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Stochastic analysis of time to treatment and treatment time of the organ in diabetic person

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Abstract

Consider the effects of diabetes on two organs. The human body's main organs are the heart and kidney. The heart and kidney are both exposed to cdps and have exponential thresholds. The organ heart is subjected to the cumulative damage process (cdp) in this model, while the organ kidney has Erlang phase 2 distributions. We provide the estimated time to organ failure based on the treatment of all damages to the two organs of a diabetic person. The cure and damage time have been determined to have various and coupled Laplace transforms. When each injury has a cure time, marginal distributions are determined.

Keywords Diabetic mellitus, cardio vascular disease, damage, cumulative damage process, Erlang phase two distributions

Introduction:

Diabetes is a global issue that causes significant morbidity and mortality. According to a WHO research, 135 million individuals were diagnosed with diabetes globally in 1995, with that figure anticipated to climb to at least 300 million by 2025. Diabetes is a metabolic condition characterized by high blood sugar levels due to inadequate insulin action. Aside from a substantial genetic susceptibility to non-insulin dependent diabetes, insulin secretion is influenced by a complex combination of host and environmental factors. Changes in lifestyle, such as decreased physical activity, increased weight, and changes in eating habits, are contributing to the rise in diabetes prevalence among adults. Cardiovascular disease is the leading cause of death among diabetics. Renal failure can result from diabetic nephropathy. Diabetic neuropathy causes people to lose feeling, especially in their feet, making them vulnerable to infections and gangrene. Refer to [4], [1], [3], [5], [6], and [8] for further information on such a deficit and disease. We investigate the example of a diabetic using the mathematical approach described in [2], [7], and [9]. The heart and kidney of a diabetic person are two organs that are subjected to damaging mechanisms in this study. Cumulative damage process (cdp) affects the organ heart, and cumulative damage process affects the organ kidney. We assume that once the threshold has been reached, the treatment centre will cure all of the harms one by one. The organ heart is exposed to cdp, and the organ kidney's life distributions are Erlang phase 2. We discovered the Laplace transform of the cure and damage time. When each injury has a cure time, marginal distributions are determined. A number of numerical examples are shown.

2. **Model Description**:

In this section we consider the case when the organ heart is exposed to cdp and the organ kidney has cumulative damage process. The organ heart is cdp and the organ heart has Erlang distribution of order 2 with parameter λ. The time to hospitalize the person (T) = min {T1, T2} where T1 and T2 are the times that the damage occurs in the organs heart and kidney respectively. Assume that the treatment time R of damage is independent of the damage magnitude. Assume that the damages occur to the heart in accordance with a renewal process with inter occurrence time distribution F(x) such that $\int_0^\infty x dF(x) < \infty$. The heart is dysfunctioning if it has N, the random number of damages occur. The heart function s even if it has k damages with an arbitrary probability P_k=P (N >k) fork = 0, 1, 2… $\{p_k\}$ is assumed to be any decreasing sequence of numbers in the unit interval such that $\sum_{k=0}^{\infty} P_k < \infty, P_0 = 1$Let p_k=$(P_{k-1} - P_k)$, the probability that heart's functional capacity decreased on k^{th}damage be given by the generating function $\varphi(s), 0 \leq s \leq 1$at time 0, the person is in normal condition, the organ heart and kidney are in good condition. The person is hospitalized if either of the organs heart or kidney is in damaged state. The damaged part of heart and the damaged part of the kidney are also curable by taking medicine in the initial stage. Treatment time of the i^{th}damage of the organ heart is R_i and R_i's, are independent and identically distributed random variables with distribution function R(y).Treatment time of the organ kidney is Assumed to be R independent of other R_i's but with distribution R(y) such that$\int_0^\infty y dR(y) < \infty$

2.1 Analysis:

We consider a treatment facility which cures the entire damages one at a time after time T. The curing time A^* is given by $A^* = R_1 + R_2 + R_3 \ldots + R_i$ where i is the numberof damages occurred during T. When the person is hospitalized only due to the dysfunctioning of heart and so$A^* = R_1 + R_2 + \ldots + R_N$. In the other case if the person is hospitalized due to damages occurred in the kidney, we have to give treatment to kidney with n(N) damages occurred to heart During the curing time is given by$A^* = R_1 + R_2 + \cdots R_{n+1}$.

The joint distribution T and A^* is given as

$P\ (T \le x, A^* \le y) = P(T_1 \le x, A^* \le y, T_2 > T_1) + P(T_2 \le x, A^* \le y, T_1 > T_2)$ $=\sum_{n=1}^{\infty} P_n \int_0^x \left(e^{-\theta z}+\theta z e^{-\theta z}\right) R_n(y) \mathrm{d}F_n(z)$

$+\sum_{n=0}^{\infty} P_n R_{n+1}(\mathrm{y}) \int_0^x \theta(\theta z)\, e^{-\theta z}[F_n(z) - F_{n+1}(z)]dz$

$+\ \sum_{n=0}^{\infty} P_n R_{n+1}(\mathrm{y}) \int_0^x \theta(\theta z)\, e^{-\theta z}[F_n(z) - F_{n+1}(z)]dz$ (1)

Here R_n(y) and F_n(y) are the n fold convolutions of R(y) and F(y) respectively.The term under the first summation of the right side of (1) is the joint probability that the person is hospitalized only due to the n^{th} damage occurs during (0, x) in the heart and $\sum_{i=1}^{n} R_i \le y$. The terms under the second symbol is the joint probability that the person is hospitalized only due to hospitalize damage occurs during (0, x), in the kidney during the treatment, heart is functioning even it has n damages occurred to it and $\sum_{i=1}^{n+1} R_I \le y$

From (1) we get

E $[e^{-\delta T} e^{\eta A^*}]$=$\int_0^{\infty}\int_0^{\infty} e^{-\delta x} e^{-\eta y} \sum_{n=1}^{\infty} P_n\ (e^{-\theta x}$+$\theta x e^{-\theta x})\ dR_n(y) dF_n(x)$

+$\int_0^{\infty}\int_0^{\infty} e^{-\delta x} e^{-\eta y} \sum_{n=0}^{\infty} P_n\ \theta(\theta x) e^{-\theta x}[F_n(x) - F_{n+1}(n)] dR_{n+1}(y) dx$

=$\sum_{n=1}^{\infty} r^{*n}\ (\eta) P_n f^{*n}(\delta+\theta) - \theta \sum_{n=1}^{\infty} r^{*n}\ (\eta) P_n \frac{\mathrm{d}}{\mathrm{dT}} f^{*n}(T)$

$+\frac{\theta^2}{(\theta+\delta)}^{2}\ r^{*n}(\eta)(1 - f^*(T) \sum_{n=0}^{\infty} P_n r^{*n}\ (\eta) f^{*n}(T)] - \frac{\theta^2}{(\theta+\delta)} r^{*n}(\eta) \sum_{n=0}^{\infty} r^{*n}\ (\eta) P_n \frac{\mathrm{d}}{\mathrm{dT}}\left[\left(1 - f^{*n}(T) f^{*n}(T)\right)\right]$ (2)

Here T= $\theta+\delta$

On simplification we get

E $[e^{-\delta T} e^{\eta A^*}]$= $\psi(r^*(\eta) f^*(\delta+\theta)$-$\theta \frac{\mathrm{d}}{\mathrm{dT}}[\psi(r^*(\eta) f^*(\mathrm{T}))]$+$\theta^2/(\theta+\delta)\ ^2 r^*(\eta)$(1-$f^*(\delta+\theta))\ \psi(r^*(\eta)\ f^*(\mathrm{T}))$-$\frac{\theta^2}{(\theta+\delta)} r^*(\eta) \frac{\mathrm{d}}{\mathrm{dT}}$[(1-$f^*(\theta+\delta))\ \psi(r^*(\eta) f^*(\mathrm{T}))]$ (3)

Here $*$ denotes Lalace $-$ stieltjes transform

Put $\eta = 0$ in equation (3) we get

E [$e^{-\delta T}$] =$\psi(f^*(\delta+\theta))$ + [1-$\psi(f^*(\delta+\theta))]\ \frac{\theta}{(\theta+\delta)^2}^{2} + (\frac{\theta^2}{\theta+\delta} - \theta)\psi'(f^*(\delta+\theta)) f^{*\prime}(\delta+\theta)$ ………… (4)

Using differentiation of equation (4)

We get

E [T] = $\frac{2(1-\psi(f^*(\theta))}{\theta}$+$\theta'(f^*(\theta)) f^{*'}(\theta)$ (5)

Put $\eta = 0$ in equation (3) and **using differentiation, we get**

E [A^*] = [$\frac{(1-\psi(f^*(\theta))}{1-f^*(\theta)}$- $\theta\ \frac{d}{d\theta}$($\frac{1-\psi(f^*(\theta))}{1-f^*(\theta)}$)] E [${A^*}_1$] (6)

2.3 Numerical illustration:

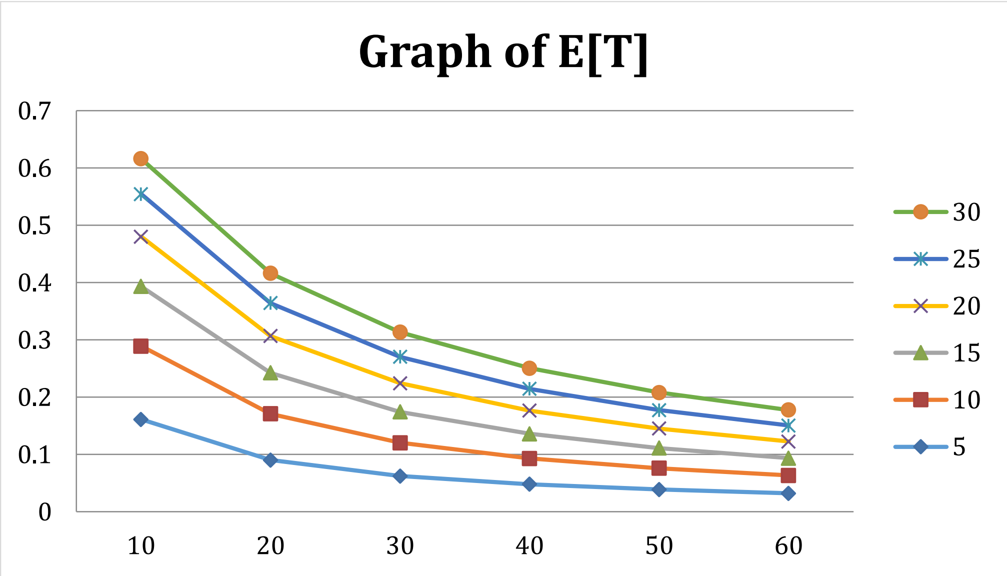
We assume that $\boldsymbol{p_1 = \frac{1}{4}}$, $\boldsymbol{p_2 = \frac{1}{4}}$, $\boldsymbol{p_3 = \frac{1}{2}}$ and E$[\boldsymbol{A^*}_{\boldsymbol{1}}] = \mathbf{20}$

Here $\psi(r) = \sum_{i=0}^{\infty} p_i r^i$

For the for the different values of $\varphi(say\ \varphi = 10{,}20{,}0{,}40{,}50{,}60)$ **and varying Parameter** $\theta = 5{,}10{,}15{,}20{,}25{,}30$ **In the equation (5) and (6)**

Table of E [T]

$\frac{\varphi \rightarrow}{\theta}$	10	20	30	40	50	60
5	0.1611	0.0903	0.0624	0.0476	0.0385	0.0323
10	0.1281	0.08060	0.0580	0.0452	0.0369	0.0312
15	0.1042	0.0717	0.0537	0.0427	0.0353	0.0301
20	0.0870	0.0641	0.0497	00403	00338	0.0290
25	0.0743	0.0576	0.0460	0.0380	0.0322	0.0279
30	0.0616	0.0521	0.0427	0.0358	0.0307	0.0269

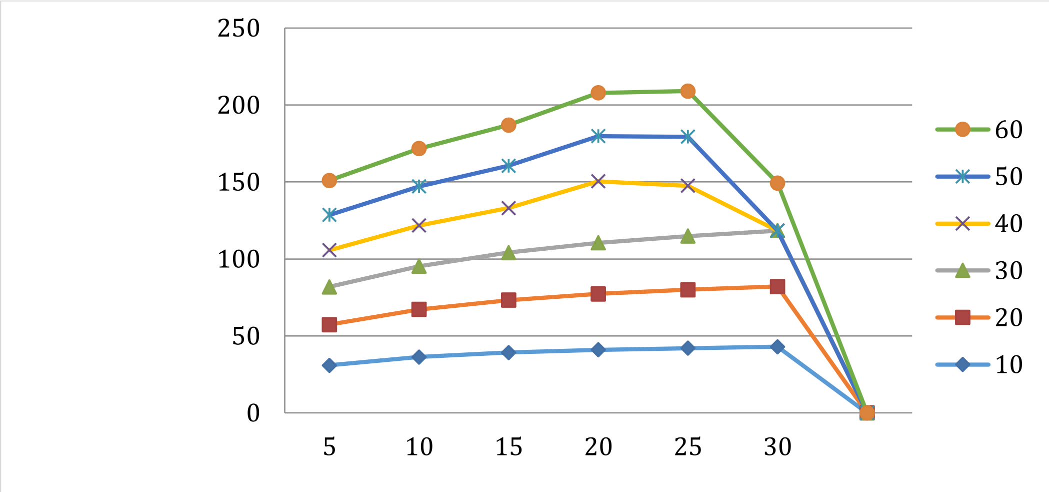


From the above table, as the value of φincreases, E [T] decreases and the value of θ increases, E [T] is also decreases. Table of E [A^*]

Table of E [$\boldsymbol{A}^*$]

$\frac{\varphi \rightarrow}{\theta}$	10	20	30	40	50	60
5	30.93	26.44	24.50	23.84	22.862	22.39
10	36.30	30.93	28.13	26.44	25.32	24.53
15	39.18	34.04	30.93	28.89	27.48	26.44
20	41.04	36.30	33.12	40.02	29.33	28.13
25	42.09	37.90	34.9	32.62	31.93	29.61
30	43.00	39.1	36.3	30.0 4	32.31	30.9

The Graph of E [$\boldsymbol{A}^*$]



From the above table, the value of φincreases. E $[A^*]$ decreases and the value of θ increases, E $[A^*]$ increases.

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