Inter-Spike-Interval Distribution of the LIF Neuron in Presence of Refractory Time Period in DDF

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Abstract – Spike plays a key role in information processing a neuorn.In temporal information encoding scheme, information has been encoded in consecutive spikes time duration. Probability distribution of these consecutive spikes duration is defined as inter-spike-interval distribution (ISI distribution). LIF neuron model in DDF (distributed delay framework) is investigated with the refractory time period. A number of different kernel functions like exponential, gamma and hypo-exponential distributed is used in the present study. Parameter values for which the ISI distribution does't show qualitative changes is observed.

Keywords –First Passage Time Problem, Distributed Delay Framework, ISI Distribution, LIF model, Neuronal Information Processing.

I. INTRODUCTION

Information encoding by a neuron is carried out in term of spike and spike sequences [1]. The spike sequence contains neuronal information which transmits across the neuronal system [9, 10]. A neuron receives information from other neurons and external environment in term of various ions and molecules which increases the potential of the neuron. After a certain value of membrane potential, an epoch is generated which is known as the spike. The membrane potential of neuron immediately reduces to reset value (the minimum value). A neuron does not start processing information immediately after emitting a spike [13, 18]. The time duration between a spike generation and starting the information processing for other spike is termed as refractory time period.Depending on the neuronal internal, external environment and neuron types, the refractory time period varies across the neurons. Kobayashi et. al. [22] has investigated the responses of olfactory receptor neurons in moth with the help of adaptive integrate-and-fire neuron model. In simulation based study, Kobayashi et. al. [22] has taken the refractory time period value as 3 milliseconds. Destexhe et. al. [8] have investigated the effect of refractoriness on information encoding mechanism into stereotypical short-term spike and suggested that refractoriness is useful for ending information for variable cortical states and also suggested the average value of refractory time period which is noticed as approximate 10 milliseconds. Teeter at. al. [23]has applied a number of refractory time period (rtp) values to classify the neuron in GLIF neuron model.

A neuron uses two information encoding techniques, namely, temporal encoding scheme and rate encoding scheme, to encode information in form of spike trains [14, 15, 18, 19]. Temporal information encoding scheme uses time duration between two spikes to encode neuronal information. This time durations are also defined as inter-spike-intervals. The probability distribution of this time interval is known asinter-spike-interval distribution (ISI distribution). The first passage time problem is useful to mathematically study the temporal information encoding scheme. FPT problem formulation can be given as [7, 15, 24, 26].

$$T = \inf\{\tau : \tau > 0, V(\tau) > V_{threshold}, V(0) = V_0 < V_{threshold}\} \quad (1)$$

Karmeshu et. al. [24] havestudied the membrane potential evolution dynamics in LIF neuron under the influence of past values of membrane potential and have suggested a distributed delay framework (DDF). Karmeshu et al. [24] have applied the temporal encoding scheme for neuronal information processing. Choudhary et al. [4, 5, 6, 7] have applied rate encoding mechanism for investigating the information processing in the LIFmodel in DDF. They have analyzed the information processing under presence of exponentially distributed kernel, gamma and hypo-exponentially distributed kernel functions.The information processing in LIF neuron model in DDF is studied in influence of the refractory time period. A number of kernel functions like exponentially distributed, gamma distributed and hypo-exponentially distributed kernel functions is analyzed and findings are compared with LIF model with stochastic input. The neuronal information encoding scheme is studied by applying temporal encoding scheme. Inclusion of refractory time period τ_{ref} modifies the FPT problem as [2, 3, 17, 26]

$$T = \inf\{\tau + \tau_{ref} : \tau > 0, V(\tau) > V_{threshold}, V(0) = V_0 < V_{threshold}\} \quad (2)$$

The article is structured in 6 sections. Section 1 contains a brief introduction about information processing and first passage time problem in neuronal models. Section 2 explains themathematical formulation of LIF model. DDF for neuron model is described in section 3. This section contains the dealing with formulation of LIF neuron model in DDF with three delay kernel functions viz.

exponentially distributed, gamma and hypo-exponentially distributed kernel functions. Simulation strategy for the considered neuron models are explained in Section 4.Simulation results and detailed study are given in Section 5.Finally, section 6 contains conclusions and future scope for the study.

II. LEAKY INTEGRATE-AND-FIRE NEURON MODEL

LIF model is the extension of integrate-and-fire (IF) neuron model [1]. LIF model is equivalent to the electrical RC-circuit [1, 11, 12]. This model is widely used for analytically investigating the neuronal activities due to its mathematical simplicity. Mathematically, the simplest form of LIF model may be formulated as [1, 2, 3, 8]

$$\frac{dV}{dt} = f(V,t) + I \tag{3}$$

Here $f(V,t)$ is a function describing the membrane potential function and I is the applied input stimulus. Different variants of LIF model can obtained from Eq. (3) by choosing a suitable value of $f(V,t)$ and *I*. The choice of $f(V,t)$ as $-\beta V$ results into leaky integrate-and-fire neuron model where it's value equal to zero results into integrate-and-fire neuron model. LIF model driven stochastic input stimulus can be described as:

$$\frac{dV}{dt} = -\beta V(t) + \mu + \xi(t) \tag{4}$$

Here β and $\mu + \xi(t)$ are known as membrane decay constant and the stochastic input. μ and $\xi(t)$ are mean value of the input stimulus and white noise.

III. LIF NEURON MODEL IN DDF

Karmeshu et. al. [24] have investigated the impact of past values of potential on its current development. They suggested that neuronal information processing depends on applied input stimulus and past values of membrane potential. Karmeshu et. al. [24] have suggested the distributed delay framework (DDF). Here, a kernel is applied to include previous membrane potential values in the model. This kernel function may also be termed as delayor memoryfunction. Let $K(t)$ is a memory kernel function then LIF model takes the form in DDF as

$$\frac{dV(t)}{dt} = -\beta \int_0^t K(t-\tau) V(t) d\tau + \mu + \xi(t) \tag{5}$$

Choice of $K(t)$ results into variants of LIF model in DDF. Inclusion of kernel function into the LIF transforms the development process into a non-Markovian process. It is a challenging task to get an analytical solution or perform a numerical simulation based study for such kind of the problem. For performingthe numerical simulation based study for such kind of problems, a linear chain trick has been suggested in literature, so that a non-Markovian may be transformed into a Markovian process.

III (A). LIF MODEL WITH EXPONENTIAL DELAY (EDD)

Karmeshu et. al. [24] have studied model given as in Eq. (5) with exponentially distributed kernel function. This kernel function has the following form.

$$K(t) = \begin{cases} \eta e^{-\eta t}; t \geq 0 \\ 0; otherwise \end{cases} \tag{6}$$

Substitution of K(t) as exponential distributed kernel function, LIF model in DDF takes the following form

$$\frac{dV}{dt} = -\beta \int_0^t \eta e^{-\eta(t-\tau)} V(\tau) d\tau + \mu + \xi(t) \tag{7}$$

Here η is the delay parameter. Incorporation of memory kernel in Eq. (7) has resulted membrane potential development process $\{V(t); t \geq 0\}$ into a non-Markovian process. Substitution of $U(t) = \int_0^t \eta e^{-\eta(t-\tau)} V(\tau) d\tau$ into Eq. (7) results as

$$\begin{aligned} \frac{dV}{dt} &= -\eta\beta U(t) + \mu + \xi(t) \\ \frac{dU(t)}{dt} &= -\eta\{U(t) - \eta V(t)\} \end{aligned} \tag{8}$$

Given $[V(t)=0$ and $U(t)=0$ at $t=0]$.

III (B). LIF MODEL WITH GAMMA DELAY (GDD)

Sharma and Karmeshu [25] have studied the LIF model as given in Eq. (5) with gamma distributed kernel function. This function has the following form.

$$K(t;\eta,m)=\frac{\eta^{m+1}t^{m}e^{-\eta t}}{m!} \tag{9}$$

For shape parameter $m=0$ Eq. (9) results into exponential distributed kernel function. Substitution K(t) as gamma distributed kernel function into Eq. (5), results the following equation.

$$\frac{dV}{dt}=-\beta\int_{0}^{t}\frac{\eta^{m+1}(t-\tau)^{m}e^{-\eta(t-\tau)}}{m!}V(\tau)d\tau+\mu+\xi(t) \tag{10}$$

The substitution of $U_{m}(t)=\int_{0}^{t}\frac{\eta^{m+1}(t-\tau)^{m}e^{-\eta(t-\tau)}}{m!}V(\tau)d\tau$ into Eq. (10) and further computation of Eq. (10) results a system of coupled linear stochastic differential equation which can be written as

$$\begin{aligned}\frac{dV}{dt}&=-\eta\beta U_{m}(t)+\mu+\xi(t)\\ \frac{dU_{i}(t)}{dt}&=-\eta\{U_{i}(t)-U_{i-1}(t)\}\\ \frac{dU_{0}(t)}{dt}&=-\eta\{U_{0}(t)-\eta V(t)\}\end{aligned} \tag{11}$$

For $i\in\{1,2,...,m\}$, with $[V(t)=0$ and $U_{j}(t)=0\forall j\in\{0,1,2,...,m\}$ at $t=0]$. Sharma and Karmeshu [25] have investigated the above system for $i=1$. This assumption simplifies Eq. (11) as given below

$$\begin{aligned}\frac{dV}{dt}&=-\eta\beta U_{1}(t)+\mu+\xi(t)\\ \frac{dU_{1}(t)}{dt}&=-\eta\{U_{1}(t)-U_{0}(t)\}\\ \frac{dU_{0}(t)}{dt}&=-\eta\{U_{0}(t)-\eta V(t)\}\end{aligned} \tag{12}$$

Sharma and Karmeshu [25] have used numerical simulation techniques for investigating the information processing mechanism of this model and noticed that the ISI distribution for such type of model exhibits the long tail behavior.

III (C). LIF MODEL WITH HYPO-EXPONENTIAL DELAY (HEDD)

Choudhary et. al. [4, 6]have considered the arrival of excitatory and inhibitory input as two separate entities which are affecting the evolution of membrane potential. These entities are modeled in term of hypo-exponentially distributed kernel function as given below [11, 14]

$$f(x)=-\frac{\lambda_{E}\lambda_{I}}{\lambda_{E}-\lambda_{I}}.(e^{-\lambda_{E}x}-e^{-\lambda_{I}x}) \tag{13}$$

Substitution of Eq. (13) into the Eq. (5) results into a variant of LIF model as given below [11, 14]

$$\frac{dV}{dt}=-\frac{\beta\lambda_{E}\lambda_{I}}{\lambda_{E}-\lambda_{I}}\int_{0}^{t}(e^{-\lambda_{E}(t-\tau)}-e^{-\lambda_{I}(t-\tau)})V(\tau)d\tau+\mu+\xi(t) \tag{14}$$

Here, $V(t)$ at $t=0$ .Substitution of $U_{1}(t)=\int_{0}^{t}e^{-\lambda_{E}(t-\tau)}V(\tau)d\tau$ and $U_{2}(t)=\int_{0}^{t}e^{-\lambda_{I}(t-\tau)}V(\tau)d\tau$ into Eq. (14) results a new model in extended space as [4, 6]

$$\begin{aligned}\frac{dV}{dt}&=-\frac{\beta\lambda_{E}\lambda_{I}}{\lambda_{I}-\lambda_{E}}(U_{1}-U_{2})+\mu+\xi(t)\\ \frac{dU_{1}(t)}{dt}&=-\lambda_{E}U_{1}+V\\ \frac{dU_{2}(t)}{dt}&=-\lambda_{I}U_{2}+V\end{aligned} \tag{15}$$

with $V(t)=U_{1}(t)=U_{2}(t)=0$ at $t=0$.

IV. SIMULATION STUDY

LIF neuron model and its variants as given in Eq. (4), Eq. (8), Eq. (12) and Eq. (15) are investigated with the simulation based study.These neuron model forms the system of coupled SDEs. Simulation based techniques are helpful for investigating such kind of problems [12]. EM numerical simulation methodis applied to study the models in distributed delay framework.

Following Euler-Maruyama scheme [12], Let $[0,t]$ is the time interval of evolution of membrane potential then divide this interval into n subinterval $[0,t_1],[t_1,t_2],...,[t_{n-1},t_n]$ of equal length.Let V_i is potential at time $t=t_i$ for neuron model defined in Eq. (15), then the potential at next time $t=t_{i+1}$ takes the form as

$$\begin{aligned} V_{i+1} &= V_i - \left(\frac{\beta\lambda_E\lambda_I}{\lambda_I-\lambda_E}(X_i - Y_i) - \mu \right) h + \sigma\sqrt{h}\xi_i \\ X_{i+1} &= X_i - (\lambda_E X_i - V_i) h \\ Y_{i+1} &= Y_i - (\lambda_I Y_i - V_i) h \end{aligned} \tag{16}$$

With $V_0=0, X_0=0$ and $Y_0=0$ .Similar simulation strategy has been applied to study the spiking behavior of simple LIF neuron with noisy input, LIF neuron with exponentially and gamma distributed kernel functions. We consider the temporal information processing in neuron and calculate inter-spike-intervals. A time duration, assumed as a refractory time, is added to the isi time. RTP is a time duration, in which a neurons do notinvolve ininformation processing activities. LIF model with stochastic input, LIF neuron with EDD, LIF neuron with GDD and LIF neuron with HEDD are investigated in three modes (i) without refractory time (ii) refractory time period with uniform distributionand (iii) refractory time period with Gaussian distribution [20, 21]. Inter-spike-interval distributions formultiple parameter values areshown in Fig. 1 to Fig. 12.Table 1 contains parameter values which are used during simulation study.In first mode, spiking activity and inter-spike-interval for considered neuron model is investigated without refractory time period.Fig 1, to Fig. 4. Fig. 5 to Fig. 8 contains ISI distribution patterns for considered neuron models with uniformly distributed rtp in between 3 ms to 5 ms time interval. In Gaussian distributed rtp, mean time is taken as 4 ms with 1ms std. deviation and ISI distributions are shown in Fig. 9 to Fig. 12.

Fig. No.	β	η	λ_1	λ_2	σ	μ
1, 5, 9	0.1	1	0.01	1	0.05	0.1
2, 6, 10	0.2	0.5	0.2	0.5	0.1	0.2
3, 7, 11	0.2	0.3	0.8	0.2	0.2	0.1
4, 8, 12	0.3	0.5	0.5	1	0.2	0.5

Table: 1 Set of parameter values used in simulation based study

V. RESULT ANAYLYSIS

Neuronal information processing and ISI distributions of LIF neuron in DDF with three different kind of delay kernel function viz. exponential, gamma and hypo-exponentially distributed kernel functionare investigated and obtained resultsare compared with LIF model with stochastic input stimulus.

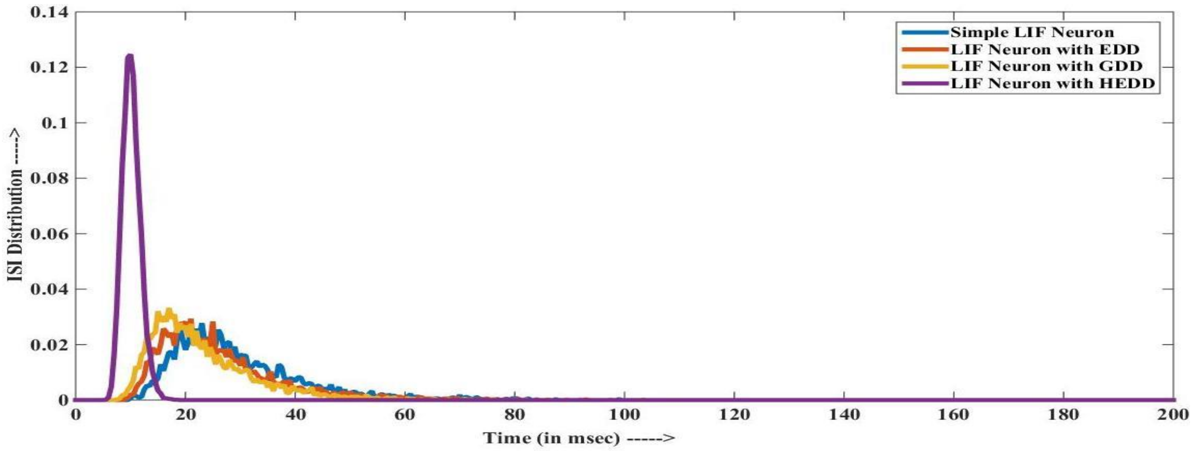

Fig. 1: ISI distribution with no rtp

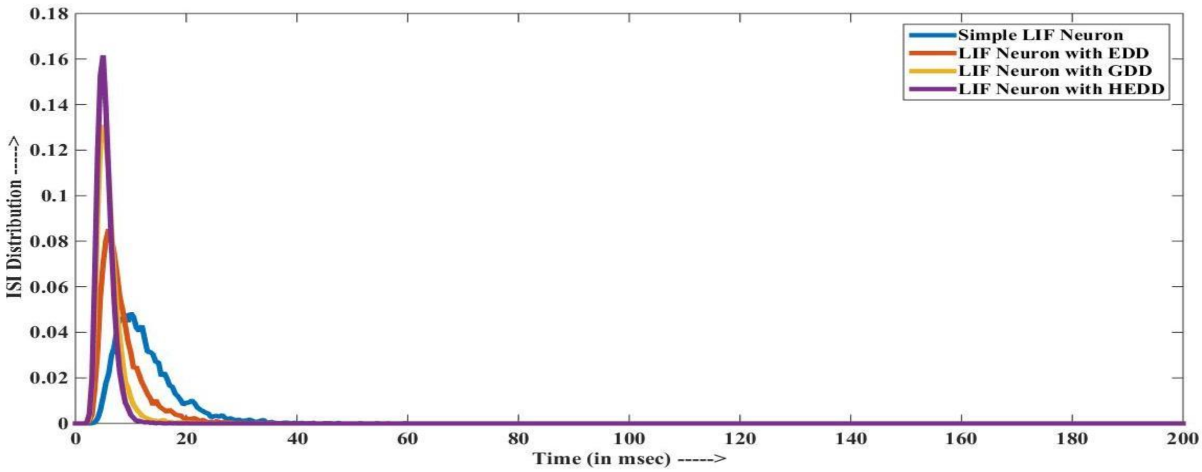

Fig. 2: ISI distribution with no rtp

Fig. 1 to Fig. 4 contains ISI distribution patterns for considered models without refractory time period. Fig. 1 illustrates ISI distribution pattern for small membrane decay constant (β = 0.1). A neuron reaches to the firing threshold quickly with small value of β and applied input stimulus. Here, LIF neuron with hypo-exponential distributed delay kernel is achieving the firing threshold more rapidly as compared to other models. This is occurring due to delay in excitatory and inhibitory potential. Fig. 2 contains ISI distribution patterns for considered neuron models having increased membrane decay constant and applied input stimulus. Here, each neuron model shows the similar behavior and reaching to its firing threshold quickly. A small spread in ISI distribution pattern for simple LIF neuron model is occurring as simple LIF neuron model has no memory. Memory, considered in distributed delay forms help a neuron to reach its firing threshold more quickly. Thus, neuron model with distributed delay pattern acquires the firing threshold more quickly as compared with simple neuron model.

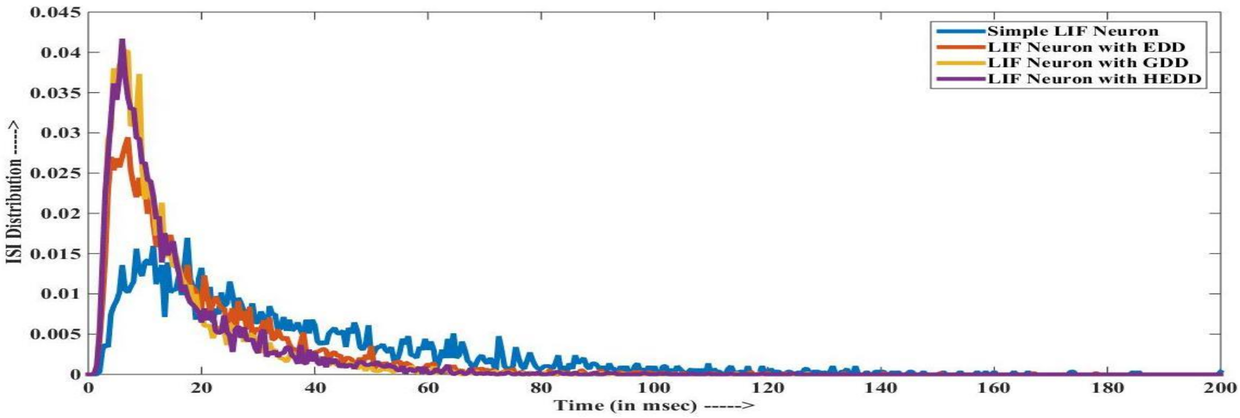

Fig. 3: ISI distribution with no rtp

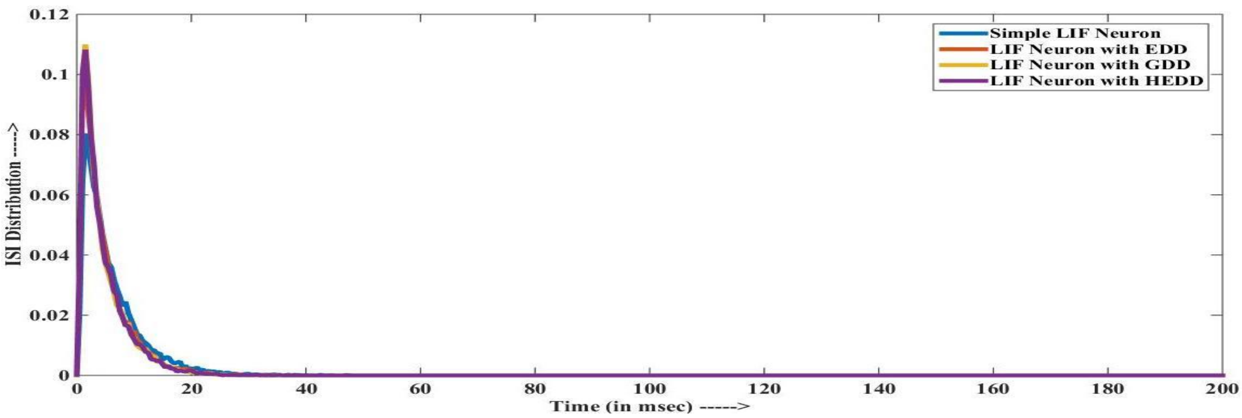

Fig. 4: ISI distribution with no rtp

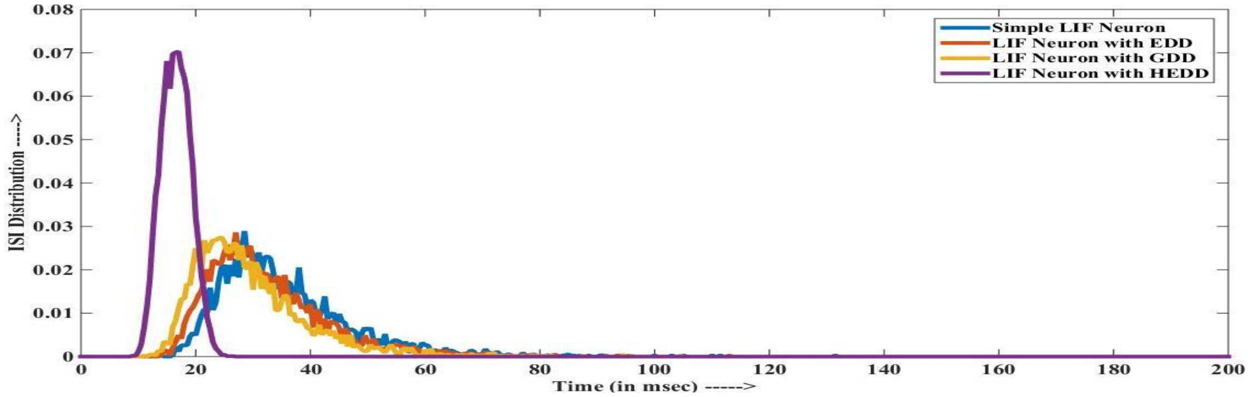

Fig. 5: ISI distribution with uniformly distributed rtp

Fig. 3 illustrates the ISI distribution patterns for more membrane decay constant, input stimulus and increased delay parameters. Here noise in applied input stimulus is playing an important role in spiking activity showing long tail behavior in spiking activity. Fig. 4 represents similar ISI distribution patterns for each neuron model. This includes the set of various parameter values for which LIF neuron with distributed delay behaves similar with simple neuron model i.e. memory has no effect on spiking activity. Fig. 5 to Fig. 8 show the ISI distribution pattern with uniformly distributed refractory time period. It is noticed that a neuron has 3 ms to 5 ms refractory time period in various experimental studies. Thus, the uniformly distributed time duration in 3 ms to 5ms time interval is considered. It is well noticed that a neuron does not participate in neuronal information processing during its refractory time period. Fig. 5 contains the ISI distribution pattern for small values of membrane decay constant and applied input stimulus. Comparison of ISI distribution pattern with Fig. 1 results a shift ISI distribution towards its right side i.e a neuron takes more time to reach its firing threshold as compared with no refractory time period. However, LIF neuron model with hypo-exponentially distributed kernel is quicker as compared with other neuron models. Fig. 6 illustartes the ISI distribution pattern with more increased membrane decay constant and applied input stimulus. Here, each neuron is showing similar qualitative behavior in ISI distribution but different quantitative behavior i.e. ISI distribution pattern is similar but number of spikes in fix simulation time varies. Simple LIF neuron has small number of spikes as compared with LIF neuron in DDF. This is happening due to memory element and refractory time periods. The refractory time period is responsible to take more time during spiking activity whereas distributed delay helps a neuron to reach its firing threshold.

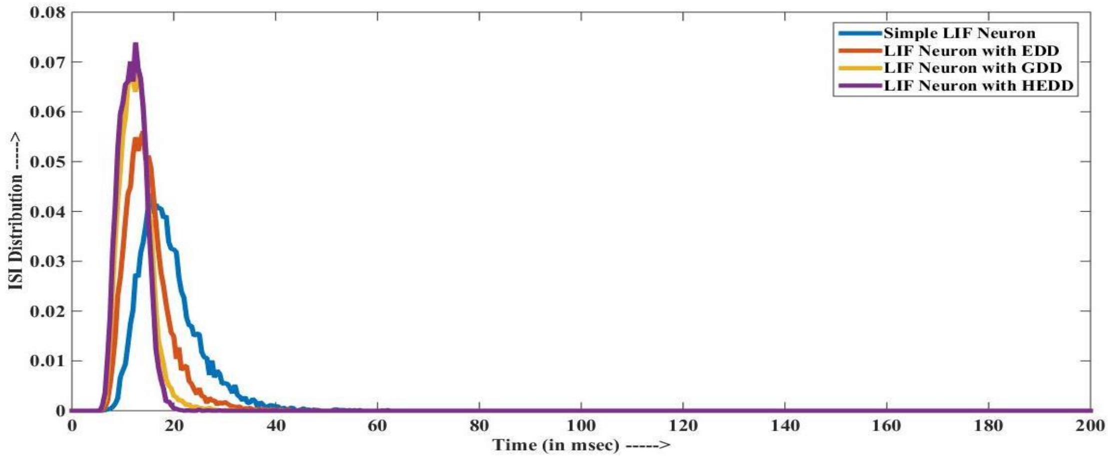

Fig. 6: ISI distribution with uniformly distributed rtp

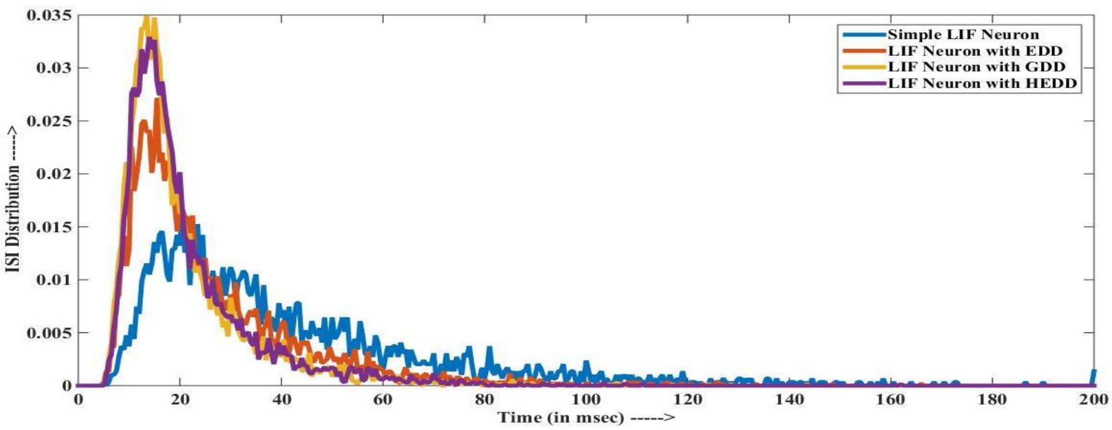

Fig. 7: ISI distribution with uniformly distributed rtp

Fig. 7 shows the ISI distribution with increased membrane decay constant and noisy input. Here, the noise in applied input stimulus and rtp is responsible to bring log tail behavior in ISI distribution patterns. Similarly, Fig. 8 represents consistent ISI distribution pattern. Here each neuron model is showing similar qualitative as well as quantitative behavior. This is happening due to the set of parameter values where the delay in the neuron model has negligible or very small effect on the neuronal information processing. Fig. 9 to 12 exhibit ISI distribution pattern for considered neuron modelsin presence of Gaussian distributed rtp. Mean and standard deviation of the refractory time period is assumed as 4ms and 1ms, respectively. Fig. 9 represents ISI distribution pattern for small membrane decay constant and applied input stimulus. As compared with Fig. 1 and Fig. 5, ISI distribution patterns in Fig. 9 have more spread due to the presence Gaussian distributed rtp. Fig 10 shows ISI distribution pattern for comparatively large membrane decay constant value and applied stochastic input stimulus. Increased membrane decay constant with Gaussian distributed rtp is making the synchronous spiking activity. The number spikes in other neuron model as compared with LIF model with hypo-exponentially distributed kernel is increased as compared with the ISI distribution patterns in Fig. 2 and Fig. 6.

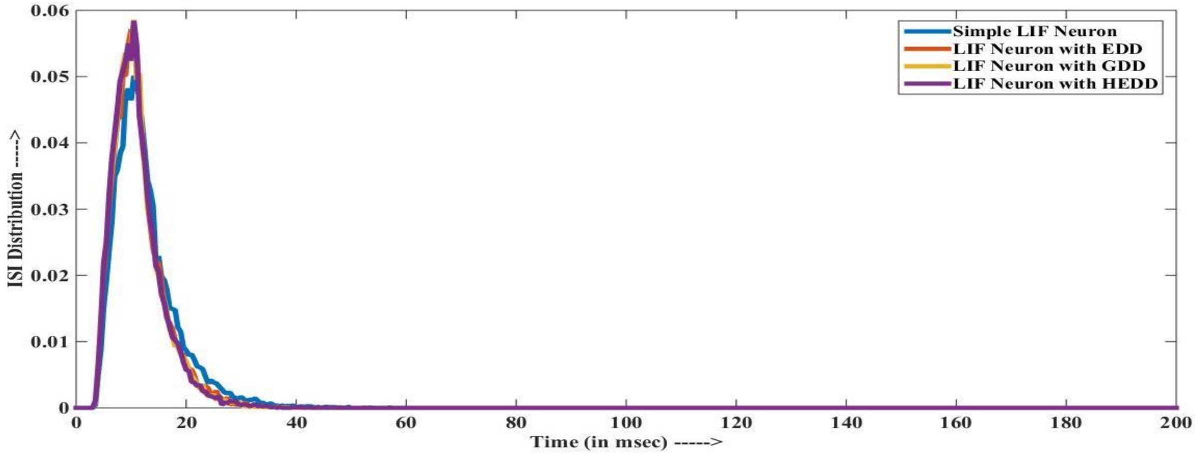

Fig. 8: ISI distribution with uniformly distributed rtp

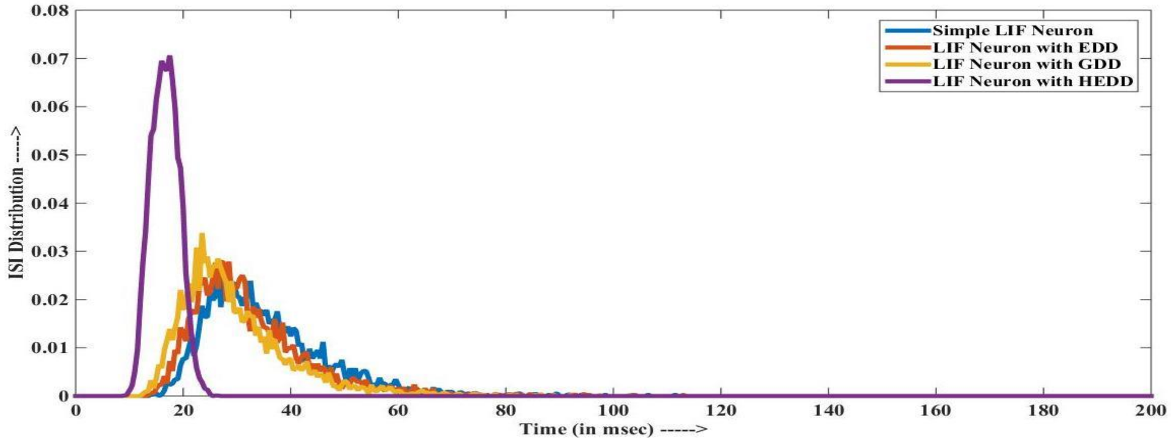

Fig. 9: ISI distribution for LIF Neuron in DDF with Gaussian distributed refractory time period

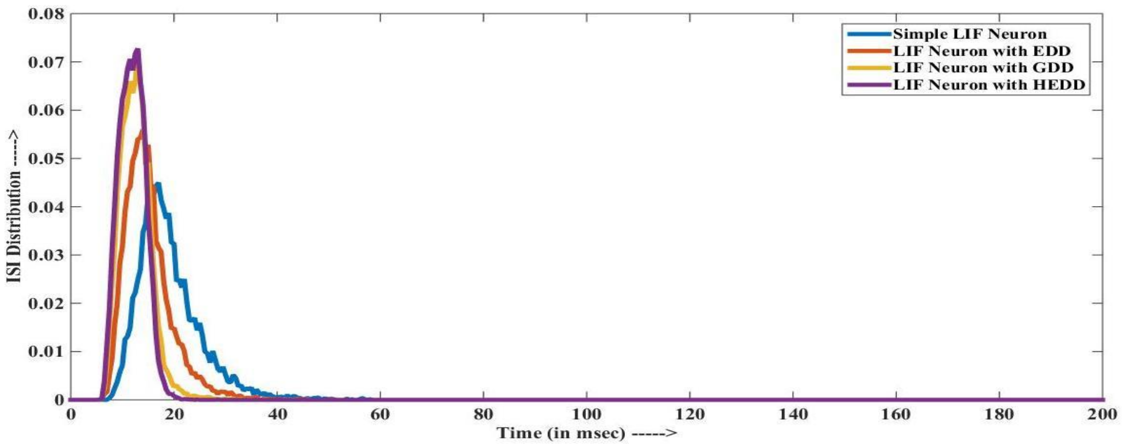

Fig. 10: ISI distribution with Gaussian distributed rtp

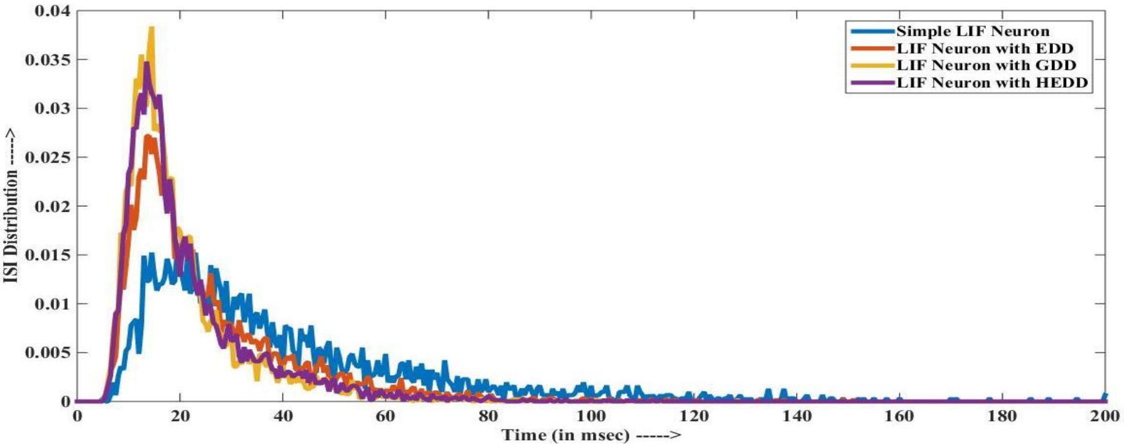

Fig. 11: ISI distribution DDF with Gaussian distributed rtp

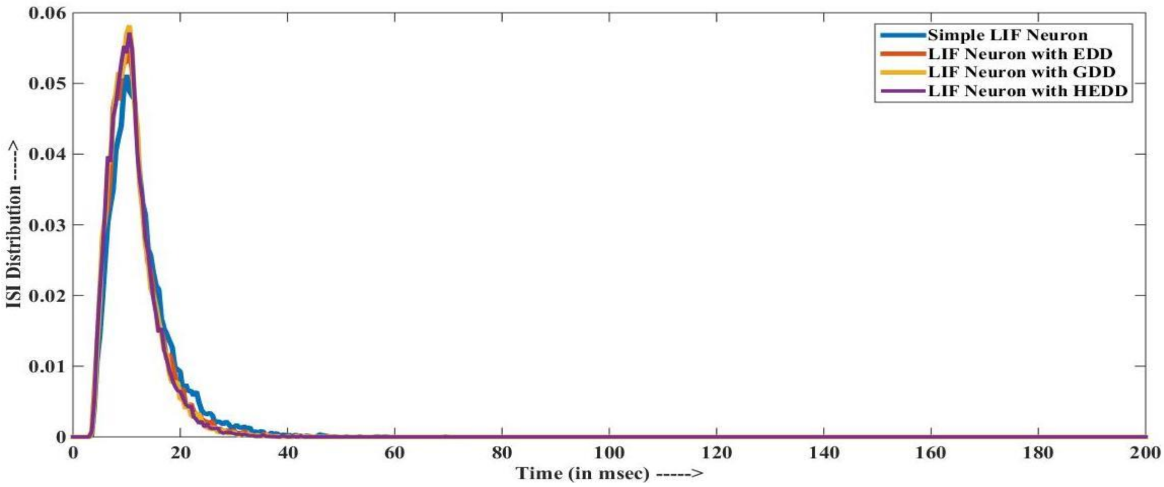

Fig. 12: ISI distribution with Gaussian distributed rtp

Fig. 11 shows the ISI distribution patterns for neuron model for comparatively more noisy input stimulus. This noise is generating the long tail behavior in ISI distribution whereas Gaussian distributed rtp is producing the similarpattern in ISI distributions. Fig. 12 illustrates the similar qualitative and quantitative behavior in ISI distribution patterns for studied neuron models. This study suggests the parameter values and refractory time period for which LIF neuron exhibits zero on its spiking behavior in distributed delay framework.

VI. CONCLUSION AND FUTURE SCOPE

LIF model is the first choice to for implementing ANN at hardware level and ANN at software level. This model has a threshold value for firing purpose. This model is widely used to analytically study the neuronal behavior as the easiness of the model. LIF neuron model in DDF is comparatively more realistic to biological neuron as compared with LIF neuron with stochastically driven input. The neuronal information processing mechanism and spiking behavior of LIF neuron model in presence of refractory time is investigated in DDF. The study is extended byincorporating three different kinds of kernel function, namely, exponentially, gamma and hypo-exponentially distributed kernel function and two different kind of refractory time period, namely, uniformly distributed and Gaussian distributed time period have used. The obtained results are compared with no refractory time period results. We notice that the Gaussian distributed refractory time period with hypo-exponentially distributed delay kernel function has ISI distribution patterns is similar with the experimental studies. We also find the combination of various parameter values in aforementioned situation where DDF has no effect on the evolution of the membrane potential and neuronal information processing.Thesefinding may be extended for other neuronal information mechanism and at ANN implementation level.

REFERENCES

[1] Abbott, L. F. and Dayan, P. "*Theoretical Neuroscience: Computational and mathematical modeling of neural systems*",The MIT press, 2001.

[2] Burkitt, A. N. "*A Review of the Integrate-and-Fire Neuron Model: I. Homogeneous Synaptic Input", Biological Cybernetics*, Vol. 95 , No.1, pp.1-19, 2006.

[3] Burkitt, A. N. "*A Review of the Integrate-and-Fire Neuron Model: II. Inhomogeneous synaptic Input and Network Properties*", Biological Cybernetics, Vol. 95, No. 2, pp. 97-112, 2006.

[4] Choudhary, S. K., Singh, K. and Bharti, S. K. "*Variability in Spiking Pattern of Leaky Integrate-and-Fire Neuron Due to Excitatory and Inhibitory Potentials*", 2nd International Conference on "Computing for Sustainable Global Development", pp. 2025-2030, March 2015.

[5] Choudhary, S. K. and Bharti, S. K., "Information Processing in Neuron with Exponential Distributed Delay*",International Journal of Machine Learning and Networked Collaborative Engineering,* Vol. 02 No. 02, 2018,pp 58-66. doi: https://doi.org/10.30991/IJMLNCE.2018v02i02.003.

[6] Choudhary, S. K. and Solanki, V. K. "*LIF Neuron with Hypo-exponential Distributed Delay: Emergence of Unimodal, Bimodal, Multi-Model ISI Distribution with long tail*" in Recent Patent on Engineering, Vol. 4, Issue, 4, 2020.

[7] Choudhary, S. K. and Solanki., V. K. "*Spiking Activity of a LIF Neuron in Distributed Delay Framework*," International Journal of Interactive Multimedia and Artificial Intelligence, Vol. 3, No. 7, pp. 70-76, June 2016.

[8] Destexhe, A. and Lilith, M. R. "*Neuronal Noise*," Springer Series in Computational Neuroscience, 2012.

[9] Gerstner, W. and Kistler, W. M. "*Spiking Neuron Models: Single Neurons, Populations, Plasticity*," Cambridge University Press, 2002.

[10] Gabbiani, F. and Koch, C. "*Principles of Spike Train Analysis. In: Koch C, Segev* I (eds) Methods in Neuronal Modeling: From Ions to Networks," MIT Press, Cambridge, 1998.

[11] Goris, R. L. T., Movshon, A. and Simoncelli, E. P. "*Partitioning Neuronal Variability*", Nature Neuroscience, Vol. 17, No. 6, pp. 858-867, 2014.

[12] Kloeden, P. E. and Platen, E, "*Numerical Solution of Stochastic Differential Equations*," Springer, Berlin, 1992.

[13] Koch, C. "*Biophysics of Computation: Information Processing in Single Neurons"*, Oxford University Press, 1998.

[14] Mansfield, M. L. "*Numerical tools for obtaining the power-law representations of heavy-tailed datasets*", The European Physical Journal B, 89:16 (2016) 1-13.

[15] McDonnell, M. D., Ikeda, S. and Manton, J. H. "*An Introductory Review of Information Theory in the Context of Computational Neuroscience*", Biological Cybernetics, Vol. 105, pp. 55-70, 2011.

[16] Richardosn, M. J. and Gerstener, W. "*Synaptic Shot Noise and Conductance Fluctuations Affect the Membrane Voltage with Equal Significance*", Neural Computation, Vol. 17, No. 4, pp. 923-947. 2005.

[17] Ross, S. M. "*Introduction to Probability Models*," 9th Edition, Academic Press, 2007.

[18] Stevens, C. F. and Zador, A. "*Information through a Spiking Neuron"*, Advances in Neural Information Processing Systems, pp. 75-81, MIT Press. 1996.

[19] Tuckwell, H. C. "*Introduction to Theoretical Neurobiology: Volume 2 Non Linear and Stochastic Theories*," Cambridge University Press, 1988.

[20] Chouhan, A.S. "A Analytical Study of Leaky Integrate-and-Fire Neuron Model Using Matlab Simulation", International Journal of Engineering Research & Technology, Vol. 2, Issue 4, pp. 2242-2245 April 2013.

[21] Telenczuk, B. Kempter, R. Curio, G. and Destexhe, A., "Refractoriness Accounts for Variable Spike Burst Responses in Somatosensory Cortex", eNeuro, Vol. 4, Issue 4, pp. 1-17 August 2017.

[22] Levakova, M. Kostal, L. Monsempes, C. Lucas, P. and Kobayashi, P., "Adaptive integrate-and-fire model reproduces the dynamics of olfactory receptor neuron responses in a moth", Journal Royal Society Interface, Vol. 16, pp. 1-10, 2019.

[23] Teeter, C. Iyer, R. Menon, V. Gouwens, N. Feng, D. Berg, J. Szafer, A. Cain, N. Zeng, H. Hawrylycz, M. Koch, C. and Mihalas, S., "Generalized leaky integrate-and-fire models classify multiple neuron types", Nature Communications, Vol. 9:709, pp. 1-15, 2019.

[24] Karmeshu, V. Gupta and K. V. Kadambari, "*Neuronal Model with Distributed Delay: Analysis and Simulation Study for Gamma Distribution Memory Kernel*," Biological Cybernetics, Vol. 104, pp. 369-383, 2011.

[25] Sharma, S. K. and Karmeshu, "*Neuronal Model With Distributed Delay: Emergence of Unimodal and Bimodal ISI Distributions*," IEEE Transactions on Nanobiosciences, Vol. 12, No. 1 pp. 1-12, 2013.

[26] Baghela, V. D .S., Choudhary, S. K. and Bharti, S. K., "*Effect of Refractory Time period on Information Processing of a Leaky Integrate-and-Fire Neuron in Distributed Delay Framework*", Advances in Mathematics: Scientific Journal, Vol. 9, No. 9, pp. 6991-6998.