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Rainbow Dominator Coloring for special Graphs

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Abstract — Rainbow vertex coloring introduced a decade ago followed by Rainbow dominator Coloring in recent years has been at tracting the researchers in graph theory. We undertake a study on rainbow vertex coloring and in particular rainbow dominator coloring for specific connected graphs namely Bull graph, Star graph, Complete graph, Helm graph and sunlet graph, Jelly fish, Jewel graph, Extended jewel graph, Triangular book, Triangular book with bookmark. We have proved that these graphs admit rainbow dominator coloring and Rainbow Dominator chromatic number $\chi_{rd}(G)$ for these graphs were also determined. Few illustrations were also shown.

Keywords — Coloring, Rainbow dominator coloring, Fan Graph, Complete Graph, Star Graph

Introduction

Graph coloring problem is assigning of colors to vertices subject to certain conditions. Graph coloring is a special case of graph labeling which finds its applications in Scheduling, Data mining, Image processing etc .Graph theory consists of 3 coloring problems, one being vertex coloring and another Edge coloring. Face coloring which is Geographical map coloring can be transformed into vertex coloring [1].It finds its application in research areas of computer science like data mining, image capturing, image segmentation clustering etc. The smallest number of colors required to color the graphs is called the Chromatic number. Many parameters were introduced and analyzed in 1987.

Rainbow edge coloring was introduced by Chatrand,John and Mckeon in the year 2008 [2] .Peterson graph,fan graph and corona graph were studied and results are found in Literature .Another concept of Rainbow vertex coloring was introduced by Krivelevich and Yuster in 2010[3] . Kulkarni Sunita Jagannatharao, S. K. Rajendra and R. Murali published their results on Rainbow dominator coloring in 2021[4].In this paper we propose to find Rainbow dominator chromatic number of some graphs like complete graph, Helm graph, Jelly fish, Jewel graph, Extended jewel graph, Triangular book, Triangular book with bookmark and bull graph.

All the graphs considered in this paper are connected, finite and undirected graphs.

I. PRELIMINARIES

The definitions required for this paper are recalled below.

Definition 1: Proper Coloring [5]

A proper coloring of a graph G is an allotment of colors to the vertices of the graph such that no two adjacent vertices have the same color and the chromatic number $\chi(G)$ of the graph is the least number of colors needed in a proper coloring of G.

Definition 2: Dominator Coloring[6]

A dominator coloring [[6]–[8], [9]] of a graph is a proper coloring such that each vertex dominates every vertex in at least one color class consisting of vertices with the same color. The chromatic number of a graph is the minimum number of colors needed in a dominator coloring of G

3: Rainbow Dominator Coloring[4]

A rainbow dominator coloring of a graph G is a proper rainbow coloring of the graph G ,in which every vertex of G dominates every vertex of some color class.The minimum number of color classes in the graph G is called the rainbow dominator chromatic number and is denoted by χ_{rd}(G).

Definition 4: Rainbow Connection Number[2]

In a connected edge colored Graph G,if any two vertices are connected by a rainbow path which is a path whose edges have distinct colors.The minimum number of colors required to make the graph rainbow connected is called Rainbow connection number .

Definition 5: Bull Graph [10]

A bull graph is the planar undirected graph with 5 vertices and 5 edges constructed by inducting two pendent vertices to any two vertices of C_3.

Definition 6: Star Graph [11]

A star $K_{1,n}$ is a tree with n vertices of degree 1and root vertex having degree n.

Definition 7: Complete Graph [5]

A graph in which, for each pair of vertices there exists unique edge that connects them. The Complete Graph is denoted by K_n.

Definition 8: Helm Graph [10]

The *Helm Graph* H_n is the graph obtained from a wheel graph $W_{1,n}$ by attaching a pendant edge at each vertex of the *n* – cycle.

Definition 8: Sunlet Graph [12]

The *Sunlet Graph* Sl_n is the graph obtained from a Cycle graph Sl_n by attaching a pendant edge at each vertex of the *n* – cycle

II. MAIN RESULTS

Theorem 1

The rainbow dominator coloring for a Bull graph G, $\chi_{rd}(G) = 3$

Proof

Let graph $G(V, E)$ be the bull graph with 5 vertices. The vertices of the graph be v_1, v_2, v_3, v_4, v_5A Rainbow dominator coloring of G is given by coloring the vertices v_1, v_4, v_5 with C_1, vertices v_2 and v_5 colored with C_2 and C_3 respectively. So, vertices v_1, v_5 dominate v_2 and vertices $v_1$with C_2, v_4 dominate v_3 with C_3.The vertex v_3 dominates v_2 and v_2 dominates v_3 with colors c_2 and c_3 respectively

Thus, any vertex of G dominates some color class. And for each pair of vertices, we can find rainbow path. The Rainbow dominator coloring for a Bull graph G is $\chi_{rd}(G) = 3$.

Example 1:

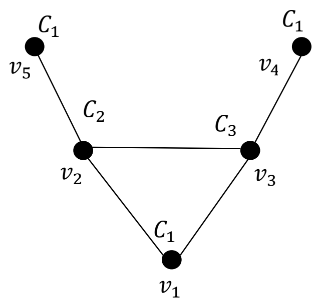

Fig. 1

Fig. 1 the bull graph

Dominating set	Dominated color class	Chromatic number
v_1, v_5	C_2	3
v_3	C_2	
v_2	C_3	

Theorem 2

For $n \geq 3$, the rainbow dominator coloring of the star graph, $\chi_{rd}(K_{1,n}) = 2$.

Proof: The vertices of the star graph be v_1 the root vertex, pendent vertices are v_{i+1}, $1 \leq i \leq n$. Assign the color C_1 to the root vertex and color C_2 to the pendent vertices v_{i+1}, $1 \leq i \leq n$.The root vertex dominates the color class $\{v_2, v_3, v_4, \dots, v_{n+1}\}$, each pendent vertex dominates color class $\{v_1\}$. And for each pair of vertices, we can find rainbow path. This guarantees that the rainbow dominator coloring is proper. The rainbow dominator coloring for the star graph $\chi_{rd}(K_{1,n}) = 2, n \geq 3$.

Example 2

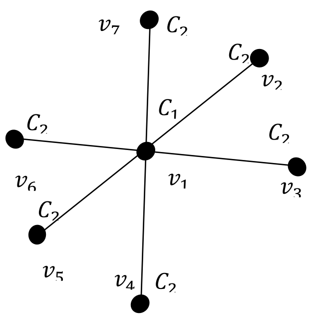

Fig. 2 the star graph $K_{1,6}$

Dominating set	Dominated color class	Chromatic number
Root vertex v_1	C_2	2

Theorem 3

For any $m \geq 3, n \geq 3$, the rainbow dominator coloring for a bistar Star $K_{1,m,n}$, $\chi_{rd}(K_{1,m,n}) = 3$.

Proof: Bistar star graph is the join of two-star graphs $K_{1,m}$and $K_{1,m}$ at the pendent vertices of P_2. The vertices of the graph are v_0, u_0 the apex vertices of $K_{1,m}$ and $K_{1,m}$, pendent vertices of $K_{1,m}$ are $v_i, 1 \leq i \leq m$,pendent vertices of $K_{1,m}$ are $u_j, 1 \leq j \leq m$. The following procedure will give proper coloring. Assign color $C_1$to v_0 and color C_2 to u_0 and color C_3 to all the other pendent vertices. The root vertices dominate itself, each pendent vertex of $k_{1,m}$ dominate color class C_1 and pendent vertex of $k_{1,m}$ dominate color class C_2.This procedure will assure that the dominator coloring is proper. Every pair of vertices has rainbow path between them. Therefore, the rainbow dominator color for Bi Star $K_{1,m,m}$, $\chi_{rd}(K_{1,m,m})$ =3.

Example 3

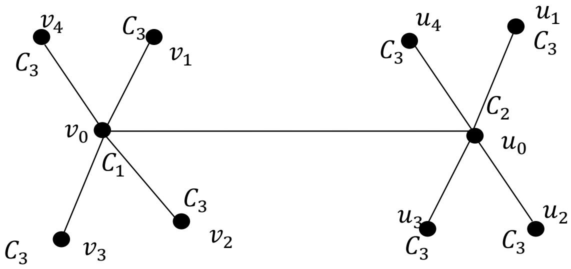

Fig. 3 the Bi-Star graph $K_{1,4,4}$

Dominating set	Dominated color classes	Chromatic number
v_0	U0	3
u_0	v_0	
$K(1,m)$	$color\ class\ C_1$	

Theorem 4

For any $n \geq 3$, the rainbow dominator coloring of the Helm graph $\chi_{rd}(H_n) = n + 1$.

Proof:

We prove this theorem by method of induction.

First, we prove the theorem for $n = 3$

Let v_1 be the central vertex and the vertices on the cycle are v_2, v_3, v_4 and u_1, u_2, u_3 be the pendent vertices adjacent to v_2, v_3, v_4 respectively. Let us color the central vertex v_1 and pendent vertices u_1, u_2, u_3 with color C_1 and v_2, v_3, v_4 with color C_2, C_3 and C_4 respectively.And color classes $\{v_2\}, \{v_3\}, \{v_4\}$ dominates itself by definition of dominator coloring and vertex v_1, dominates the color classes $\{v_2\}, \{v_3\}, \{v_4\}$. Each pendent vertex dominates the adjacent vertex. This will ensure proper dominator coloring for the helm graph, when n is 3. And for every pair of vertices we can find rainbow path between them. the rainbow dominator coloring of the Helm graph $\chi_{rd}(H_3) = 4$.

We prove that the rainbow dominator coloring for the helm graph H_{k+1}, $\chi_{rd}(H_{k+1}) = k + 2$

Let v_1 be the central vertex and the vertices on the rim be $v_2, v_3, v_4, \ldots, v_{k+1}$ and $u_1, u_2, u_3, u_4, \ldots, u_k$ be the pendent vertices adjacent to $v_2, v_3, v_4, \ldots, v_{k+1}$. Let us color the central vertex v_1 and pendent vertices $u_1, u_2, u_3, u_4, \ldots, u_k$ with color C_1, and $v_2, v_3, v_4, \ldots v_{k+2}$ with color C_2, C_3, C_4,…, C_{k+2} respectively. The vertex v_1, dominates the color classes $\{v_2\}, \{v_3\}, \{v_4\}, \ldots, \{v_{k+2}\}$. Each pendent vertex dominates the adjacent vertex with the color class $\{v_i\}, 2 \leq i \leq k + 2$. The above procedure provides a proper dominator coloring. And for every pair of vertices, we can find rainbow path between them. The rainbow dominator coloring of the Helm graph $\chi_{rd}(H_k) = k + 1$. □

Example 4

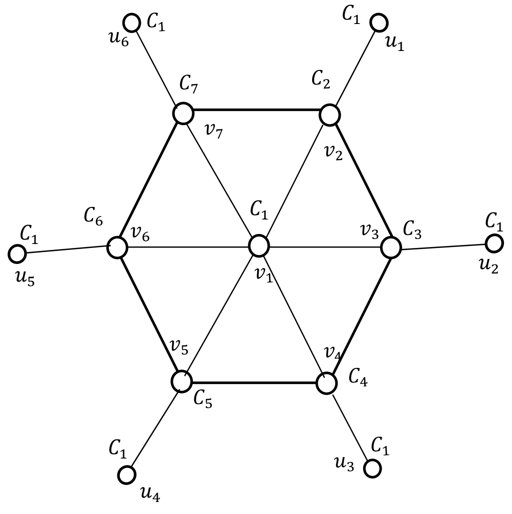

Fig. 4 the helm graph H_6

Dominating set	Dominated vertices	Chromatic number
v_1	v_2 , $v_{3,}v_4$, $v_{5,}v_6$, $v_{7,}$	K+1
u_i	v_{i+1}	

Theorem 5

For any $n \geq 3$, the rainbow dominator coloring of the Sunlet graph $\chi_{rd}(Sl_n) = n + 1$

Proof

Let v_1 be the central vertex and the vertices on the rim be $v_2, v_3, v_4, \ldots, v_n$ and $u_1, u_2, u_3, u_4, \ldots, u_n$ be the pendent vertices adjacent to $v_1, v_3, v_4, \ldots, v_n$. Let us color the pendent vertices $u_1, u_2, u_3, u_4, \ldots, u_n$ with color C_1nd $v_1, v_2, v_3, v_4, \ldots v_n$ with color C_2, C_3, C_4,…, C_{n+1} respectively. The vertex $\{v_1\}$, $\{v_2\}$, $\{ v_3\}$, $\{v_4\}$, …, $\{v_n\}$ dominates the themselves. Each pendent vertex dominates the adjacent vertex with the color class $\{v_i\}, 1 \leq i \leq n + 1$. The above procedure provides a proper dominator coloring. And for every pair of vertices, we can find rainbow path between them. The rainbow dominator coloring of the Sunlet graph $\chi_{rd}(Sl_n) = n + 1$.

EXAMPLE 5

Dominating set	Dominated vertices	Chromatic number
v_i i = 1 to n	itself	n

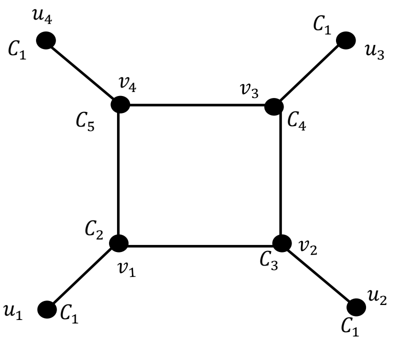

Fig. 5 the sunlet graph Sl_4

Dominating set	Dominated color classes	Chromatic number
v_i i = 1 to n	itself	n+1
Pendant vertex	Adjacent vertices	

Theorem 6

For any $n \geq 3$, the rainbow dominator coloring of the Compete graph $\chi_{rd}(K_n) = n$

Proof

Let the vertices on the Complete graph be $v_1, v_2, v_3, v_4, \ldots, v_n$. Let us color the pendent vertices $v_1, v_2, v_3, v_4, \ldots v_n$ with color C_1, C_2, C_3, C_4,…, C_n respectively. The vertex $\{v_1\}, \{v_2\}, \{ v_3\}, \{v_4\}, \ldots, \{v_n\}$ dominates themselves.The above procedure provides a proper dominator coloring. And for every pair of vertices, we can find rainbow path between them. The rainbow dominator coloring of the complete graph $\chi_{rd}(Sl_n) = n + 1$.

EXAMPLE 6

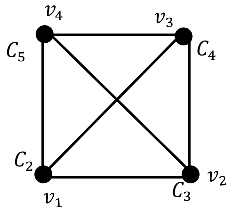

Fig. 6 complete graph K_4

Theorem 7

For any $m, n \geq 3$, the rainbow dominator coloring of the Jelly fish graph $V\left(J(m,n)\right) is\ 4$

Proof

Let V (J(m,n) = { Vi : 1≤ I ≤ 4 ,u_i : 1≤ I ≤ m,w_i : 1≤ I ≤ n be the vertex set and EJ_n ($v_1 v_2$, $v_1 v_4$, $v_2 v_3$, $v_3 v_4$, v_2 ui: 1≤ I ≤ m , v_4 w_1: edge set of jelly fish graph .It is obtained by a 4 cycle v_1, v_2 , v_3, v_4, by joining v_1 and v_3 with an edge and appending i+n pendant edges to v_2 and n pendant edges to v_4. The vertices are named as v_1, v_2,v_3, v_4, The pendant vertices are $u_1, u_2, u_3, u_4, w_1, w_2, w_3, w_4$. Assign the color c_1, c_2, c_3, c_4 to the vertices v_1 , v_2 , v_3 , v_4 respectively. The m pendent vertices $u_1, u_2, u_3, \ldots, u_m$ affixed with v_1, colored with c_2, similarly n pendent vertices $w_1, w_2, w_3, \ldots, w_n$ affixed with v_3, colored with c_2. Thus, every vertex will dominate atleast one color class. And for every pair of vertices there exists a rainbow path. The rainbow dominator chromatic number for the Jelly fish graph is 4

Example 7

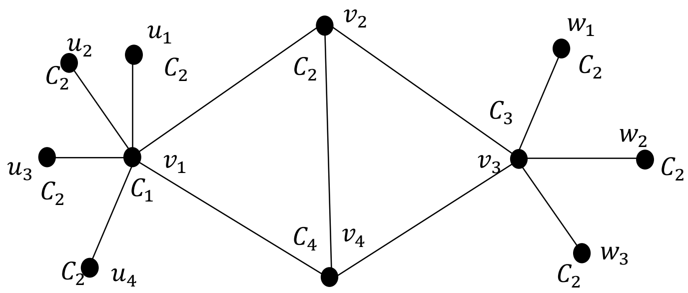

Fig. 7 Jelly fish graph $JF(4,3)$

Dominating set	Dominated color classes	Chromatic number
v_i	Vertices with Color class c_i	4
u_i	Vertices with Color class c_2	
w_i	Vertices with Color class c_2	

Theorem 8

$For\ m, n \geq 3$, the rainbow dominator coloring of the Jewel graph $\chi_{rd}(\mathrm{V(J_n)}) = 3$

Proof:

Let Jn be the Jewel graph with vertex set V (Jn) = {u,x,v,y,vi : 1 ≤ i ≤ n} and the edge set E(Jn) = {ux, vx, uy, vy, xy, uvi, vvi : 1 ≤ i ≤ n}. We color the vertices as follows

X as c_1 , u and v as c_3 ,y as c_2,$v_1$as c_1,v_2 as c_2 and v_3 as c_1.

Y dominates color class c_3. x dominates color class C_3 and u_1,u_2,u_3 dominates color class c_3

Thus, every vertex will dominate atleast one color class. And for every pair of vertices there exists a rainbow path. The rainbow dominator chromatic number for the Jewel graph V(J_n) is 3 $\chi_{rd}(\mathrm{V(J_n)}) = 3$.

Example 8

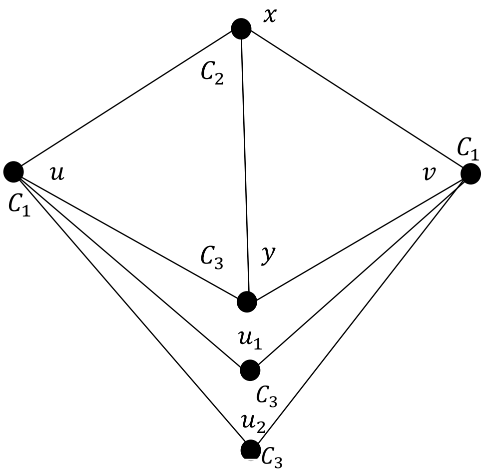

Fig. 8 Jewel graph J_2

Dominating set	Dominated color classes	Chromatic number
x	C_3	4
y	C_3	
u_1, u_2, u_3	C_3	

Theorem 9

For any $n \geq 3$, the rainbow dominator coloring of the Triangular book graph $\chi_{rd}(B(3, n) = 3$

Proof

Let the Triangular Book $B(3,n)$ is a graph with vertex set $V(B(3,n)) = \{v_1, v_2, v_3, v_i : 4 \leq i \leq n+2\}$ and the edge set $E(B(3,n)) = \{v_1v_2, v_2v_3, v_1v_3, v_1v_i, v_2v_i : 4 \leq i \leq n+2\}$. The triangular book graph B(3,n) is colored as follows. Assign the color c_1 to the vertex v_1, color c_2 to the vertex v_2. The vertices $v_3, v_4, \ldots, v_{n+2}$ joined with v_1 and v_2 colored with c_3.

The vertex v_1 as c_1, v_2 as c_2 and v_3 ,v_4,v_5 ,v_6 as c_3.v_1 dominates v_2 .The vertex v_2 dominates color C_1.The vertices v_3 ,v_4,v_5 ,v_6 dominates color class c_1. The vertex v_1 dominates color C_2. Thus, every vertex will dominate atleast one color class. And for every pair of vertices there exists a rainbow path. The rainbow dominator chromatic number for the Triangular book graph $B(3,n)$ is 3 $\chi_{rd}(B(3, n) = 3$.

Example 9

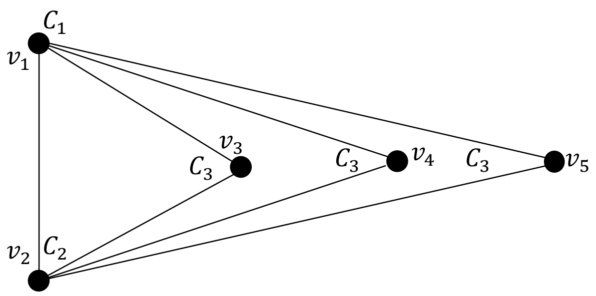

Fig.9 Triangular Book Graph $B(3,3)$

Dominating set	Dominated color classess	Chromatic number
V_1	C_2	3
V2	C_1	
v_3 ,v_4,v_5 ,v_6	V1	

Theorem 10

For any $n \geq 3$, the rainbow dominator coloring of the Triangular book graph with book mark χ_{rd}(TB_n) is 3

Proof

Let the Triangular Book with bookmark TB_n is a graph with vertex set $V(TB_n) = \{v_1, v_2, v_3, v_i : 4 \leq i \leq n+3\}$. The following procedure confirms the power dominator coloring. Assign the color c_1 to the vertex v_1, color c_2 to the vertex v_2. The vertices $v_3, v_4, \ldots, v_{n+2}, v_{n+3}$ are colored with c_3. The vertex v_1 as c_1, v_2 as c_2 and v_3 ,v_4,v_5 ,v_6 as c_3 and v_7 as c_3. V_1 and v_7 dominates each other .The vertex v_2 dominates v_1.The vertices v_3,v_4,v_5 ,v_6 dominates v_1. Thus, every vertex will dominate atleast one color class. And for every pair of vertices there exists a rainbow path. The rainbow dominator chromatic number for the Triangular book graph with book mark χ_{rd}(TB_n) is 3

Example 10

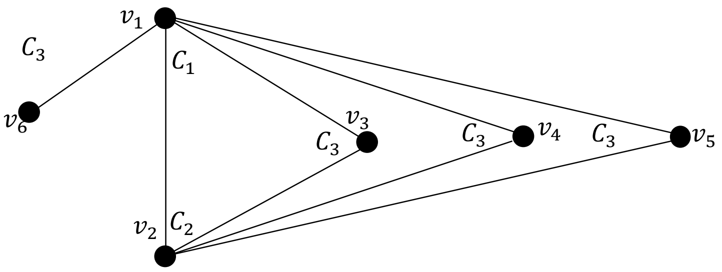

Fig. 10 Triangular Book with bookmark TB_3

Dominating set	Dominated color classes	Chromatic number
V_1	C_2	3
V2	C_1	
v_3 ,v_4,v_5 ,v_6	C3	
V_7	C_3	

CONCLUSION

The idea of this paper is to find rainbow dominator coloring of connected and undirected finite graphs.The rainbow dominator chromatic number for graphs like Complete Graph, Helm graph, sunlet graph, Jelly fish, Jewel graph, Extended jewel graph, Triangular book, Triangular book with bookmark and bull graph. and bull graph which is denoted as χ_{rd} are determined. The basic parameters for this Rainbow dominator coloring is the existing concepts, rainbow path and dominator coloring of every vertex. It is verified that every pair of vertices has a rainbow path and also satisfying the condition that every vertex dominates the other .The researchers can find rainbow coloring for more number of connected finite graphs .

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