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A Study on n-Connected Total Perfect k-Domination in Fuzzy Graphs with Application

T.Gunasekar[1] and K.Elavarasan[2]

[2]Research Scholar, [1,2]Department of Mathematics, Vel Tech Rangarajan Dr.Sagunthala R&D Institute of Science and Technology, Chennai – 62, Tamil Nadu, India.

[1]Corresponding Author

Abstract

In this paper, we introduce the concept of n-connected total perfect k-domination in fuzzy graphs. We have generalized triple connected total perfect dominating set to n-connected total perfect k-dominating set. New concepts are compared with the old one and generalized values are calculated for some of the major classes like cycles and trees in fuzzy graphs. Also n-connected total perfect k-dominating number of fuzzy graphs is obtained.

Keywords: Perfect k-dominating set, n-connected total perfect dominating set, n-connected total perfect k-dominating set and number.

1. INTRODUCTION

Fuzzy graph theory has become a burgeoning field of research study today. The idea of fuzzy relation and fuzzy set are initiated in 1965 by L.A.Zadeh[17]. In 1975, Rosenfeld[14] initiated the idea of fuzzy graph and theoretical ideas such as paths, loops and connectivity. In 1998, the theory of dominance in fuzzy graphs started with A.Somasundaram and S.Somasundaram [16]. The idea of perfect domination and total dominating set initiated by Cokayne et al[4]. Revathi et al[11,12] initiated about the idea of connected total perfect domination of fuzzy graph. The most important and applicable concept in fuzzy graph theory is connectivity. We redefine about the idea of triple connected total perfect dominating set and number in fuzzy graphs. Chaluvaraju et al[3] generalized perfect domination to perfect k-domination in graphs. The motive of the present paper is to initiate the concept of n-connected total perfect k-dominating set and number in a fuzzy graph and interpret some results of fuzzy graph. Also n-connected total perfect k-dominating set has been defined. Section 2 contains preliminaries, section 3 some results on n-connected total perfect domination in fuzzy graphs and n-connected total perfect k-dominating set and number are some of the topics tackled in this section and finally section 4 discussed application to our results. All the fuzzy graphs considered here are finite, undirected with no loops and multiple edges. As usual p $= |V|$ and m$= |E|$ denote the number of vertices and edges of a fuzzy graph G.

2. PRELIMINARIES

Definition 2.1[17]

The fuzzy set of a base set or reference set V is specified by its function of membership σ, where $\sigma : V \to [0,1]$ assigning to each $u \in V$ the degree or grade to which u belongs to σ.

Definition 2.2[17]

There are two fuzzy sets σ and τ of a set V, then the set σ is called a fuzzy subset of τ, if $\tau(u) \leq \sigma(u)$ each $u \in V$.

Definition 2.3[14]

Let $G = (\sigma, \mu)$ is called a fuzzy graph, if there exist a set of functions of membership $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$ such that $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$.

Definition 2.4[14]

If $\tau(u) \leq \sigma(u)$ where $u \in V$ and $(u, v) \leq (u, v)$ for every $u, v \in V$, then $H = (\tau, \rho)$ is said to be a fuzzy subgraph of a fuzzy graph G.

Definition 2.5[14]

If $\tau(u) \leq \sigma(u)$ where $u \in V$ and $(u, v) \leq (u, v)$ for every $u, v \in V$, then H is called a spanning fuzzy subgraph of a fuzzy graph G.

Definition 2.6[8]

Order $p = \sum_{(u \in V)} \sigma(u)$ and size $q = \sum_{((u,v) \in E)} \mu(u, v)$.

Definition 2.7[6]

If $\mu^{\infty}(u, v) \leq (u, v)$ for every $u, v \in V$, then arc (u, v) is called a strong arc. Where $\mu^{\infty}(u, v)$ be the strongest path strength and the vertex u is said to be a strong neighbor to v, otherwise it is called weak arc. The vertex u is called a isolated in G if $(u, v) = 0$ every $v \neq u$, $v \in V$.

Definition 2.8[9]

$d_N(v) = \sum_{(u \in N_s(v))} \sigma(u)$, $\delta_N(G) = \min\{d_N(u): u \in V(G)\}$ and $\Delta_N(G) = \max\{d_N(u): u \in V(G)\}$.

Definition 2.9[9]

If $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ for every $u, v \in V$, then the fuzzy graph G is called a complete fuzzy graph. It is described by K_σ.

Definition 2.10[9]

There is a bipartition V_1 and V_2 of G. If every vertex in V_1 has a strong neighbor in V_2 and also V_2 has a strong neighbor in V_1, then the bipartition (V_1, V_2) is called a complete bipartite fuzzy graph of G. It is identified by $K_{(m,n)}$.

Definition 2.11[9]

An edge $xy \in \mu^*$ is called a fuzzy bridge if its removal reduces the strength of connectedness between some pair of vertices in G. similarly a vertex $v \in \sigma^*$ is said to be a fuzzy cut vertex of G if its removal decreases the strength of connectedness between some other pair of vertices.

Definition 2.12[6]

A connected fuzzy graph G (σ, μ) is a fuzzy tree if it has a fuzzy spanning subgraph F= (σ, v) which is a tree, where for all edges xy not in F, there exists a path from x to y in F, whose strength is more then $\mu(xy)$.

Definition 2.13[13]

The fuzzy vertex connectivity of a connected fuzzy graph G is defined as the minimum strong weight of fuzzy vertex cuts of G. It is denoted by N(G). Similarly the fuzzy edge connectivity of a connected fuzzy graph G is defined as the minimum strong weight of fuzzy edge cuts of G. It is denoted by M(G).

Definition 2.14[2,4,10,16]

If (u, v) be a strong arc, then the node u dominates the node v for every node $u, v \in V$of G. A subset D of V is called a 2-dominating set of G if for every node v ∈ V−D there exist atleast two strong neighbours in D.

Definition 2.15[1]

A set D⊆ V of a fuzzy graph G = (V,σ, μ) is a fuzzy k-dominating set of G if for every node u ∊ V-D there exist at least k strong arcs (u,v) for v ∊ D. The minimum fuzzy cardinality of a fuzzy k-dominating set in G is called the fuzzy k-dominating number $\gamma_{kD}$of G.

Definition 2.16[3]

If for every node v not in a subset P of Vwhich dominated by absolutely k nodes of P, then P is called a perfect k-dominating set of G. It is identified by P_{kD}. The minimum cardinality of a perfect k-dominating set of G is the perfect k-domination number γ_{PkD}(G).

Definition 2.17[13]

If there is a subgraph P_c of G which is connected and induced by P_D of G, then P_c is said to be connected P_D.

Definition 2.18[7]

If for each node of G be dominates to at least a node of P_t of G, then P_t is called a total P_D of G.

Definition 2.19[13]

A total P_D of G is called a connected total P_D if the induced subgraph total P_D is connected. It is identified by $c$tp(G).

Definition 2.20[13]

A ctp of G is called a minimal $c$tp(G) if for all node in , $ctp - \{v\}$ is not $ctp(G)$.

Definition 2.21[13]

$\gamma_{ctp}(G) = \min\{ctp(G)\}$ and $\Gamma_{ctp}(G) = \max\{ctp(G)\}$.

Definition 2.22[5,15]

If there are three nodes connected and lying on a path T_c of G, then $T_c(G)$ called triple connected fuzzy graph.

3. MAIN RESULTS

a) n-Connected total perfect k-domination in fuzzy graph

In the present section, we initiate the new concept of n-connected total perfect k-domination to n-connected total perfect k-dominating set (k_{nctp}) and number (γk_{nctp}) of fuzzy graph, which is the generalization of Tctp(G).

Definition 3.1

A ctp(G) is called a triple connected total perfect dominating set if the induced subgraph <ctp(G)> is triple connected. It is denoted by Tctp(G).

Definition 3.2

Let G $= (\sigma, \mu)$ is said to be n-connected for $n \in (0, \infty)$ if N(G)$\geq n$. That is, G is n-connected if there exists no fuzzy vertex cut with strong weights less then n.

Definition 3.3

A connected total perfect k- dominating set is said to be n-connected total perfect k-dominating set if the induced subgraph $< kctp(G) >$ is n-connected, denoted by k_{nctp}(G). The smallest number of vertices in n-connected total perfect k-dominating set of G is called its number, denoted by γk_{nctp}(G).

Observation 3.1. Every vertex with degree at most k-1 belongs to any n-connected total perfect k-dominating set for every fuzzy graph G and positive integer k.

Observation 3.2. Any n-connected total perfect k-dominating set is a perfect k-dominating set, and hence γ_{PkD}(G)$\leq \gamma k_{nctp}$(G) for every graph G and positive integer k.

Theorem 3.1. Let n, k be positive integers and G be a fuzzy graph such that γk_{nctp}(G) $\geq \frac{n+1}{n}(k-1)$. Then, γk_{nctp}(G) $\leq \frac{np}{n+1}$.

Proof: Let G= (σ, μ) be a fuzzy graph with the underlying crisp graph G^*= (V,E)

Where σ: V → [0,1] and μ: V x V →[0,1], E ⊆ V x V are mapping.

Assume that |V| = p.

Now $v_1, v_2, \ldots v_{n+1}$ be a partition of V(G) into n+1 subsets. Now, Consider the fuzzy spanning sub graph H(σ′, μ′) of G= (σ, μ) such that σ′ = σ′ and μ′ ≤ μ.

Then σ′(u) = σ′(u) for u in V and μ′(e) ≤ μ(e)for every e in E.

Now we consider E(H) = E(G) – $\bigcup_{i=1}^{n+1}$ E(⟨v_i⟩) and ⟨v_i⟩ is the sub graph induced by the node v_i.

For every vertex x in V, We know that $kctp_H(G) \geq k$ __________ (3.1)

Consider without loss of generality that $|V_1| = max_{1\leq i\leq n+1}|V_i|$

From (3.1) $V(G) - V_1$ be a $kctp(G)$ dominating set.

Therefore γk_{nctp}(G) $= |V(G) - V_1|$

$$\leq |V(G)| - |V_1|$$

$$\leq p - \frac{p}{n+1}$$

Hence, γk_{nctp}(G) $\leq \frac{np}{n+1}$.

Theorem 3.2. If there is γk_{nctp}(G) in a fuzzy graph with maximum degree Δ, then

$$\frac{p}{2(\Delta+1)} \leq \gamma k_{nctp}(G) \leq 2q - p + 1.$$

Proof: For lower bound, each membership values of k_{nctp}(G) can dominate at most maximum degree Δ membership values and itself.

Hence $\frac{p}{2(\Delta+1)} \leq \gamma k_{nctp}$(G)………………(3.2)

Now we consider the upper bound, if there is γk_{ctp}(G) then,

γk_{nctp}(G) $\leq p-1$

γk_{nctp}(G) $\leq 2q-p+1$…………… (3.3)

From the equations (3.2) and (3.3), we have proved the theorem.

Theorem 3.3. If k_{nctp}(G) is a fuzzy graph without isolated edge, then γk_{nctp}(G) $\leq \frac{q}{\Delta(G)+1}$.

Proof: let D be an n-edge connected total perfect k-dominating set of G

Since, $|D|\ \Delta(G) \leq \sum_{e\in D} d_E(e) = \sum_{e\in D}|N(e)|$

Thus, γk_{nctp}(G) $\leq \frac{q}{\Delta(G)+1}$.

Observation 3.3. In a fuzzy graph G if γk_{nctp}(G) $\geq 2n-1$, then γk_{nctp}(G) $\leq \frac{p}{2}$.

Observation 3.4. Let D be a n-connected total perfect k-dominating set of a fuzzy graph G then no bridge exist between any two vertices of k_{nctp}(G) .

Observation 3.5. If there is H be a k-connected spanning subgraph of k_{nctp}(G),

then γk_{nctp}(G) $\leq$ γk_{nctp}(H).

Theorem 3.4. If ƙ = Δ(G) − 1, then G is a k_{nctp}(G) if and only if it satisfies one of the conditions below

(i) At least two adjacent vertices u, v exist, such that deg(u) = deg(v) = Δ(G).

(ii) There is a vertex u for which deg(u) = Δ(G) − 1.

Proof: Assume that G is a k_{nctp}(G) in fuzzy graph with ƙ = Δ(G) − 1. We must prove condition (i), if there are no vertices of degree Δ(G) − 1 in k_{nctp}(G). Let D is n-connected total perfect k-dominating set in fuzzy graph G. Then in D, there is at least one vertex v ∈ V −D with Δ(G)−1 neighbours. However, we assume that no vertex in G has degree Δ(G) − 1, therefore deg(v) > Δ(G) − 1. Hence deg(v) = Δ(G). Thus there is a vertex w ∈ V − D adjacent to v and clearly w has degree Δ(G). Thus satisfies (i). Conversely, If two adjacent vertices of degree Δ(G) exist, we can construct Δ(G) − 1 in k_{nctp}(G) in fuzzy graph by using all other vertices as elements of D. Now, V (G) −{u} is a Δ(G) − 1 in k_{nctp}(G) set in fuzzy graph if there is a vertex u of degree Δ(G) exist. For ƙ = Δ(G) − 1 in either instance, G is a k_{nctp}(G) in fuzzy graph.

Theorem 3.5. If there exist k_{nctp}(G) and fuzzy edge cut E, H be a fuzzy graph obtained by adding a new vertex x to G and joining x to the end vertex of E. let $\mu(v_i x) = \min\{\mu(v_i u) : v_i u$ is a strong edge in G. then H is n-connected total perfect k-dominating set.

Theorem 3.6. Assume that T is a k_{nctp}(G) -tree. Then one or more of the following applies:

(i) In T, there is at least one vertex with degree ƙ.

(ii) There exist at least two vertices u and v of degree ƙ + 1 such that every interior vertex in the $u-v$ path has degree greater than ƙ + 1.

Proof: Let D is n-connected total perfect k-dominating tree T that has a k_{nctp}(G) set in fuzzy graph. Assume that (i) is false. We must then prove that (ii) is correct. As a result, the following situations are considered:

Case (i) In $V-D$, there is at least one vertex of degree ƙ + 1. Let u be a vertex in $V-D$ with degree ƙ + 1. u is next to a vertex $u_1 \in V-D$ since u $\in V-D$ and deg(u) = ƙ + 1. Since $u_1 \in V-D$, deg(u_1) = ƙ + 1 once more. Case (ii) holds if deg(u_1) = ƙ + 1. If not, at least one vertex $u_2 \in V-D$ is next to u_1. Continuing the same arguments, we can prove that (ii) holds since T is a finite tree.

Case (ii) A vertex of degree ƙ + 1 does not exist in $-D$. Let u be a vertex with a degree larger than ƙ + 1 $\in V-D$. Then in $V-D$, u has at least two neighbours. Assume u_1 is one of these neighbours. u_1 has a neighbour $u_2 \in V-D$ and deg(u_2) > ƙ + 1 , because $u_1 \in V-D$ and deg(u_1) > ƙ + 1. This argument clearly never ends, which contradicts the assumption that T is finite. As a result, the only option is Case (i), for which (ii) holds.

Observation 3.6. Every n-connected total perfect k-dominating set of a fuzzy graph G is connected total perfect k-dominating set of a fuzzy graph G.

Remark 3.1. There is no k_{nctp}(G) set, if G is a K_σ.

Remark 3.1. There is no k_{nctp}(G) set, if G is a $K_{(m,n)}$.

4. APPLICATION

Fuzzy graph models are now being used in a variety of fields of science and technology, including computer science, topology, operations research, biology, and social sciences. It has been found that in a social group, some people can influence others, and this can only happen when they have a strong bond. The nodes represent a person's level of authority within a particular social group. True membership and false membership are used to determine a person's level of power. True membership can be defined as how much power a person has, whereas false membership can be defined as how much power a person loses. The percentage of true and false membership influence can be understood as the degree of true and false membership of edges. As a result, dominance can assist us in identifying individuals with strong social connections. Also, imagine we have a group of small villages in a faraway location. We would like to establish radio stations in a few of these towns so that massages can be broadcast to the entire region. We must employ multiple radio stations to reach all towns because each radio station has a limited broadcasting range. However, due to the high expense of radio stations, we wish to locate as few as feasible that can reach all other settlements. Allow a vertex to represent each village. The distance between two villages is labeled on an edge between two villages, say in kilometers. We may effectively delete all edges in the graph that show a distance of more than fifty kilometers because we have assumed that a radio station has a broadcast range of only fifty kilometers. We only need to locate a dominating set in fuzzy graph with n-connected perfect k-domination. Three radio stations would be sufficient if we could afford radio stations with a broadcast range of seventy kilometers.

5. CONCLUSIONS

The majority of the principles in fuzzy graph theory, which was newly established, can be immediately applied to issues that were previously addressed algorithmically. In fuzzy graphs, this article attempts to generalize some of the existing triple connected total perfect domination results. Also n-connected total perfect k-dominating set and number are obtained. In addition, some applications to n-connected total perfect k-domination of fuzzy graphs are discussed.

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