{3}\theta(g_{ij} + v_i v_j)$$

Thus, we have

$$\sigma_1^1 = g^{11}\sigma_{11} = \frac{1}{3A}\left[\frac{2A_4}{A} - \frac{B_4}{B} - \frac{C_4}{C}\right]$$

$= \beta\frac{cosh\beta T}{3bsinh^2(\beta T)}$ …………(23)

$$\sigma_2^2 = g^{22}\sigma_{22} = \frac{1}{3A}\left[\frac{2B_4}{B} - \frac{A_4}{A} - \frac{C_4}{C}\right]$$

$= \frac{\beta}{3bsinh^2\beta T}\left[\frac{3N}{2} - \frac{cosh\beta T}{2}\right]$ …………(24)

$$\sigma_3^3 = g^{33}\sigma_{33} = \frac{1}{3A}\left[\frac{2C_4}{C} - \frac{A_4}{A} - \frac{B_4}{B}\right]$$

$= \frac{\beta}{3bsinh^2\beta T}\left[-\frac{3N}{2} - \frac{cosh\beta T}{2}\right]$ …………(25)

$\sigma_4^4 = g^{44}\sigma_{44} = 0$ …………(26)

Now shear σ is given by

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ji} = \frac{1}{2}[(\sigma_1^1)^2 + (\sigma_2^2)^2 + (\sigma_3^3)^2 + (\sigma_4^4)^2]$$

$= \frac{\beta}{6bsinh^2\beta T}(3cosh^2\beta T + 9N^2)^{1/2}$……..……(27)

Now,

$\frac{\sigma}{\theta} = \frac{1}{12cosh\beta T}(3cosh^2\beta T + 9N^2)^{1/2}$………..(28)

$$\neq 0$$

We define average scale factor R as,

$R^3 = A^2BC$

The deceleration parameter q for this model is,

$q = \frac{-R\ddot{R}}{\dot{R}^2}$ ……………..(29)

4. Conclusion:

The spatial volume increase with time. Since $\frac{\sigma}{\theta} \neq 0$, hence the model represents anisotropic space time. The deceleration parameter q<0 indicates that the model represent accelerating universe. The model starts expanding with big bang at T = 0 and the expansion in the model decreases with time. The reality condition $\rho + p > 0$, $\rho + 3p > 0$ is given by Ellis (1971) are satisfied.

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