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INVENTORY MODEL WITH TRUNCATED WEIBULL DECAY UNDER PERMISSIBLE DELAY IN PAYMENTS AND INFLATION HAVING BOTH TIME AND SELLING PRICE DEPENDENT DEMAND

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Abstract

The prime disserta of inventory modelling is developing suitable postulates for the model charactering the practical situation closer to the reality. This paper develops an EOQ model with the assumption that the life time of commodity follows a truncated Weibull probability distribution and delay in payments are permitted under inflation. It is further considered that the demand is a function of both time and selling price. With suitable postulates on the demand, deterioration, replenishment and inflation the instantaneous level of inventory at any given time is derived. The optimal ordering and pricing policies of the model are obtained through sensitivity analysis. The effect of changes in cost and parameters on optimal ordering policies is discussed. It is observed that the delay in decay characterized by truncated Weibull distribution has significant influence on ordering quantity and optimal selling price. This model includes decays of increasing/decreasing/constant rates of decay. It is further observed the inflation rate significantly effect the optimal policies of the model. Allowing permissible delay in payments can influence the policies of the inventory system. This model also includes some of the earlier models as special/particular cases.

Keywords: Truncated Weibull distribution, time and selling price dependent demand, EOQ model, rate of inflation, sensitivity analysis.

1. INTRODUCTION

One of the major considerations for the inventory control and management is life time of the commodity. Several authors have focused on developing the inventory models with different assumptions on life time of the commodity K Srinivasa Rao et al (2017). The literature review of mathematical model for storage with random life time of commodity are presented in the papers of Namhias S (1982), Raafat (1991), Goyal and Giri (2001), Raxaein le et al (2010). Recently Eswara Rao et al (2015), Madhulatha et al (2017), Nagendra et al (2018) Srinivasa Rao et al (2020) and others have invented and presented several inventory models with the consideration that the life of the commodity is random and follows a specific probability distribution depending on the nature of the commodity.

However, in all these papers they assumed the deterioration starts immediately that is the inception of the commodity. The deterioration has a significant influence on the life time of the commodity. Hence it is needed to characterize with a suitable probability distribution for the life time of the commodity. Otherwise, the model gives falsification in predicting the performance of the system and scheduling the orders of inventory optimally. For example, in many products the deterioration of the commodity starts only after a specified period of time but not immediately of its inception. The delay in decay can be well characterized by the truncated probability distribution. It is also to be observed that the rate of decay may be constant, increasing or decreasing. This variable rate of delayed decay can be effetely modeled by a truncated Weibull probability distribution. Another basic assumption in inventory model is regarding the payments. Most of the papers consider that the supplier will get the payment immediately after fulfilling the order. However, in this competitive business environment the supplier permits a fixed period of time for settlements of the accounts without changing any interest. This type of inventory models is known as models for EOQ with permissible delay in payments. Some work has been reported in literature with respect to EOQ models with delay in payments, Sarker (2000), Chang and Liao (2004) and Ouyang (2005). However, they assume the money value remains constant over time. But in reality, money value changes over time. Ignoring inflation may not provide accurate modeling in inventory models Srinivasa Rao et al (2015). Another important consideration in inventory modeling is regarding the nature of demand. Recently, Amulya et al (2021) have developed an inventory model with truncated Weibull decay under permissible delay in payments and inflation having time dependent demand. But for many commodities the demand is not only a function of time but also depends up on the selling price. If the selling price is more then the demand is less and selling price is less the demand is more. Hence to analyze the inventory system associated with market yards such as textiles, edible oils it is needed to develop an inventory model with truncated Weibull decay having both time and selling price dependent demand under permissible delay in payments with inflation. This paper addresses the gap in the area of research and provide an efficient model.

The rest of the papers is formulated as follows:

Section 2 delas with the assumptions and notations of the model. Section 3 deals with the differential equations governing the inventory system and its solution. Section 4 deals with derivation of optimal ordering and pricing policies of the modeling. Section 5 deals with the numerical illustration and sensitivity analysis. Section 6 is for summarizing the results with the conclusion.

2. ASSUMPTIONS

For developing the Economic Order Quantity model, the following assumptions are made

i. Deterioration start time is γ.

ii. Weibull distribution is the life time distribution of the commodity. Its p.d.f is

$$f(t) = \alpha\beta(t-\gamma)^{\beta-1}e^{-\alpha(t-\gamma)^{\beta}}$$

where α is the scale parameter, β is the shape parameter and γ is the location parameter.

The instantaneous deterioration rate is

$$h(t) = \alpha(t-\gamma)^{\beta}\ ,\ t \geq \gamma$$

iii. Demand function is

$$R(p(t)) = a + b_1\left[\frac{-\theta t^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right] - b_2(t) = a + b_1\left[\frac{-\theta t^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right] - b_2 p e^{rt},$$

which is selling price dependent demand.

Where, a is the fixed demand, $a > 0$, b_1 is the demand parameter, $b_2 > 0$, and $p(t)$ is the selling price of an item at time t and p is the selling price of the item at time $t = 0$.

iv. Rate of inflation is $r,\ 0 < r < 1$

v. Allowing shortages is not permitted.

vi. Zero lead time.

vii. During the permissible delay period (M), the account is not settled, the generated sales revenue is deposited in an interest-bearing account. At the end of the trade credit period, the customer pays off for all the units ordered.

viii. There is no repair or replacement of the deteriorated units during the cycle time.

NOTATION

H : Finite horizon length.

$R(p(t))$: Demand per unit time as a function of selling price.

h : Holding costof inventory per unit time after excluding interest.

r : Rate of inflation.

$p(t) = pe^{rt}$: Per unit selling price.

$g(t) = ge^{rt}$: Purchase cost of a unit at time t.

$A(t) = Ae^{rt}$: Per order cost at time t.

I_C : Interest charged per Rs. INR in stock per a year by the supplier.

I_e : Interest earned in Rs. INR per a year.

M : Permissible delay period which is allowed in settling the account.

Q : Order quantity per a cycle.

T : Cycle length

$I(t)$: On-hand inventory at time $t, 0 \leq t \leq T$.

$TC(p,T)$: Total cost over $(0, H)$.

$NP(p,T)$: Net profit rate function over planning period.

3. INVENTORY MODEL

Let Q be the inventory level of the system at time $t = 0$. During $(0, \gamma)$ inventory will decrease due to demand and during (γ, T) inventory will decrease due to demand and deterioration. Since no shortages are allowed, at time T the inventory level reaches zero, the stock is replenished instantaneously. The schematic diagram representing the inventory level is shown in Figure-3.1.

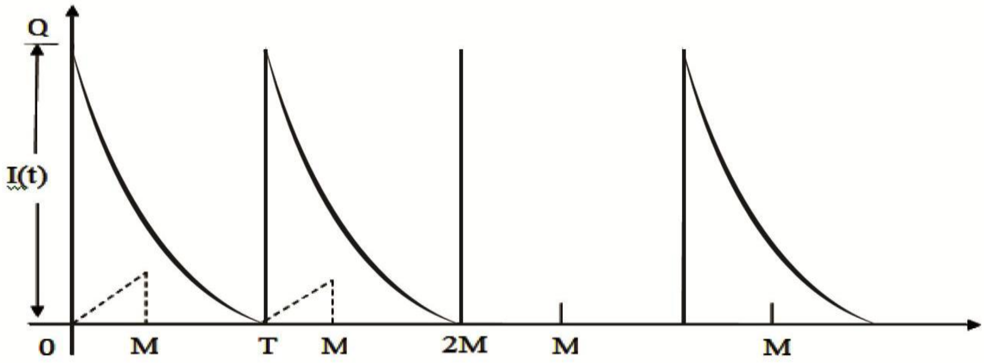


Figure -3.1: Schematic diagram representing the inventory level of both time and selling price dependent demand model

Let $I(t)$ be the on-hand inventory at time t. The differential equations governing the on-hand inventory at time t are

$$\frac{d}{dt}I(t) = -R(p,t) \qquad 0 \le t \le \gamma \qquad (1)$$

$$\frac{d}{dt}I(t) + h(t)I(t) = -R(p,t) \qquad \gamma \le t \le T \qquad (2)$$

where $h(t) = \alpha\beta(t-\gamma)^{\beta-1} \qquad \gamma \le t \le T$

and $R(p,t) = a + b_1\left[\frac{-\theta t^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right] - b_2 p(t) = a + b_1\left[\frac{-\theta t^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right] - b_2 p e^{rt}$

with initial conditions $I(0) = Q \; and \; I(T) = 0$.

Solving equation (1) and using the initial condition $I(0) = Q$, we get

$$I(t) = Q - at + \frac{b_1\theta}{T^{\frac{1}{n}}}t^{\frac{1}{n}} + \frac{b_2 p}{r}(e^{rt} - 1) \qquad 0 \le t \le \gamma \qquad (3)$$

Solving equation (2) and using the initial condition $I(T) = 0$, we get

$$I(t) = e^{-\alpha(t-\gamma)^\beta}\left[a\int_t^T e^{\alpha(u-\gamma)^\beta}du - \frac{b_1\theta}{nT^{\frac{1}{n}}}\int_t^T u^{\frac{1}{n}-1}e^{\alpha(u-\gamma)^\beta}du - b_2 p\int_t^T e^{ru+\alpha(u-\gamma)^\beta}du\right]$$

$$\gamma \le t \le T \qquad (4)$$

Equating equations (3) and (4) when $t = \gamma$, we get

$$Q = a\gamma - \frac{b_1\theta}{T^{\frac{1}{n}}}\gamma^{\frac{1}{n}} - \frac{b_2 p}{r}(e^{r\gamma} - 1) + a\int_\gamma^T e^{\alpha(u-\gamma)^\beta}du$$

$$-\frac{b_1\theta}{nT^{\frac{1}{n}}}\int_\gamma^T u^{\frac{1}{n}-1}e^{\alpha(u-\gamma)^\beta}du - b_2 p\int_\gamma^T e^{ru+\alpha(u-\gamma)^\beta}du \qquad (5)$$

Substituting Q in equation (3), we get

$$I(t) = a(\gamma - t) + \frac{b_1\theta}{T^{\frac{1}{n}}}\left(t^{\frac{1}{n}} - \gamma^{\frac{1}{n}}\right) + \frac{b_2 p}{r}(e^{rt} - e^{r\gamma}) + a\int_\gamma^T e^{\alpha(u-\gamma)^\beta}du$$

$$-\frac{b_1\theta}{nT^{\frac{1}{n}}}\int_\gamma^T u^{\frac{1}{n}-1}e^{\alpha(u-\gamma)^\beta}du - b_2 p\int_\gamma^T e^{ru+\alpha(u-\gamma)^\beta}du \qquad 0 \le t \le \gamma \qquad (6)$$

Since the length of time intervals are all the same, we have

$I(jT + t) =$

$$
\begin{cases} a(\gamma - t) + \frac{b_1\theta}{T^{\frac{1}{n}}}\left(t^{\frac{1}{n}} - \gamma^{\frac{1}{n}}\right) + \frac{b_2 p}{r}(e^{rt} - e^{r\gamma}) + a\int_{\gamma}^{T} e^{\alpha(u-\gamma)^{\beta}} du \\ -\frac{b_1\theta}{nT^{\frac{1}{n}}}\int_{\gamma}^{T} u^{\frac{1}{n}-1} e^{\alpha(u-\gamma)^{\beta}} du - b_2 p\int_{\gamma}^{T} e^{ru+\alpha(u-\gamma)^{\beta}} du & 0 \le t \le \gamma \\ e^{-\alpha(t-\gamma)^{\beta}}\left[a\int_{t}^{T} e^{\alpha(u-\gamma)^{\beta}} du - \frac{b_1\theta}{nT^{\frac{1}{n}}}\int_{t}^{T} u^{\frac{1}{n}-1} e^{\alpha(u-\gamma)^{\beta}} du - b_2 p\int_{t}^{T} e^{ru+\alpha(u-\gamma)^{\beta}} du\right] & \gamma \le t \le T \end{cases} \tag{7}
$$

4. THE OPTIMAL ORDERING AND PRICING POLICIES

Total cost function is the sum of Ordering Cost (OC), Cost Deterioration (CD), Inventory Carrying Cost (ICC), Interest Charged (IC_1) and Interest Earned (IE_1).

Each cost component is computed as follows:

Ordering Cost, OC is

$$
OC = A(0) + A(T) + A(2T) + \ldots + A(n-1)T) = A\left[\frac{e^{rH} - 1}{e^{rT} - 1}\right] \tag{8}
$$

Cost Deterioration, CD is

$$
CD = \sum_{j=0}^{n-1} g e^{rjT}\left[Q - \int_{0}^{T}\left[a + b_1\left(\frac{-\theta t^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right) - b_2 p e^{rt}\right] dt\right]
$$

where, Q is as given in equation (5).

On simplification, we get

$$
CD = g\left[a(\gamma - T) - \frac{b_1\theta}{T^{\frac{1}{n}}}\gamma^{\frac{1}{n}} + b_1\theta + \frac{b_2 p}{r}(e^{rT} - e^{r\gamma}) + a\int_{\gamma}^{T} e^{\alpha(u-\gamma)^{\beta}} du \right.
$$
$$
\left. - \frac{b_1\theta}{nT^{\frac{1}{n}}}\int_{\gamma}^{T} u^{\frac{1}{n}-1} e^{\alpha(u-\gamma)^{\beta}} du - b_2 p\int_{\gamma}^{T} e^{ru+\alpha(u-\gamma)^{\beta}} du\right]\left[\frac{e^{rH} - 1}{e^{rT} - 1}\right] \tag{9}
$$

Inventory Carrying Cost, ICC is

$$
ICC = h\sum_{j=0}^{n-1} g(jT)\left[\int_{0}^{T} I(jT + t) dt\right]
$$
$$
= hg\left[\frac{a\gamma^2}{2} - \frac{b_1\theta}{(n+1)T^{\frac{1}{n}}}\gamma^{\frac{1}{n}+1} + \frac{b_2 p}{r^2}[e^{r\gamma}(1 - r\gamma) - 1]\right.
$$
$$
+ \gamma\left[a\int_{\gamma}^{T} e^{\alpha(u-\gamma)^{\beta}} du - \frac{b_1\theta}{nT^{\frac{1}{n}}}\int_{\gamma}^{T} u^{\frac{1}{n}-1} e^{\alpha(u-\gamma)^{\beta}} du - b_2 p\int_{\gamma}^{T} e^{ru+\alpha(u-\gamma)^{\beta}} du\right]
$$
$$
\left. + \int_{\gamma}^{T} e^{-\alpha(t-\gamma)^{\beta}}\left[a\int_{t}^{T} e^{\alpha(u-\gamma)^{\beta}} du - \frac{b_1\theta}{nT^{\frac{1}{n}}}\int_{t}^{T} u^{\frac{1}{n}-1} e^{\alpha(u-\gamma)^{\beta}} du - b_2 p\int_{t}^{T} e^{ru+\alpha(u-\gamma)^{\beta}} du\right] dt\right]
$$
$$
\times\left[\frac{e^{rH} - 1}{e^{rT} - 1}\right] \tag{10}
$$

For computing interest charged and earned, there are two possibilities based on the customer's choice. Interest Charges (IC) for unsold items at the initial time or after the permissible delay period M and interest Earned (IE) from the sales revenue during the permissible delay period.

Case (i): Optimum cycle length T is larger than or equal to M i.e., $T \ge M$

Interest Charged in $(0, H)$, IC_1 is

$$IC_1 = I_c \sum_{j=0}^{n-1} g(jT) \left[\int_M^T I(jT+t)dt \right]$$

$$= I_c g \left[\frac{a}{2}(\gamma^2 + M^2 - 2M\gamma) + \frac{b_1\theta}{T^{\frac{1}{n}}} \left[\frac{n}{n+1} \left(\gamma^{\frac{1}{n}+1} - M^{\frac{1}{n}+1} \right) - \gamma^{\frac{1}{n}}(\gamma - M) \right] \right.$$

$$+ \frac{b_2 p}{r^2} \left[e^{r\gamma}\big(1 - r(\gamma - M)\big) - e^{rM} \right]$$

$$+(\gamma - M) \left[a \int_\gamma^T e^{\alpha(u-\gamma)^\beta} du - \frac{b_1\theta}{nT^{\frac{1}{n}}} \int_\gamma^T u^{\frac{1}{n}-1} e^{\alpha(u-\gamma)^\beta} du - b_2 p \int_\gamma^T e^{ru+\alpha(u-\gamma)^\beta} du \right]$$

$$\left. + \int_\gamma^T e^{-\alpha(t-\gamma)^\beta} \left[a \int_t^T e^{\alpha(u-\gamma)^\beta} du - \frac{b_1\theta}{nT^{\frac{1}{n}}} \int_t^T u^{\frac{1}{n}-1} e^{\alpha(u-\gamma)^\beta} du - b_2 p \int_t^T e^{ru+\alpha(u-\gamma)^\beta} du \right] dt \right]$$

$$\times \left[\frac{e^{rH} - 1}{e^{rT} - 1} \right] \qquad (11)$$

Interest Earned in $(0, H)$, IE_1 is

$$IE_1 = I_e \sum_{j=0}^{n-1} p(jT) \left[\int_0^M \left[a + b_1 \left(\frac{-\theta t^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}} \right) - b_2 p e^{rt} \right] t dt \right]$$

$$= I_e p \left[\frac{aM^2}{2} - \frac{b_1\theta}{(n+1)T^{\frac{1}{n}}} M^{\frac{1}{n}+1} - \frac{b_2 p}{r^2} \left[e^{rM}(rM - 1) + 1 \right] \right] \left[\frac{e^{rH} - 1}{e^{rT} - 1} \right] \qquad (12)$$

The total cost over $(0, H)$ is $TC(p, T)$ and is given by

$$TC(p,T) = OC + CD + ICC + IC_1 - IE_1 \qquad (13)$$

Substituting equations (8), (9), (10), (11) and (12) in (13), we get

$$TC(p,T) = \left[A + g \left[a(\gamma - T) - \frac{b_1\theta}{T^{\frac{1}{n}}} \gamma^{\frac{1}{n}} + b_1\theta + \frac{b_2 p}{r}(e^{rT} - e^{r\gamma}) + a \int_\gamma^T e^{\alpha(u-\gamma)^\beta} du \right. \right.$$

$$\left. - \frac{b_1\theta}{nT^{\frac{1}{n}}} \int_\gamma^T u^{\frac{1}{n}-1} e^{\alpha(u-\gamma)^\beta} du - b_2 p \int_\gamma^T e^{ru+\alpha(u-\gamma)^\beta} du \right]$$

$$+hg \left[\frac{a\gamma^2}{2} - \frac{b_1\theta}{(n+1)T^{\frac{1}{n}}} \gamma^{\frac{1}{n}+1} + \frac{b_2 p}{r^2} \left[e^{r\gamma}(1 - r\gamma) - 1 \right] \right.$$

$$+ \gamma \left[a \int_\gamma^T e^{\alpha(u-\gamma)^\beta} du - \frac{b_1\theta}{nT^{\frac{1}{n}}} \int_\gamma^T u^{\frac{1}{n}-1} e^{\alpha(u-\gamma)^\beta} du - b_2 p \int_\gamma^T e^{ru+\alpha(u-\gamma)^\beta} du \right]$$

$$\left. + \int_\gamma^T e^{-\alpha(t-\gamma)^\beta} \left[a \int_t^T e^{\alpha(u-\gamma)^\beta} du - \frac{b_1\theta}{nT^{\frac{1}{n}}} \int_t^T u^{\frac{1}{n}-1} e^{\alpha(u-\gamma)^\beta} du - b_2 p \int_t^T e^{ru+\alpha(u-\gamma)^\beta} du \right] dt \right]$$

$$+I_c g \left[\frac{a}{2}(\gamma^2 + M^2 - 2M\gamma) + \frac{b_1\theta}{T^{\frac{1}{n}}} \left[\frac{n}{n+1} \left(\gamma^{\frac{1}{n}+1} - M^{\frac{1}{n}+1} \right) - \gamma^{\frac{1}{n}}(\gamma - M) \right] \right.$$

$$+ \frac{b_2 p}{r^2} \left[e^{r\gamma}\big(1 - r(\gamma - M)\big) - e^{rM} \right]$$

$$+(\gamma - M) \left[a \int_\gamma^T e^{\alpha(u-\gamma)^\beta} du - \frac{b_1\theta}{nT^{\frac{1}{n}}} \int_\gamma^T u^{\frac{1}{n}-1} e^{\alpha(u-\gamma)^\beta} du - b_2 p \int_\gamma^T e^{ru+\alpha(u-\gamma)^\beta} du \right]$$

$$+\int_{\gamma}^{T} e^{-\alpha(t-\gamma)^{\beta}}\left[a\int_{t}^{T} e^{\alpha(u-\gamma)^{\beta}}du-\frac{b_1\theta}{nT^{\frac{1}{n}}}\int_{t}^{T} u^{\frac{1}{n}-1}e^{\alpha(u-\gamma)^{\beta}}du-b_2p\int_{t}^{T} e^{ru+\alpha(u-\gamma)^{\beta}}du\right]dt\Bigg]$$

$$-I_e p\left[\frac{aM^2}{2}-\frac{b_1\theta}{(n+1)T^{\frac{1}{n}}}M^{\frac{1}{n}+1}-\frac{b_2p}{r^2}\left[e^{rM}(rM-1)+1\right]\right]\Bigg]\left[\frac{e^{rH}-1}{e^{rT}-1}\right] \quad (14)$$

The net profit is the difference of gross revenue and total cost.

The gross revenue is $(pe^{rT}-ge^{rT})\left[a-\frac{b_1\theta}{nT}-b_2pe^{rT}\right]$

Hence, the net profit is

$$NP(p,T)=(pe^{rT}-ge^{rT})\left[a-\frac{b_1\theta}{nT}-b_2pe^{rT}\right]-TC(p,T) \quad (15)$$

where, $TC(p,T)$ is as given in (14)

For obtaining the optimal policies of the model, maximize $NP(p,T)$ with respect to T and p. The conditions for obtaining optimality are

$$\frac{\partial NP(p,T)}{\partial T}=0,\frac{\partial NP(p,T)}{\partial p}=0 \quad and \quad D=\begin{vmatrix}\frac{\partial^2 NP(p,T)}{\partial p^2} & \frac{\partial^2 NP(p,T)}{\partial T\partial p}\\ \frac{\partial^2 NP(p,T)}{\partial T\partial p} & \frac{\partial^2 NP(p,T)}{\partial T^2}\end{vmatrix}<0$$

where D is the determinant of Hessian matrix

$\frac{\partial NP(p,T)}{\partial T}=0,$

$$(pe^{rT}-ge^{rT})\left[\left[\frac{b_1\theta}{n^2T^{\frac{1}{n}+1}}-b_2pe^{rT}\right]+\left[a-\frac{b_1\theta}{nT^{\frac{1}{n}}}-b_2pe^{rT}\right]r\right]-\left\{\left[\frac{e^{rH}-1}{e^{rT}-1}\right]\right.$$

$$\left[g\left[-a-\frac{b_1\theta}{nT^{\frac{1}{n}+1}}\gamma^{\frac{1}{n}}+b_2pe^{rT}+ae^{\alpha(T-\gamma)^{\beta}}-\frac{b_1\theta}{nT}e^{\alpha(T-\gamma)^{\beta}}-b_2pe^{rT+\alpha(T-\gamma)^{\beta}}\right]\right.$$

$$+hg\left[\frac{b_1\theta}{nT}\frac{\gamma^{\frac{1}{n}+1}}{n+1}+\gamma\left[ae^{\alpha(T-\gamma)^{\beta}}-\frac{b_1\theta}{nT}e^{\alpha(T-\gamma)^{\beta}}-b_2pe^{rT+\alpha(T-\gamma)^{\beta}}\right]\right.$$

$$\left.+\int_{\gamma}^{T} e^{-\alpha(t-\gamma)^{\beta}}\left[ae^{\alpha(T-\gamma)^{\beta}}-\frac{b_1\theta}{nT}e^{\alpha(T-\gamma)^{\beta}}-b_2pe^{rT+\alpha(T-\gamma)^{\beta}}\right]dt\right]$$

$$+I_c g\left[\frac{b_1\theta}{nT^{\frac{1}{n}+1}}\left[\frac{n}{n+1}\left(\gamma^{\frac{1}{n}+1}-M^{\frac{1}{n}+1}\right)-\gamma^{\frac{1}{n}}(\gamma-M)\right]\right.$$

$$+(\gamma-M)\left[ae^{\alpha(T-\gamma)^{\beta}}-\frac{b_1\theta}{nT}e^{\alpha(T-\gamma)^{\beta}}-b_2pe^{rT+\alpha(T-\gamma)^{\beta}}\right]$$

$$\left.+\int_{\gamma}^{T} e^{-\alpha(t-\gamma)^{\beta}}\left[ae^{\alpha(T-\gamma)^{\beta}}-\frac{b_1\theta}{nT}e^{\alpha(T-\gamma)^{\beta}}-b_2pe^{rT+\alpha(T-\gamma)^{\beta}}\right]dt\right]+I_e p\left[\frac{b_1\theta}{nT^{\frac{1}{n}+1}}\frac{M^{\frac{1}{n}+1}}{n+1}\right]\Bigg]$$

$$+\left[A+g\left[a(\gamma-T)-\frac{b_1\theta}{T^{\frac{1}{n}}}\gamma^{\frac{1}{n}}+b_1\theta+\frac{b_2p}{r}(e^{rT}-e^{r\gamma})+a\int_{\gamma}^{T} e^{\alpha(u-\gamma)^{\beta}}du\right.\right.$$

$$\left.-\frac{b_1\theta}{nT^{\frac{1}{n}}}\int_{\gamma}^{T} u^{\frac{1}{n}-1}e^{\alpha(u-\gamma)^{\beta}}du-b_2p\int_{\gamma}^{T} e^{ru+\alpha(u-\gamma)^{\beta}}du\right]$$

$$+hg\left[\frac{a\gamma^2}{2}-\frac{b_1\theta}{(n+1)T^{\frac{1}{n}}}\gamma^{\frac{1}{n}+1}+\frac{b_2p}{r^2}\left[e^{r\gamma}(1-r\gamma)-1\right]\right.$$

$$+\gamma\left[a\int_{\gamma}^{T} e^{\alpha(u-\gamma)^{\beta}}du-\frac{b_1\theta}{nT^{\frac{1}{n}}}\int_{\gamma}^{T} u^{\frac{1}{n}-1}e^{\alpha(u-\gamma)^{\beta}}du-b_2p\int_{\gamma}^{T} e^{ru+\alpha(u-\gamma)^{\beta}}du\right]$$

$$+\int_{\gamma}^{T} e^{-\alpha(t-\gamma)^{\beta}}\left[a\int_{t}^{T} e^{\alpha(u-\gamma)^{\beta}}du-\frac{b_1\theta}{nT^{\frac{1}{n}}}\int_{t}^{T} u^{\frac{1}{n}-1}e^{\alpha(u-\gamma)^{\beta}}du-b_2p\int_{t}^{T} e^{ru+\alpha(u-\gamma)^{\beta}}du\right]dt\Bigg]$$

$$+I_cg\left[\frac{a}{2}(\gamma^2+M^2-2M\gamma)+\frac{b_1\theta}{T^{\frac{1}{n}}}\left[\frac{n}{n+1}\left(\gamma^{\frac{1}{n}+1}-M^{\frac{1}{n}+1}\right)-\gamma^{\frac{1}{n}}(\gamma-M)\right]\right.$$

$$+\frac{b_2p}{r^2}\left[e^{r\gamma}\left(1-r(\gamma-M)\right)-e^{rM}\right]$$

$$+(\gamma-M)\left[a\int_{\gamma}^{T} e^{\alpha(u-\gamma)^{\beta}}du-\frac{b_1\theta}{nT^{\frac{1}{n}}}\int_{\gamma}^{T} u^{\frac{1}{n}-1}e^{\alpha(u-\gamma)^{\beta}}du-b_2p\int_{\gamma}^{T} e^{ru+\alpha(u-\gamma)^{\beta}}du\right]$$

$$+\int_{\gamma}^{T} e^{-\alpha(t-\gamma)^{\beta}}\left[a\int_{t}^{T} e^{\alpha(u-\gamma)^{\beta}}du-\frac{b_1\theta}{nT^{\frac{1}{n}}}\int_{t}^{T} u^{\frac{1}{n}-1}e^{\alpha(u-\gamma)^{\beta}}du-b_2p\int_{t}^{T} e^{ru+\alpha(u-\gamma)^{\beta}}du\right]dt\Bigg]$$

$$-I_ep\left[\frac{aM^2}{2}-\frac{b_1\theta}{(n+1)T^{\frac{1}{n}}}M^{\frac{1}{n}+1}-\frac{bp}{r^2}\left[e^{rM}(rM-1)+1\right]\right]\Bigg]\left[\frac{e^{rH}-1}{(e^{rT}-1)^2}\right]re^{rT}\Bigg\}=0 \quad (16)$$

$\frac{\partial NP(p,T)}{\partial p}=0$ implies,

$$e^{rT}\left(a-\frac{b_1\theta}{nT}-2pb_2e^{rT}+gb_2e^{rT}\right)-\Bigg\{g\left[\frac{b_2}{r}(e^{rT}-e^{r\gamma})-b_2\int_{\gamma}^{T} e^{ru+\alpha(u-\gamma)^{\beta}}du\right]$$

$$+hg\left[\frac{b_2}{r^2}\left[e^{r\gamma}(1-r\gamma)-1\right]-b_2\gamma\left[\int_{\gamma}^{T} e^{ru+\alpha(u-\gamma)^{\beta}}du\right]\right.$$

$$\left.-b_2\int_{\gamma}^{T} e^{-\alpha(t-\gamma)^{\beta}}\left[\int_{t}^{T} e^{ru+\alpha(u-\gamma)^{\beta}}du\right]dt\right]+I_cg\left[\frac{b_2}{r^2}\left[e^{r\gamma}\left(1-r(\gamma-M)\right)-e^{rM}\right]\right.$$

$$\left.-b_2(\gamma-M)\left[\int_{\gamma}^{T} e^{ru+\alpha(u-\gamma)^{\beta}}du\right]-b_2\int_{\gamma}^{T} e^{-\alpha(t-\gamma)^{\beta}}\left[\int_{t}^{T} e^{ru+\alpha(u-\gamma)^{\beta}}du\right]dt\right]$$

$$-I_e\left[\frac{aM^2}{2}-\frac{b_1\theta}{(n+1)T^{\frac{1}{n}}}M^{\frac{1}{n}+1}-\frac{2bp}{r^2}\left[e^{rM}(rM-1)+1\right]\right]\Bigg\}\left[\frac{e^{rH}-1}{e^{rT}-1}\right]=0 \quad (17)$$

For given values of the parameters and costs, equations (16) and (17) are solved using MATHCAD to get the optimal cycle length $T=T_1$ and selling price $p=p_1$. Substituting the optimal values T_1 and p_1 in equation (14) we get the minimum total cost. Substituting this minimum total cost, T_1 and p_1 in equation (15), we get the maximum profit as

$$NP^*(p_1,T_1)=(p_1e^{rT_1}-ge^{rT_1})\left[a-\frac{b_1\theta}{nT_1}-b_2p_1e^{rT_1}\right]$$

$$-\Bigg[A+g\left[a(\gamma-T_1)-\frac{b_1\theta}{T_1^{\frac{1}{n}}}\gamma^{\frac{1}{n}}+b_1\theta+\frac{b_2p_1}{r}(e^{rT_1}-e^{r\gamma})+a\int_{\gamma}^{T_1} e^{\alpha(u-\gamma)^{\beta}}du\right.$$

$$\left.-\frac{b_1\theta}{nT_1^{\frac{1}{n}}}\int_{\gamma}^{T_1} u^{\frac{1}{n}-1}e^{\alpha(u-\gamma)^{\beta}}du-b_2p_1\int_{\gamma}^{T_1} e^{ru+\alpha(u-\gamma)^{\beta}}du\right]$$

$$+hg\left[\frac{a\gamma^2}{2}-\frac{b_1\theta}{(n+1)T_1^{\frac{1}{n}}}\gamma^{\frac{1}{n}+1}+\frac{b_2p_1}{r^2}\left[e^{r\gamma}(1-r\gamma)-1\right]\right.$$

$$+\int_{\gamma}^{T_1} e^{-\alpha(t-\gamma)^\beta}\left[a\int_{t}^{T_1} e^{\alpha(u-\gamma)^\beta}du-\frac{b_1\theta}{nT_1^{\frac{1}{n}}}\int_{t}^{T_1} u^{\frac{1}{n}-1}e^{\alpha(u-\gamma)^\beta}du-b_2p_1\int_{t}^{T_1} e^{ru+\alpha(u-\gamma)^\beta}du\right]dt\Bigg]$$

$$+I_cg\left[\frac{a}{2}(\gamma^2+M^2-2M\gamma)+\frac{b_1\theta}{T_1^{\frac{1}{n}}}\left[\frac{n}{n+1}\left(\gamma^{\frac{1}{n}+1}-M^{\frac{1}{n}+1}\right)-\gamma^{\frac{1}{n}}(\gamma-M)\right]\right.$$

$$+\frac{b_2p_1}{r^2}\left[e^{r\gamma}\left(1-r(\gamma-M)\right)-e^{rM}\right]$$

$$+(\gamma-M)\left[a\int_{\gamma}^{T_1} e^{\alpha(u-\gamma)^\beta}du-\frac{b_1\theta}{nT_1^{\frac{1}{n}}}\int_{\gamma}^{T_1} u^{\frac{1}{n}-1}e^{\alpha(u-\gamma)^\beta}du-b_2p_1\int_{\gamma}^{T_1} e^{ru+\alpha(u-\gamma)^\beta}du\right]$$

$$+\int_{\gamma}^{T_1} e^{-\alpha(t-\gamma)^\beta}\left[a\int_{t}^{T_1} e^{\alpha(u-\gamma)^\beta}du-\frac{b_1\theta}{nT_1^{\frac{1}{n}}}\int_{t}^{T_1} u^{\frac{1}{n}-1}e^{\alpha(u-\gamma)^\beta}du-b_2p_1\int_{t}^{T_1} e^{ru+\alpha(u-\gamma)^\beta}du\right]dt\Bigg]$$

$$-I_ep\left[\frac{aM^2}{2}-\frac{b_1\theta}{(n+1)T_1^{\frac{1}{n}}}M^{\frac{1}{n}+1}-\frac{b_2p_1}{r^2}\left[e^{rM}(rM-1)+1\right]\right]\Bigg]\left[\frac{e^{rH}-1}{e^{rT_1}-1}\right] \qquad (18)$$

Case (ii): Cycle Length T is smaller than M i.e., $T<M$

Interest Earned, IE_2 is

$$IE_2=I_e\sum_{j=0}^{n-1}p(j,T)\left\{\int_0^T R(p(t))tdt+R(p(T))[T(M-T)]\right\}$$

$$=pI_e\left\{\int_0^T\left(a+b_1\left(\frac{-\theta t^{\frac{1}{n}-1}}{nT^{\frac{1}{n}}}\right)-bpe^{rt}\right)tdt+\left[a+b_1\left(\frac{-\theta}{nT}\right)-b_2pe^{rT}\right][T(M-T)]\right\}\left[\frac{e^{rH}-1}{e^{rT}-1}\right]$$

$$=pI_e\left[\frac{aT^2}{2}-\frac{b_1\theta T}{n+1}-\frac{b_2p}{r^2}\left[e^{rT}(rT-1)-1\right]+\left[a+b_1\left(\frac{-\theta}{nT}\right)-b_2pe^{rT}\right][T(M-T)]\right]\left[\frac{e^{rH}-1}{e^{rT}-1}\right] \qquad (19)$$

Thus, the total cost over $(0,H)$ is $TC(p,T)$

$$TC(p,T)=OC+CD+ICC-IE_2 \qquad (20)$$

Substituting equations (8), (9), (10) and (19) in (20), we get

$$TC(p,T)=\Bigg[A+g\left[a(\gamma-T)-\frac{b_1\theta}{T^{\frac{1}{n}}}\gamma^{\frac{1}{n}}+b_1\theta+\frac{b_2p}{r}(e^{rT}-e^{r\gamma})+a\int_{\gamma}^{T} e^{\alpha(u-\gamma)^\beta}du\right.$$

$$\left.-\frac{b_1\theta}{nT^{\frac{1}{n}}}\int_{\gamma}^{T} u^{\frac{1}{n}-1}e^{\alpha(u-\gamma)^\beta}du-b_2p\int_{\gamma}^{T} e^{ru+\alpha(u-\gamma)^\beta}du\right]$$

$$+hg\left[\frac{a\gamma^2}{2}-\frac{b_1\theta}{(n+1)T^{\frac{1}{n}}}\gamma^{\frac{1}{n}+1}+\frac{b_2p}{r^2}\left[e^{r\gamma}(1-r\gamma)-1\right]\right.$$

$$+\gamma\left[a\int_{\gamma}^{T} e^{\alpha(u-\gamma)^\beta}du-\frac{b_1\theta}{nT^{\frac{1}{n}}}\int_{\gamma}^{T} u^{\frac{1}{n}-1}e^{\alpha(u-\gamma)^\beta}du-b_2p\int_{\gamma}^{T} e^{ru+\alpha(u-\gamma)^\beta}du\right]$$

$$\left.+\int_{\gamma}^{T} e^{-\alpha(t-\gamma)^\beta}\left[a\int_{t}^{T} e^{\alpha(u-\gamma)^\beta}du-\frac{b_1\theta}{nT^{\frac{1}{n}}}\int_{t}^{T} u^{\frac{1}{n}-1}e^{\alpha(u-\gamma)^\beta}du-b_2p\int_{t}^{T} e^{ru+\alpha(u-\gamma)^\beta}du\right]dt\right]$$

$$-pI_e\left[\frac{aT^2}{2}-\frac{b_1\theta T}{n+1}-\frac{b_2p}{r^2}\left[e^{rT}(rT-1)-1\right]+\left[a+b_1\left(\frac{-\theta}{nT}\right)-b_2pe^{rT}\right][T(M-T)]\right]\left[\frac{e^{rH}-1}{e^{rT}-1}\right] \qquad (21)$$

The net profit is the difference of gross revenue and total cost.

The gross revenue is $(pe^{rT}-ge^{rT})\left[a-\frac{b_1\theta}{nT}-b_2pe^{rT}\right]$

Hence, the net profit is

$$NP(p,T)=(pe^{rT}-ge^{rT})\left[a-\frac{b_1\theta}{nT}-b_2pe^{rT}\right]-TC(p,T) \tag{22}$$

where, $TC(p,T)$ is as given in equation (21)

For obtaining the optimal policies of the model we maximize $NP(p,T)$ with respect to T and p. The conditions for obtaining optimality are

$$\frac{\partial NP(p,T)}{\partial T}=0, \frac{\partial NP(p,T)}{\partial p}=0 \quad and \quad D=\begin{vmatrix}\frac{\partial^2 NP(p,T)}{\partial p^2} & \frac{\partial^2 NP(p,T)}{\partial T\partial p}\\ \frac{\partial^2 NP(p,T)}{\partial T\partial p} & \frac{\partial^2 NP(p,T)}{\partial T^2}\end{vmatrix}<0$$

where D is the determinant of Hessian matrix

$\frac{\partial NP(p,T)}{\partial T}=0$ implies,

$$(pe^{rT}-ge^{rT})\left[\left[\frac{b_1\theta}{n^2T^{\frac{1}{n}+1}}-b_2pe^{rT}\right]+\left[a-\frac{b_1\theta}{nT^{\frac{1}{n}}}-b_2pe^{rT}\right]r\right]-\left\{\left[\frac{e^{rH}-1}{e^{rT}-1}\right]\right.$$

$$\left[g\left[-a-\frac{b_1\theta}{nT^{\frac{1}{n}+1}}\gamma^{\frac{1}{n}}+b_2pe^{rT}+ae^{\alpha(T-\gamma)^\beta}-\frac{b_1\theta}{nT}e^{\alpha(T-\gamma)^\beta}-b_2pe^{rT+\alpha(T-\gamma)^\beta}\right]\right.$$

$$+hg\left[\frac{b_1\theta}{nT}\frac{\gamma^{\frac{1}{n}+1}}{n+1}+\gamma\left[ae^{\alpha(T-\gamma)^\beta}-\frac{b_1\theta}{nT}e^{\alpha(T-\gamma)^\beta}-b_2pe^{rT+\alpha(T-\gamma)^\beta}\right]\right.$$

$$\left.+\int_\gamma^T e^{-\alpha(t-\gamma)^\beta}\left[ae^{\alpha(T-\gamma)^\beta}-\frac{b_1\theta}{nT}e^{\alpha(T-\gamma)^\beta}-b_2pe^{rT+\alpha(T-\gamma)^\beta}\right]dt\right]$$

$$\left.-I_ep\left[aT-\frac{b_1\theta}{n+1}-bpTe^{rT}+\left[a-\frac{b_1\theta}{nT}-bpe^{rT}\right](M-2T)+(MT-T^2)\left(\frac{b_1\theta}{nT^2}-b_2pre^{rT}\right)\right]\right]$$

$$+\left[A+g\left[a(\gamma-T)-\frac{b_1\theta}{T^{\frac{1}{n}}}\gamma^{\frac{1}{n}}+b_1\theta+\frac{b_2p}{r}(e^{rT}-e^{r\gamma})+a\int_\gamma^T e^{\alpha(u-\gamma)^\beta}du\right.\right.$$

$$\left.-\frac{b_1\theta}{nT^{\frac{1}{n}}}\int_\gamma^T u^{\frac{1}{n}-1}e^{\alpha(u-\gamma)^\beta}du-b_2p\int_\gamma^T e^{ru+\alpha(u-\gamma)^\beta}du\right]$$

$$+hg\left[\frac{a\gamma^2}{2}-\frac{b_1\theta}{(n+1)T^{\frac{1}{n}}}\gamma^{\frac{1}{n}+1}+\frac{b_2p}{r^2}[e^{r\gamma}(1-r\gamma)-1]\right.$$

$$+\gamma\left[a\int_\gamma^T e^{\alpha(u-\gamma)^\beta}du-\frac{b_1\theta}{nT^{\frac{1}{n}}}\int_\gamma^T u^{\frac{1}{n}-1}e^{\alpha(u-\gamma)^\beta}du-b_2p\int_\gamma^T e^{ru+\alpha(u-\gamma)^\beta}du\right]$$

$$\left.+\int_\gamma^T e^{-\alpha(t-\gamma)^\beta}\left[a\int_t^T e^{\alpha(u-\gamma)^\beta}du-\frac{b_1\theta}{nT^{\frac{1}{n}}}\int_t^T u^{\frac{1}{n}-1}e^{\alpha(u-\gamma)^\beta}du-b_2p\int_t^T e^{ru+\alpha(u-\gamma)^\beta}du\right]dt\right]$$

$$\left.\left.-I_ep\left[\frac{aT^2}{2}-\frac{b_1\theta T}{n+1}-\frac{b_2p}{r^2}[e^{rT}(rT-1)-1]+\left[a+b_1\left(\frac{-\theta}{nT}\right)-b_2pe^{rT}\right][T(M-T)]\right]\right]\right\}$$

$$\left[\frac{e^{rH}-1}{(e^{rT}-1)^2}\right]re^{rT}=0 \tag{23}$$

$\frac{\partial NP(p,T)}{\partial p}=0$ implies,

$$e^{rT}\left[a-\frac{b_1\theta}{nT}-2pb_2e^{rT}+gb_2e^{rT}\right]-\left\{g\left[\frac{b_2}{r}(e^{rT}-e^{r\gamma})-b_2\int_{\gamma}^{T}e^{ru+\alpha(u-\gamma)^{\beta}}du\right]\right.$$

$$+hg\left[\frac{b_2}{r^2}[e^{r\gamma}(1-r\gamma)-1]-b_2\gamma\left[\int_{\gamma}^{T}e^{ru+\alpha(u-\gamma)^{\beta}}du\right]\right.$$

$$\left.-b_2\int_{\gamma}^{T}e^{-\alpha(t-\gamma)^{\beta}}\left[\int_{t}^{T}e^{ru+\alpha(u-\gamma)^{\beta}}du\right]dt\right]+-b_2(\gamma-M)\left[\int_{\gamma}^{T}e^{ru+\alpha(u-\gamma)^{\beta}}du\right]$$

$$-b_2\int_{\gamma}^{T}e^{-\alpha(t-\gamma)^{\beta}}\left[\int_{t}^{T}e^{ru+\alpha(u-\gamma)^{\beta}}du\right]d\,t\right]$$

$$-I_e\left[\frac{aT^2}{2}-\frac{b_1\theta T}{n+1}-\frac{2b_2p}{r^2}[e^{rT}(rT-1)-1]\right.$$

$$\left.\left.+\left[a+b_1\left(\frac{-\theta}{nT}\right)-2b_2pe^{rT}\right][T(M-T)]\right]\right\}\left[\frac{e^{rH}-1}{e^{rT}-1}\right]=0 \qquad (24)$$

For given values of the parameters and costs, equations (23) and (24) are solved using MATHCAD to get the optimal cycle length $T=T_2$ and selling price $p=p_2$. Substituting the optimal values of T_2 and p_2 in equation (21), we get the minimum total cost. Substituting this minimum total cost, T_2 and p_2 in equation (22), we get the maximum profit as

$$NP^*(p_2,T_2)=(p_2e^{rT_2}-ge^{rT_2})\left[a-\frac{b_1\theta}{nT_2}-b_2p_2e^{rT_2}\right]$$

$$-\left[A+g\left[a(\gamma-T_2)-\frac{b_1\theta}{T_2^{\frac{1}{n}}}\gamma^{\frac{1}{n}}+b_1\theta+\frac{b_2p_2}{r}(e^{rT_2}-e^{r\gamma})+a\int_{\gamma}^{T_2}e^{\alpha(u-\gamma)^{\beta}}du\right.\right.$$

$$\left.-\frac{b_1\theta}{nT_2^{\frac{1}{n}}}\int_{\gamma}^{T_2}u^{\frac{1}{n}-1}e^{\alpha(u-\gamma)^{\beta}}du-b_2p_2\int_{\gamma}^{T_2}e^{ru+\alpha(u-\gamma)^{\beta}}du\right]$$

$$+hg\left[\frac{a\gamma^2}{2}-\frac{b_1\theta}{(n+1)T_2^{\frac{1}{n}}}\gamma^{\frac{1}{n}+1}+\frac{b_2p_2}{r^2}[e^{r\gamma}(1-r\gamma)-1]\right.$$

$$+\gamma\left[a\int_{\gamma}^{T_2}e^{\alpha(u-\gamma)^{\beta}}du-\frac{b_1\theta}{nT^{\frac{1}{n}}}\int_{\gamma}^{T_2}u^{\frac{1}{n}-1}e^{\alpha(u-\gamma)^{\beta}}du-b_2p_2\int_{\gamma}^{T_2}e^{ru+\alpha(u-\gamma)^{\beta}}du\right]$$

$$\left.+\int_{\gamma}^{T_2}e^{-\alpha(t-\gamma)^{\beta}}\left[a\int_{t}^{T_2}e^{\alpha(u-\gamma)^{\beta}}du-\frac{b_1\theta}{nT^{\frac{1}{n}}}\int_{t}^{T_2}u^{\frac{1}{n}-1}e^{\alpha(u-\gamma)^{\beta}}du-b_2p_2\int_{t}^{T_2}e^{ru+\alpha(u-\gamma)^{\beta}}du\right]dt\right]$$

$$-p_2I_e\left[\frac{aT_2^{\,2}}{2}-\frac{b_1\theta T_2}{n+1}-\frac{b_2p_2}{r^2}[e^{rT_2}(rT_2-1)-1]\right.$$

$$\left.\left.+\left[a+b_1\left(\frac{-\theta}{nT_2}\right)-b_2p_2e^{rT_2}\right][T_2(M-T_2)]\right]\right]\left[\frac{e^{rH}-1}{e^{rT_2}-1}\right] \qquad (25)$$

5. NUMERICAL ILLUSTRATION

The optimal values of selling price (p) and cycle length (T) are obtained by using the equation (16) and (17) or (23) and (24). The optimal values of T are taken as $T=T_1\ if\ T_1\geq M$ and $T=T_2\ if\ T_2<M$.

To illustrate the developed model of Case (i) i.e, if $T_1\geq M$, a numerical example with the following parameter values is considered. The deteriorating parameters $\alpha,\beta\ and\ \gamma$ vary from 0.020 to 0.024, 0.06 to 0.72 and 0.06 to 0.72 respectively. The values of the other parameters and costs are considered as follows:

$a = 1500\ to\ 1800,\ b_1 = 0.15\ to\ 0.18$ units, $b_2 = 0.80\ to\ 0.96$ units, A = Rs. 250.0 to 300.0, $g = Rs. 0.20\ to\ 0.24$, h =

	b_1	b_2	α	β	γ	A	g	h	I_c	I_e	M	θ	n	r	H	Q	T	p	NP
1500	0.1500	0.80	0.02	0.6	0.6	250	0.2	0.1	0.15	0.12	0.5	50	0.60	0.01	12	2643.123	1.755	3.591	2008.510
1575																2764.562	1.748	3.509	2048.262
1650																2887.902	1.742	3.433	2087.726
1725																3012.882	1.738	3.363	2127.028
1800																3139.279	1.735	3.298	2166.256
	0.1575															2648.629	1.759	3.589	2008.179
	0.1650															2654.133	1.762	3.587	2007.848
	0.1725															2659.633	1.766	3.585	2007.517
	0.1800															2665.130	1.770	3.583	2007.187
		0.84														2642.285	1.754	3.591	2008.552
		0.88														2641.448	1.754	3.592	2008.595
		0.92														2640.612	1.754	3.592	2008.637
		0.96														2639.777	1.753	3.593	2008.679
			0.021													2748.122	1.822	3.549	2003.067
			0.022													2854.985	1.891	3.512	1997.810
			0.023													2963.667	1.961	3.477	1992.758
			0.024													3074.124	2.031	3.446	1987.930
				0.63												2780.954	1.845	3.536	2002.864
				0.66												2928.277	1.941	3.485	1997.520
				0.69												3085.658	2.044	3.438	1992.603
				0.72												3253.666	2.153	3.396	1988.257
					0.63											2565.521	1.705	3.657	2002.945
					0.66											2492.533	1.658	3.725	1998.127
					0.69											2423.804	1.613	3.796	1993.957
					0.72											2359.008	1.571	3.869	1990.352
						262.5										2514.405	1.671	3.698	2001.188
						275.0										2395.384	1.593	3.813	1995.571
						287.5										2285.170	1.521	3.938	1991.357
						300.0										2182.966	1.454	4.070	1988.299

Rs. 0.100 to 0.120, $I_c = Rs. 0.150$ to 0.180, $I_e = Rs. 0.120$ to 0.144, $M = 15\ days = \frac{15}{30} = 0.500\ to\ 0.600$, $\theta = 50\ to\ 60$, $n = 0.6\ to\ 0.72$, $r = 0.010$ to 0.012, $H = 12.0$ to 14.4 months.

By substituting the above values in equations (16) and (17) and solving, the optimal values of cycle length T and selling price p are obtained. Substituting the optimal values of cycle length T and selling price p in equations (5) and (15), the optimal values of Order quantity Q and net profit NP are obtained and presented in Table-1.

a	b_1	b_2	α	β	γ	A	g	h	I_c	I_e	M	θ	n	r	H	Q	T	p	NP
							0.21									2564.694	1.703	3.669	1973.005
							0.22									2491.072	1.655	3.749	1939.354
							0.23									2421.917	1.610	3.831	1907.423
							0.24									2356.914	1.568	3.916	1877.082
								1.105								2648.295	1.758	3.592	1997.091
								1.110								2653.655	1.762	3.593	1985.636
								1.115								2659.203	1.765	3.594	1974.144
								1.120								2664.940	1.769	3.595	1962.614
									0.158							2645.674	1.756	3.590	1995.964
									0.165							2648.582	1.758	3.589	1983.383
									0.172							2651.844	1.760	3.587	1970.764
									0.180							2655.460	1.763	3.586	1958.105
										0.126						2675.373	1.776	3.563	2010.995
										0.132						2708.000	1.797	3.535	2013.627
										0.138						2741.006	1.819	3.509	2016.410
										0.144						2774.392	1.840	3.484	2019.350
											0.525					2697.267	1.790	3.536	2018.056
											0.550					2755.292	1.828	3.482	2028.601
											0.575					2817.303	1.868	3.429	2040.265
											0.600					2883.405	1.911	3.377	2053.179
												52.5				2648.629	1.759	3.589	2008.179
												55.0				2654.133	1.762	3.587	2007.848
												57.5				2659.633	1.766	3.585	2007.517
												60.0				2665.130	1.770	3.583	2007.187
													0.63			2640.736	1.753	3.591	2008.740
													0.66			2638.820	1.752	3.591	2008.939
													0.69			2637.266	1.751	3.592	2009.113
													0.72			2635.995	1.750	3.592	2009.267
														0.011		2631.263	1.747	3.598	2009.565
														0.011		2619.449	1.739	3.606	2010.628
														0.012		2607.681	1.732	3.614	2011.697
														0.012		2595.958	1.724	3.622	2012.774
															12.6	2457.889	1.634	3.753	1972.060
															13.2	2293.856	1.526	3.929	1941.024
															13.8	2148.134	1.431	4.518	1914.491
															14.4	3995.705	2.635	4.653	1668.949

Table-1: Optimal values of Q, NP, T and p for different values of parameters and costs

From Table-1, it is observed that when the parameter 'a' is increasing from 1500 to 1800 units, the optimal ordering quantity 'Q' and net profit 'NP' are increasing from 2643.123 to 3139.279 units and Rs.2008.51 to Rs.2166.256 respectively , the cycle length ‘T’ and the unit selling price 'p' are decreasing from 1.755 to 1.735 and Rs. 3.591 to Rs.3.298, when other parameters and costs are fixed.

When the parameter 'b_1' is increasing from 0.015 to 0.018 units, the optimal ordering quantity 'Q' and cycle length 'T' are increasing from 2643.123 to 2665.130 and 1.755 to 1.770, selling price 'p' and the net profit 'NP' are decreasing from 3.591 to 3.583 and Rs.2008.510 to Rs.2007.187 respectivelly, when other parameters and costs are fixed.

When the parameter 'b_2' is increasing from 0.80 to 0.96 units, the optimal ordering quantity 'Q' and cycle length 'T' are decreasing from 2643.123 to 2639.777 and 1.755 to 1.753, selling price 'p' and the net profit 'NP' are increasing from 3.591 to 3.593 and Rs. 2008.510 to Rs. 2008.679 respectivelly, when other parameters and costs are fixed.

As the deterioration parameter α is increasing from 0.020 to 0.024, the optimal ordering quantity 'Q' and cycle length 'T' are increasing from 2643.123 to 3074.124 and 1.755 to 2.031, selling price 'p' and the net profit 'NP' is decreasing from 3.591 to 3.446 and Rs. 2008.51 to Rs. 1987.93 respectivelly, when other parameters and costs are fixed.

When the parameter β is increasing from 0.60 to 0.72, the optimal ordering quantity 'Q' and cycle length 'T' are increasing from 2643.123 to 3253.666 and 1.755 to 2.153, selling price 'p' and the net profit 'NP' are decreasing from 3.591 to 3.396 and Rs. 2008.51 to Rs. 1988.257 respectivelly, when other parameters and costs are fixed.

As the deterioration parameter γ is increasing from 0.60 to 0.72, the optimal ordering quantity 'Q', cycle length 'T' and the net profit 'NP' are decreasing from 2643.123 to 2359.008, 1.755 to 1.571 and Rs. 2008.51 to Rs. 1990.352, the selling price 'p' is increasing from 3.591 to 3.869 respectivelly, when other parameters and costs are fixed.

If the ordering cost 'A' increases from Rs.250 to 300, the optimal ordering quantity 'Q', cycle length 'T' and the net profit 'NP' are decreasing from 2643.123 to 2182.966, 1.755 to 1.454 and Rs. 2008.51 to Rs. 1988.299, the selling price 'p' is increasing from 3.591 to 4.07 respectivelly, when other parameters and costs are fixed.

When the unit cost 'g' is increasing from Rs.0.20 to 0.24, the optimal ordering quantity 'Q', cycle length 'T' and the net profit 'NP' are decreasing from 2643.123 to 2356.914, 1.755 to 1.568 and Rs. 2008.51 to Rs. 1877.082, the selling price 'p' is increasing from 3.591 to 3.916 respectivelly, when other parameters and costs are fixed.

When holding cost 'h' is increasing from Rs.0.100 to 0.120, the optimal ordering quantity 'Q', cycle length 'T' and selling price 'p' are increasing from 2643.123 to 2664.94, 1.755 to 1.769 and 3.591 to 3.595, the net profit 'NP' is decreasing from Rs. 2008.51 to Rs. 1962.614 respectivelly, when other parameters and costs are fixed.

When interest charged 'I_C' increases from Rs.0.150 to 0.180, the optimal ordering quantity 'Q' and cycle length 'T' are increasing from 2643.123 to 2655.46 and 1.755 to 1.763, selling price 'p' and the net profit 'NP' are decreasing from 3.591 to 3.586 and Rs. 2008.51 to Rs. 1958.105 respectivelly, when other parameters and costs are fixed.

If interest charged 'I_e' increases from Rs.0.120 to 0.144, the optimal ordering quantity 'Q', cycle length 'T' and the net profit 'NP' are increasing from 2643.123 to 2774.392 and Rs. 2008.51 to Rs. 2019.35, the selling price 'p' is decreasing from 3.591 to 3.484 respectivelly, when other parameters and costs are fixed.

If the permissible delay period 'M' increases from 0.5 months to 0.6 months, the optimal ordering quantity 'Q', cycle length 'T' and the net profit 'NP' are increasing from 2643.123 to 2883.405, 1.755 to 1.911 and Rs. 2008.51 to Rs. 2053.179, the selling price 'p' is decreasing from 3.591 to 3.377 respectivelly, when other parameters and costs are fixed.

If the parameter 'θ' increases from 50 to 60, the optimal ordering quantity 'Q' and cycle length 'T' are increasing from 2643.123 to 2665.13 and 1.755 to 1.770, the selling price 'p' and the net profit 'NP' is decreasing 3.591 to 3.583 and Rs. 2008.51 to Rs. 2007.187 respectivelly, when other parameters and costs are fixed.

If the parameter 'n' increases from 0.60 to 0.70, the optimal ordering quantity 'Q' and cycle length 'T' are decreasing from 2643.123 to 2635.995 and 1.755 to 1.750, the selling price 'p' and the net profit 'NP' is increasing from 3.591 to 3.592 and Rs. 2008.510 to Rs. 2009.267 respectivelly, when other parameters and costs are fixed.

The inflation rate 'r' increases from 0.010 to 0.0120, the optimal ordering quantity 'Q' and cycle length 'T' are decreasing from 2643.123 to 2595.958 and 1.755 to 1.724, the selling price 'p' and the net profit 'NP' are increasing from 3.591 to 3.622 and Rs. 2008.51 to Rs. 2012.774 respectivelly, when other parameters and costs are fixed.

When the time horizon 'H' increases from 12 months to 13.8, the optimal ordering quantity 'Q', cycle length 'T' and selling price 'p' are increasing from 2643.123 to 3995.705 1.755 to 2.635 and 3.591 to 4.653, the net profit 'NP' is decreasing from Rs. 2008.51 to Rs. 1668.949 respectivelly, when other parameters and costs are fixed.

SENSITIVITY ANALYSIS

To study the effect of changes in the model parameters and costs on the optimal values of the order quantity, cycle length, selling price and net profit, the sensitivity analysis is carried by considering $a = 1500,\ b_1 = 0.15$ units, $b_2 = 0.80$ units, $\alpha = 0.02, \beta = 0.60, \gamma = 0.60,\ \mathrm{A} = \mathrm{Rs.}250,\ g = Rs.0.20, \mathrm{h} = \mathrm{Rs.}0.100,\ I_c = Rs.0.150, I_e = Rs.0.120,\ M = 0.500,\ \theta = 50,\ n = 0.6,\ r = 0.01,\ H = 12$ months. Table-2 summarizes these results for variations of -15%, -10%, -5%, 0, 5%, 10%, 15% of the parameters and costs.

As the parameter a increases from -15% to +15%, the optimal order quantity 'Q' is increases from 2293.811 to 3012.882, cycle length ‘T’ decreases from 1.793 to 1.738, selling price ‘p’ decreases from Rs. 3.883to Rs. 3.363 and the net profit increases from Rs. 1885.358 to Rs. 2127.028.

When the total demand during the cycle period b_1 increases from -15% to +15%, the optimal order quantity 'Q' increases from 2626.585 to 2659.633, cycle length 'T' is increases from 1.743 to 1.766, selling price 'p' is decreasing from 3.597 to 3.585 and the net profit 'NP' decreases from Rs. 2009.505 to Rs. 2007.517.

When the total demand during the cycle period b_2 increases from -15% to +15%, the optimal order quantity 'Q' decreases from 2645.642 to 2640.612, cycle length 'T' is decreases from 1.756 to 1.754, selling price 'p' is increases from 3.589 to 3.592 and the net profit 'NP' increases from Rs. 2008.383 to Rs. 2008.637.

As the deterioration parameter α increases from -15% to +15%, the optimal order quantity 'Q' increases from 2339.726 to 2963.667, cycle length 'T' increases from 1.558 to 1.961, selling price 'p' decreases from Rs. 3.739 to Rs. 3.477 and the net profit 'NP' decreases from Rs. 2025.765 to Rs. 1992.758.

If the parameter β increases from -15% to +15%, the optimal order quantity 'Q' increases from 2281.124 to 3085.658, cycle length 'T' increases from 1.517 to 2.044, selling price 'p' decreases from Rs. 3.778 to Rs.3.438 and the net profit 'NP' decreases from Rs. 2026.289 to Rs. 1992.603.

When the deterioration parameter γ increases from -15% to +15%, the optimal order quantity 'Q' decreases from 2907.751 to 2423.804, cycle length 'T' decreases from 1.925 to 1.613, selling price 'p' increases from Rs. 3.411 to Rs. 3.796 and the net profit 'NP' decreases from Rs. 2030.948 to Rs. 1993.957.

When the ordering cost A increases from -15% to +15%, the optimal order quantity 'Q' decreases from 3098.036 to 2285.17, cycle length 'T' decreases from 2.051 to 1.521, selling price 'p' increases from Rs. 3.328 to Rs3.938 and the net profit 'NP' decreases from Rs. 2044.856 to Rs. 1991.357.

As the unit cost g increases from -15% to +15%, the optimal order quantity 'Q' decreases from 2911.128 to 2421.917, cycle length 'T' decreases from 1.929 to 1.610, selling price 'p' increases from Rs. 3.372 to Rs. 3.831 and the net profit 'NP' decreases from Rs. 2127.576 to Rs. 1907.423.

As the holding cost h increases from -15% to +15%, the optimal order quantity 'Q' increases from 2628.727 to 2659.203, cycle length 'T' increases from 1.745 to 1.765, selling price 'p' increases from Rs.3.587 to Rs. 3.594 and the net profit decreases from Rs.2042.567 to Rs. 1974.144.

When the interest charged I_c increases from -15% to +15%, the optimal order quantity 'Q' increases from 2637.62 to 2651.844, cycle length 'T' increases from 1.751 to 1.760, selling price 'p' decreases from Rs. 3.593 to Rs. 3.587 and the net profit decreases from Rs. 2045.961 to Rs. 1970.764.

If the interest earned I_e increases from -15% to +15%, the optimal order quantity 'Q' increases from 2548.608 to 2741.006, cycle length 'T' increases from 1.693 to 1.819, selling price 'p' decreases from Rs. 3.682 to Rs. 3.509 and the net profit 'NP' increases from Rs. 2001.891 to Rs. 2016.41.

When the permissible delay period M increases from -15% to +15%, the optimal order quantity 'Q' increases from 2502.944 to 2817.303, cycle length 'T' increases from 1.663 to 1.868, selling price 'p' decreases from Rs. 3.757 to Rs.3.429 and the net profit 'NP' increases from Rs. 1984.755 to Rs. 2040.265.

If the total demand during the entire cycle θ increases from -15% to +15%, the optimal order quantity 'Q' increases from 2626.585 to 2659.633, cycle length 'T' increases from 1.743 to 1.766, selling price 'p' decreases from Rs. 3.597 to Rs. 3.585 and the net profit 'NP' decreases from Rs. 2009.505 to Rs 2007.517.

If the demand index parameter n increases from -15% to +15%, the optimal order quantity 'Q' decreases from 2654.94 to 2637.266, cycle length 'T' decreases from 1.762 to 1.751, selling price 'p' increases from Rs 3.588 to Rs. 3.592 and the net profit 'NP' increases from Rs. 2007.542 to Rs. 2009.113.

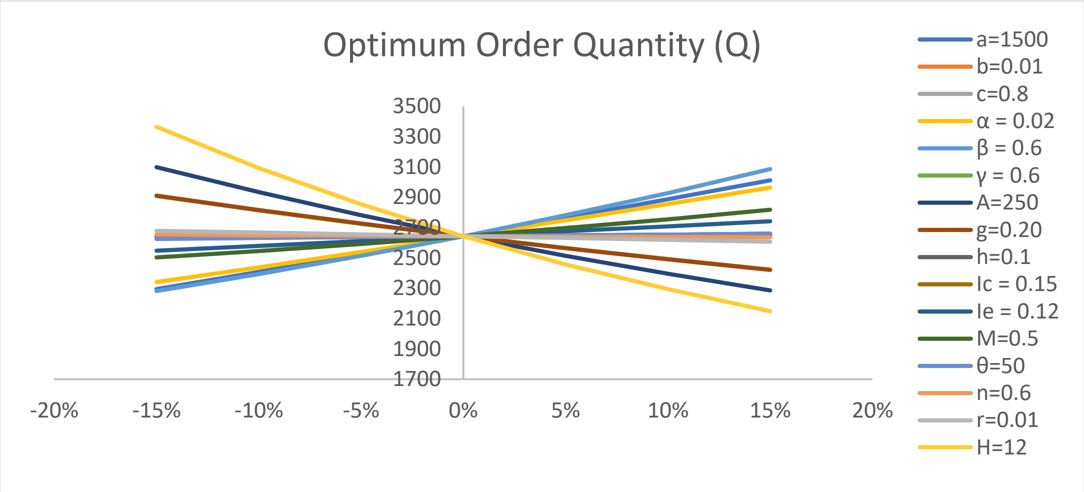
If the inflation rate r increases from -15% to +15%, the optimal order quantity 'Q' decreases from 2678.978 to 2607.681, cycle length 'T' decreases from 1.778 to 1.732, selling price 'p' increases from Rs. 3.568 to Rs 3.614 and the net profit 'NP' increases from Rs. 2005.389 to Rs. 2011.697.

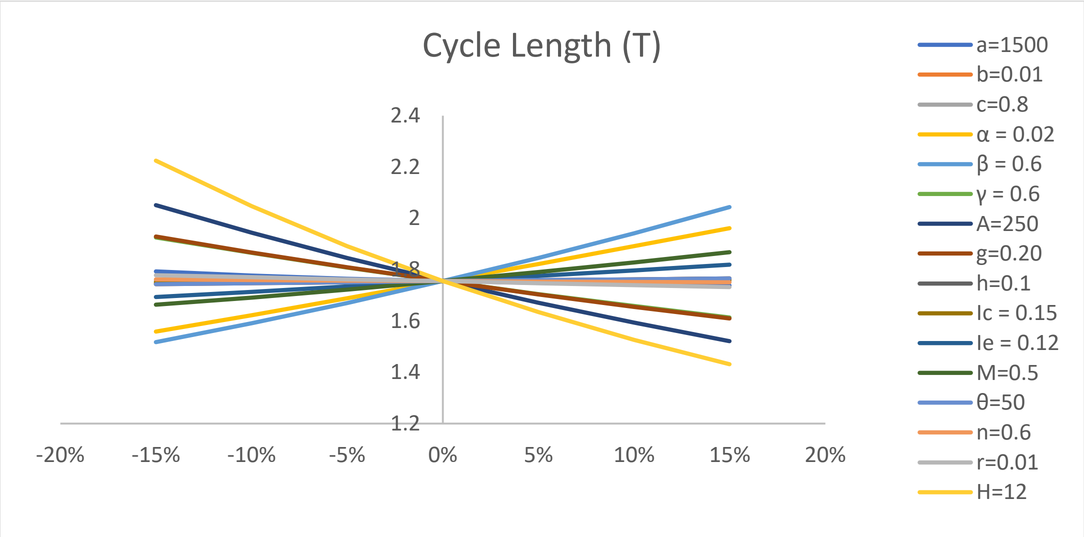
When the time horizon H increases from -15% to +15%, the optimal order quantity 'Q' decreases from 3363.593 to 2148.134, cycle length 'T' decreases from 2.224 to 1.431, selling price 'p' increases from Rs. 3.205 to Rs. 4.518 and the net profit 'NP' decreases from Rs. 2162.271 to Rs. 1914.491.

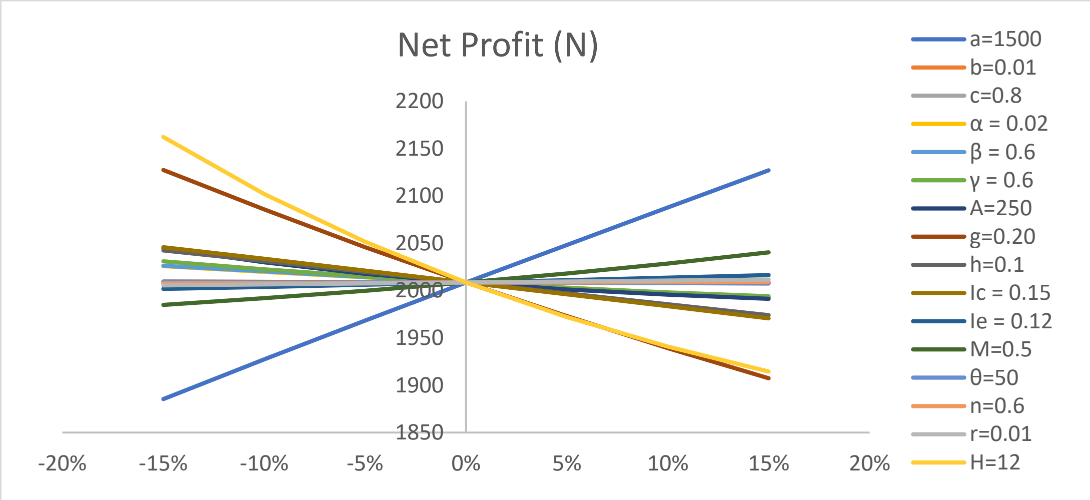
Table-2: EFECT ON OPTIMAL VALUES WITH RESPECT TO PARAMETERS VARIATION

Variation Parameters		Percentage change in parameter						
		-15	-10	-5	0	5	10	15
a	Q	2293.811	2407.306	2523.906	2643.123	2764.562	2887.902	3012.882
	T	1.793	1.777	1.764	1.755	1.748	1.742	1.738
	p	3.883	3.777	3.68	3.591	3.509	3.433	3.363
	NP	1885.358	1927.364	1968.294	2008.51	2048.262	2087.726	2127.028
b_1	Q	2626.585	2632.1	2637.613	2643.123	2648.629	2654.133	2659.633
	T	1.743	1.747	1.751	1.755	1.759	1.762	1.766
	p	3.597	3.595	3.593	3.591	3.589	3.587	3.585
	NP	2009.505	2009.173	2008.842	2008.51	2008.179	2007.848	2007.517
b_2	Q	2645.642	2644.801	2643.961	2643.123	2642.285	2641.448	2640.612
	T	1.756	1.755	1.755	1.755	1.754	1.754	1.754
	p	3.589	3.59	3.59	3.591	3.591	3.592	3.592
	NP	2008.383	2008.425	2008.468	2008.51	2008.552	2008.595	2008.637
α	Q	2339.726	2438.885	2540.03	2643.123	2748.122	2854.985	2963.667
	T	1.558	1.622	1.688	1.755	1.822	1.891	1.961
	p	3.739	3.685	3.636	3.591	3.549	3.512	3.477
	NP	2025.765	2019.878	2014.12	2008.51	2003.067	1997.81	1992.758
β	Q	2281.124	2393.74	2514.233	2643.123	2780.954	2928.277	3085.658
	T	1.517	1.591	1.67	1.755	1.845	1.941	2.044
	p	3.778	3.713	3.65	3.591	3.536	3.485	3.438
	NP	2026.289	2020.304	2014.353	2008.51	2002.864	1997.52	1992.603
γ	Q	2907.751	2813.779	2725.731	2643.123	2565.521	2492.533	2423.804
	T	1.925	1.865	1.808	1.755	1.705	1.658	1.613
	p	3.411	3.468	3.528	3.591	3.657	3.725	3.796
	NP	2030.948	2022.364	2014.938	2008.51	2002.945	1998.127	1993.957
A	Q	3098.036	2933.755	2782.534	2643.123	2514.405	2395.384	2285.17
	T	2.051	1.944	1.846	1.755	1.671	1.593	1.521
	p	3.328	3.405	3.493	3.591	3.698	3.813	3.938
	NP	2044.856	2029.836	2017.908	2008.51	2001.188	1995.571	1991.357
g	Q	2911.128	2815.918	2726.729	2643.123	2564.694	2491.072	2421.917
	T	1.929	1.867	1.809	1.755	1.703	1.655	1.61
	p	3.372	3.442	3.515	3.591	3.669	3.749	3.831
	NP	2127.576	2085.651	2046.011	2008.51	1973.005	1939.354	1907.423
h	Q	2628.727	2633.339	2638.138	2643.123	2648.295	2653.655	2659.203
	T	1.745	1.748	1.751	1.755	1.758	1.762	1.765
	p	3.587	3.588	3.59	3.591	3.592	3.593	3.594
	NP	2042.567	2031.247	2019.895	2008.51	1997.091	1985.636	1974.144
Variation Parameters		**Percentage change in parameter**						
		-15	-10	-5	0	5	10	15
I_c	Q	2637.62	2639.094	2640.929	2643.123	2645.674	2648.582	2651.844
	T	1.751	1.752	1.753	1.755	1.756	1.758	1.76
	p	3.593	3.592	3.592	3.591	3.59	3.589	3.587
	NP	2045.961	2033.506	2021.023	2008.51	1995.964	1983.383	1970.764
I_e	Q	2548.608	2579.743	2611.247	2643.123	2675.373	2708	2741.006
	T	1.693	1.713	1.734	1.755	1.776	1.797	1.819
	p	3.682	3.65	3.62	3.591	3.563	3.535	3.509
	NP	2001.891	2003.962	2006.167	2008.51	2010.995	2013.627	2016.41

M	Q	2502.944	2546.06	2592.754	2643.123	2697.267	2755.292	2817.303
	T	1.663	1.691	1.722	1.755	1.79	1.828	1.868
	p	3.757	3.702	3.646	3.591	3.536	3.482	3.429
	NP	1984.755	1991.961	1999.847	2008.51	2018.056	2028.601	2040.265
θ	Q	2626.585	2632.1	2637.613	2643.123	2648.629	2654.133	2659.633
	T	1.743	1.747	1.751	1.755	1.759	1.762	1.766
	p	3.597	3.595	3.593	3.591	3.589	3.587	3.585
	NP	2009.505	2009.173	2008.842	2008.51	2008.179	2007.848	2007.517
n	Q	2654.94	2649.966	2646.129	2643.123	2640.736	2638.82	2637.266
	T	1.762	1.759	1.757	1.755	1.753	1.752	1.751
	p	3.588	3.589	3.59	3.591	3.591	3.591	3.592
	NP	2007.542	2007.924	2008.242	2008.51	2008.74	2008.939	2009.113
r	Q	2678.978	2666.98	2655.029	2643.123	2631.263	2619.449	2607.681
	T	1.778	1.77	1.762	1.755	1.747	1.739	1.732
	p	3.568	3.576	3.583	3.591	3.598	3.606	3.614
	NP	2005.389	2006.422	2007.462	2008.51	2009.565	2010.628	2011.697
H	Q	3363.593	3091.53	2852.981	2643.123	2457.889	2293.856	2148.134
	T	2.224	2.047	1.892	1.755	1.634	1.526	1.431
	p	3.205	3.315	3.445	3.591	3.753	3.929	4.518
	NP	2162.271	2102.211	2051.463	2008.51	1972.06	1941.024	1914.491







6. Conclusion

In this paper we have praposed and analyzed an EOQ model for deteriorating items with truncated Weibull distribution having permissible delay in payments and inflation. In inventory control, permissible delay in payments has significance influence in obtaining the optimal pricing and ordering policies. The truncated Weibull distribution is one of the most significant life time distribution for items such as food and vegetables markets, market yards and chemical industries, etc., where the deterioration is skewed and having long upper tail. The truncated Weibull distribution includes exponential distribution as a particular case. We have considered that the demand of items is a function of both time and selling price. The sensitivity analysis of the model revealed that the pricing and ordering are highly influenced by the parameters and costs. The model with constraints on warehouse capacity and budget can also be developed with permissible delay in payment and truncated Weibull decay.

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