Ranking of Linear and Non-Linear Hexagonal Fuzzy Number through Haar Wavelet

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Abstract: Fuzzy number is a multi valued quantity and it plays a crucial role in all the decision making problems to represent the value for the linguistic terms. In the previous decade, triangular, trapezoidal and pentagonal fuzzy numbers were used to solve fuzzy decision making problems. In this article, Linear and Non-linear Hexagonal fuzzy number are derived to overcome the fuzziness arisen in six different points. And recently, most of the researchers have focused on different types of ranking method on fuzzy number. Hence, Haar Wavelet ranking method for hexagonal fuzzy number is proposed. Numerical Illustration is also given to check the validity of the ranking method. Fuzzy assignment problem has been taken to determine the minimum transportation cost for proper disposal of solid wastages.

Keywords: Hexagonal Fuzzy Number, α -cut, Haar ranking method, Solid Waste.

1 Introduction

Fuzzy sets have been introduced by (Zadeh, 1965) to tackle imprecision data in real life problems. In 2003 (Coxe & Reiter, 2003), used fuzzy automata on a hexagonal background using simple arithmetic combinations of nearby fuzzy values. In 2013, (Rajarajeswari & Sudha, 2013) used interval arithmetic in a new operation for addition, subtraction and multiplication of Hexagonal Fuzzy number on the basis of alpha cut sets of fuzzy numbers. (Rajarajeswari & Sudha, 2014) generalized hexagonal fuzzy numbers by rank, mode, divergence and spreads to optimize the decision making, approximation and risk analysis. In the year of 2015, (Dhurai & Karpagam, 2016) used interval arithmetic to introduce a new membership function and satisfied the operation of addition, subtraction and multiplication of hexagonal fuzzy number on the basis of alpha cut sets of fuzzy numbers. In the year of 2015, (Thamaraiselvi & Santhi, 2015) introduced a fuzzy transportation problem with hexagonal fuzzy numbers by fuzzy zero point method. In the year of 2016, (Sudha & Revathy, 2016) used the centroid formula of triangle and rectangle in A new Ranking Fuzzy On Hexagonal numbers [6]. In the year of 2016, (Karpagam, 2016) used hexagonal fuzzy numbers in fuzzy optimal solution for fully fuzzy linear programming problems. In the year of 2016, Parvathi & Gajalakshmi, 2016) have used Verhulst's demand, which calculated the demand from initial stage to the maturity age of that product. In the year of 2016, (Animesh &Arnab, 2016) have developed a model to minimize the net system cost of sorting and transporting the wastes and to maximize the revenue generated from different treatment facilities. Ordering cost for the buyer, holding cost for the buyer, fixed backorder cost and order quantities were taken as hexagonal fuzzy numbers. In the year of 2017, (Christi & Devi, 2017) used Fuzzy geometric programming approach to determine the optimal solution of a multi-objective two stage fuzzy transportation problem in which supplies, demands were hexagonal fuzzy numbers and fuzzy membership of the objective function was defined to find out the best compromise solution among the set of feasible solutions for the multi-objective two stage transportation problem. In the year of 2017, (Ghadle & Pathade, 2017) used zero suffix method in results in a ranking method for generalized hexagonal and generalized octagonal fuzzy numbers. (Thorani & Ravi Shankar, 2017) have proposed Multi-objective Assignment model and is tested by considering an example with three parameters as fuzzy cost, fuzzy time and fuzzy quality with generalized LR trapezoidal fuzzy numbers and a practical data is tested with two parameters fuzzy cost and fuzzy time as generalized LR trapezoidal fuzzy numbers for a civil construction process by various cases. Fuzzy Transportation Problem has been converted into crisp values. In the year 2016, (Hussain & Priya, 2016) used minmax – maxmin principle in some operations of hexagonal fuzzy number and solve a fuzzy game problem with hexagonal fuzzy numbers.

2 Preliminaries

The following definitions are required in order to understand the fuzzy set, fuzzy number, and membership functions.

Definition 2.1 A fuzzy set (Zadeh, 1965) $\tilde{A}$ is a subset of a universe of discourse X, which is characterized by a membership function $\mu_{\tilde{A}}(x)$ representing a mapping $\mu_{\tilde{A}}: X \to [0,1]$. The function value of $\mu_{\tilde{A}}(x)$ is called the membership value, which represents the degree of truth that x is an element of the fuzzy set $\tilde{A}$.

Definition 2.2 A fuzzy set $\tilde{A}$ defined on the set of real numbers R is said to be a fuzzy number and its membership function $\tilde{A}: R \to [0,1]$ has the following characteristics,

(i) $\tilde{A}$ is convex.

$\mu_{\tilde{A}}\left(\lambda x_1 + (1-\lambda)x_2\right) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)),\ \forall x \in [x_1, x_2], \lambda \in [0,1].$

(ii) $\tilde{A}$ is normal if $\max \mu_{\tilde{A}}(x) = 1$.

(iii) $\tilde{A}$ is piecewise continuous.

Definition 2.3 The α-cut of the fuzzy set $\tilde{A}$ of the universe of discourse X is defined as $\tilde{A}_\alpha = \{x \in X / \mu_{\tilde{A}}(x) \geq \alpha\}$, where $\alpha \in [0,1]$.

Definition 2.4 A triangular fuzzy number $\tilde{A}$ can be defined as a triplet (*l*, *m*, *r*) and the membership function $\mu_{\tilde{A}}(x)$ is defined as:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x < l \\ \left(\dfrac{x-l}{m-l}\right) & l \leq x \leq m \\ \left(\dfrac{r-x}{r-m}\right) & m \leq x \leq r \\ 0 & x > r \end{cases}$$

Where *l*, *m*, *r* are real numbers and $l \leq m \leq r$.

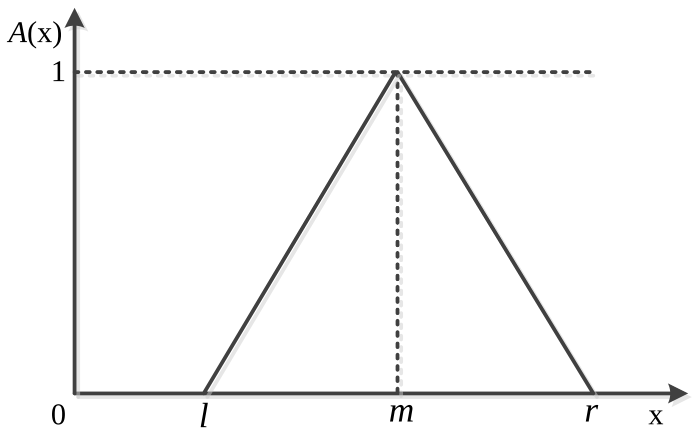

Figure 1: Triangular Fuzzy Number

Definition 2.5 A trapezoidal fuzzy number $\tilde{A}$ can be defined as $(a_1, a_{2,}, a_3, a_4)$ and the membership function is defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} \left(\dfrac{x-a_1}{a_2-a_1}\right) & a_1 \leq x \leq a_2 \\ 1 & a_2 \leq x \leq a_3 \\ \left(\dfrac{a_4-x}{a_4-a_3}\right) & a_3 \leq x \leq a_4 \\ 0 & x \leq a_1 \ \&\ x \geq a_4 \end{cases}$$

Where a_1, a_2, a_3, a_4 are real numbers $a_1 \le a_2 \le a_3 \le a_4$.

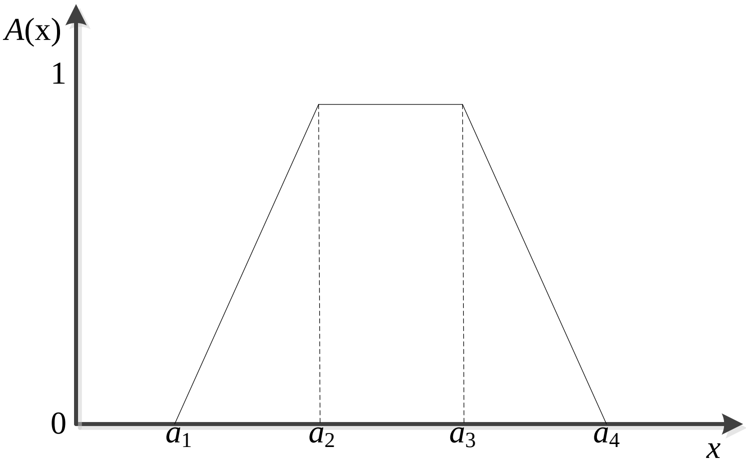

Figure 2: Trapezoidal Fuzzy Number

3 Hexagonal fuzzy number and its variations

In the following section, different types of hexagonal fuzzy numbers in different viewpoint are developed.

3.1 Hexagonal Fuzzy Number

A hexagonal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6)$ must satisfy the following conditions

1. $\mu_{\tilde{A}}(x)$ is normal and convex.
2. $\mu_{\tilde{A}}(x)$ is a continuous function in the closed interval [0, 1].
3. $\mu_{\tilde{A}}(x)$ is strictly increasing and continuous function on $[a_1, a_2]$, and $[a_2, a_3]$.
4. $\mu_{\tilde{A}}(x)$ is strictly decreasing and continuous function on $[a_4, a_5]$, and $[a_5, a_6]$.

3.1.1 Equality of two Hexagonal Fuzzy Numbers: Two hexagonal fuzzy numbers $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6)$ and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5, b_6)$ are equal if and only if $a_1 = b_1, a_2 = b_2, a_3 = b_3, a_4 = b_4, a_5 = b_5, a_6 = b_6$.

Next, new types of hexagonal fuzzy numbers in various forms are derived.

3.2 Linear Hexagonal Fuzzy Number with Symmetry

3.2.1 Linear Hexagonal Fuzzy Number with Symmetry: A linear hexagonal fuzzy number is written as $\tilde{A}_{LS} = (a_1, a_2, a_3, a_4, a_5, a_6; h)$ whose membership function is written as

$$
\mu_{\tilde{A}_{LS}}(x)=\begin{cases} h\left(\dfrac{x-a_1}{a_2-a_1}\right), & a_1 \le x \le a_2 \\ h+(1-h)\left(\dfrac{x-a_2}{a_3-a_2}\right), & a_2 \le x \le a_3 \\ 1, & a_3 \le x \le a_4 \\ h+(1-h)\left(\dfrac{a_5-x}{a_5-a_4}\right), & a_4 \le x \le a_5 \\ h\left(\dfrac{a_6-x}{a_6-a_5}\right), & a_5 \le x \le a_6 \\ 0, & x \le a_1 \ and \ x \ge a_6 \end{cases}
$$

3.2.2 Alpha cut of Linear Hexagonal Fuzzy Number with Symmetry:

$$
A_\alpha=\begin{cases} A_{1L}(\alpha)=a_1+\dfrac{\alpha}{h}(a_2-a_1) \ for \ \alpha\in[0,h] \\ A_{2L}(\alpha)=a_2+\left(\dfrac{\alpha-h}{1-h}\right)(a_3-a_2) \ for \ \alpha\in[h,1] \\ A_{2R}(\alpha)=a_5-\left(\dfrac{\alpha-h}{1-h}\right)(a_5-a_4) \ for \ \alpha\in[h,1] \\ A_{1R}(\alpha)=a_6-\dfrac{\alpha}{h}(a_6-a_5) \ for \ \alpha\in[0,h] \end{cases}
$$

Where $A_{1L}(\alpha)$, $A_{2L}(\alpha)$ are increasing functions with respect to α and $A_{2R}(\alpha)$, $A_{1R}(\alpha)$ decreasing functions with respect to α.

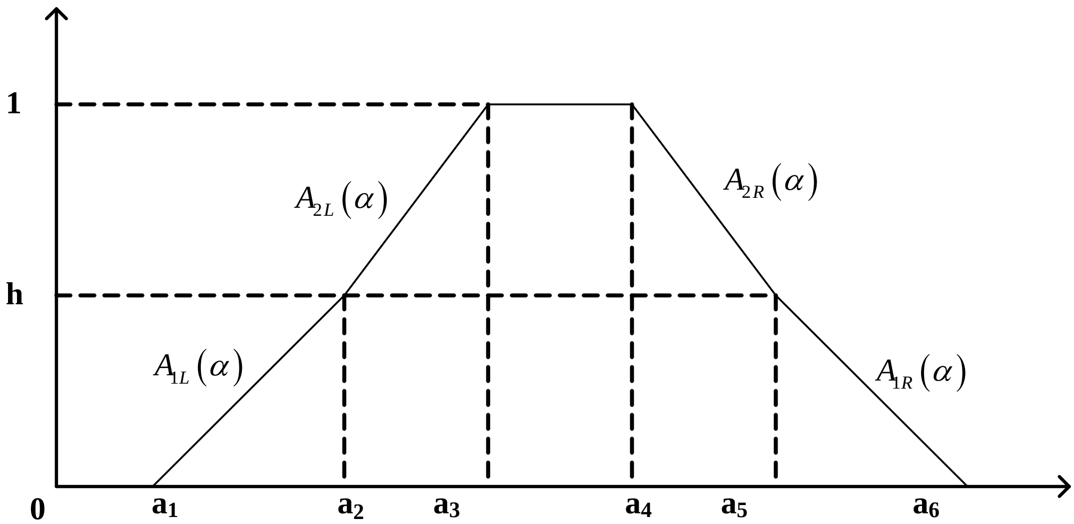

Figure 3: Linear Hexagonal Fuzzy Number with Symmetry

3.3 Linear Hexagonal Fuzzy Number with Asymmetry

3.3.1 Linear Hexagonal Fuzzy Number with Asymmetry: A linear hexagonal fuzzy number with asymmetry is written as $\tilde{A}_{LAS}=(a_1,a_2,a_3,a_4,a_5,a_6;h,k)$ whose membership function is written as

$$\mu_{\tilde{A}_{LAS}}(x)=\begin{cases} h\left(\dfrac{x-a_1}{a_2-a_1}\right), & a_1\le x\le a_2 \\ h+(1-h)\left(\dfrac{x-a_2}{a_3-a_2}\right), & a_2\le x\le a_3 \\ 1, & a_3\le x\le a_4 \\ k+(1-k)\left(\dfrac{a_5-x}{a_5-a_4}\right), & a_4\le x\le a_5 \\ k\left(\dfrac{a_6-x}{a_6-a_5}\right), & a_5\le x\le a_6 \\ 0, & x\le a_1 \text{ and } x\ge a_6 \end{cases}$$

Note:

1. If $h = k$ the asymmetry hexagonal fuzzy number becomes symmetry hexagonal fuzzy number.
2. For asymmetry hexagonal fuzzy number $h \neq k$.

3.3.2 Alpha cut of Linear Hexagonal Fuzzy Number with Asymmetry:

$$A_\alpha=\begin{cases} A_{1L}(\alpha)=a_1+\dfrac{\alpha}{h}(a_2-a_1) \text{ for } \alpha\in[0,h] \\ A_{2L}(\alpha)=a_2+\left(\dfrac{\alpha-h}{1-h}\right)(a_3-a_2) \text{ for } \alpha\in[h,1] \\ A_{2R}(\alpha)=a_5-\left(\dfrac{\alpha-k}{1-k}\right)(a_5-a_4) \text{ for } \alpha\in[1,k] \\ A_{1R}(\alpha)=a_6-\dfrac{\alpha}{k}(a_6-a_5) \text{ for } \alpha\in[0,k] \end{cases}$$

Where $A_{1L}(\alpha)$, $A_{2L}(\alpha)$ are increasing functions with respect to α and, $A_{2R}(\alpha)$ and $A_{1R}(\alpha)$ are decreasing functions with respect to α.

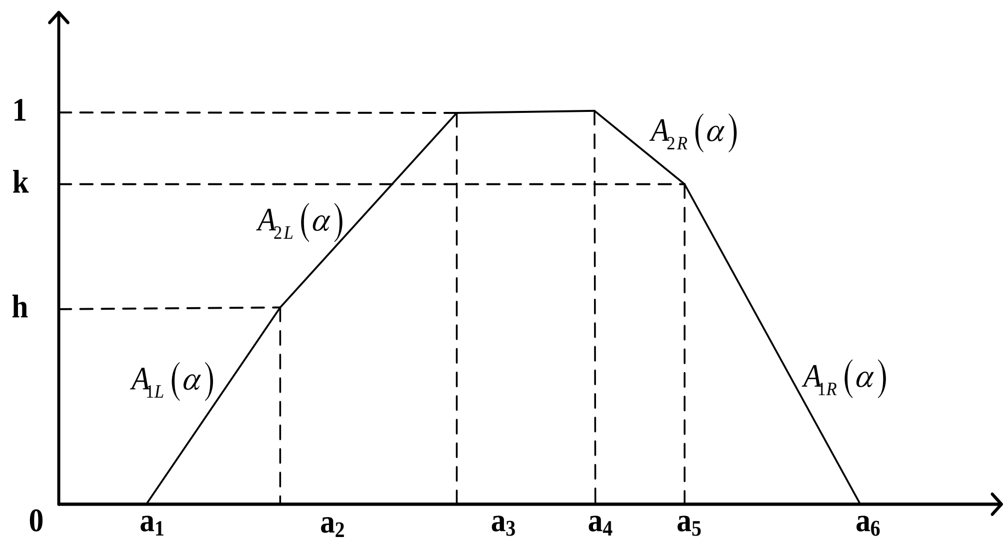

Figure 4: Linear Hexagonal Fuzzy Number with Asymmetry

3.4 Non Linear Hexagonal Fuzzy Number with Symmetry

3.4.1 Non Linear Hexagonal Fuzzy Number with Symmetry: A non linear hexagonal fuzzy number with symmetry is written as $\tilde{A}_{NLS} = (a_1, a_2, a_3, a_4, a_5, a_6; m, n,)_{(n_1, n_2; m_1, m_2)}$ whose membership function is written as

$$\mu_{\tilde{A}_{NLS}}(x) = \begin{cases} h\left(\frac{x-a_1}{a_2-a_1}\right)^{n_1}, & a_1 \le x \le a_2 \\ h+(1-h)\left(\frac{x-a_2}{a_3-a_2}\right)^{n_2}, & a_2 \le x \le a_3 \\ 1, & a_3 \le x \le a_4 \\ h+(1-h)\left(\frac{a_5-x}{a_5-a_4}\right)^{m_1}, & a_4 \le x \le a_5 \\ h\left(\frac{a_6-x}{a_6-a_5}\right)^{m_2}, & a_5 \le x \le a_6 \\ 0, & x \le a_1 \ and \ x \ge a_6 \end{cases}$$

3.4.2 Alpha cut of Non Linear Hexagonal Fuzzy Number with symmetry

$$\tilde{A}_\alpha = \begin{cases} A_{1L}(\alpha) = a_1 + \left(\frac{\alpha}{h}\right)^{n_1}(a_2 - a_1) \ for \ \alpha \in [0,h] \\ A_{2L}(\alpha) = a_2 + \left(\frac{\alpha - h}{1-h}\right)^{n_2}(a_3 - a_2) \ for \ \alpha \in [h,1] \\ A_{2R}(\alpha) = a_5 - \left(\frac{\alpha - h}{1-h}\right)^{m_1}(a_5 - a_4) \ for \ \alpha \in [1,h] \\ A_{1R}(\alpha) = a_6 - \left(\frac{\alpha}{h}\right)^{m_2}(a_6 - a_5) \ for \ \alpha \in [0,h] \end{cases}$$

Where $A_{1L}(\alpha)$, $A_{2L}(\alpha)$ are increasing functions with respect to α and $A_{2R}(\alpha)$ and $A_{1R}(\alpha)$ are decreasing functions with respect to α.

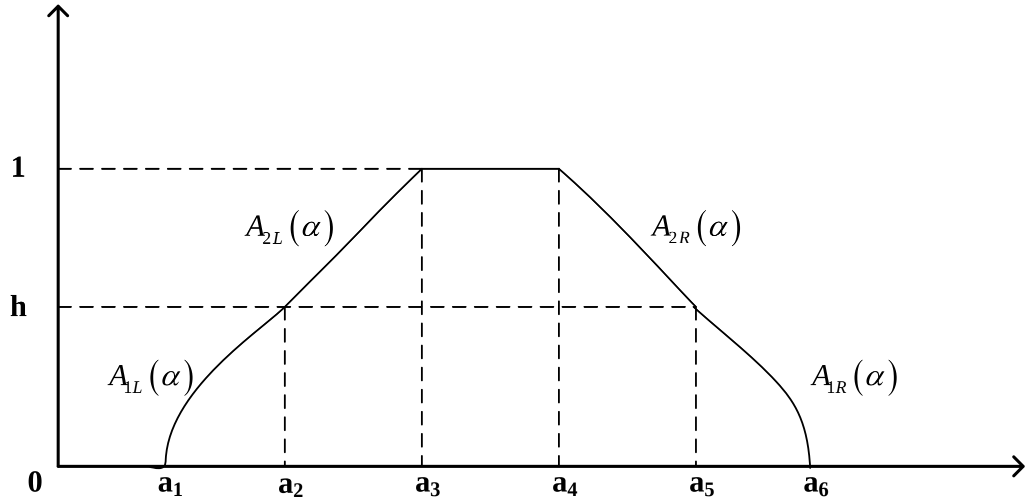

Figure 5: Non Linear Hexagonal Fuzzy Number with Symmetry

3.5 Non Linear Hexagonal Fuzzy Number with Asymmetry

3.5.1 Non Linear Hexagonal Fuzzy Number with Asymmetry

A non linear hexagonal fuzzy number with asymmetry is written as $\tilde{A}_{NLAS} = (a_1, a_2, a_3, a_4, a_5, a_6; m, n)_{(n_1, n_2; m_1, m_2)}$ whose membership function is written as

$$\mu_{\tilde{A}_{NLAS}}(x) = \begin{cases} h\left(\dfrac{x-a_1}{a_2-a_1}\right)^{n_1}, & a_1 \le x \le a_2 \\ h+(1-h)\left(\dfrac{x-a_2}{a_3-a_2}\right)^{n_2}, & a_2 \le x \le a_3 \\ 1, & a_3 \le x \le a_4 \\ k+(1-k)\left(\dfrac{a_5-x}{a_5-a_4}\right)^{m_1}, & a_4 \le x \le a_5 \\ k\left(\dfrac{a_6-x}{a_6-a_5}\right)^{m_2}, & a_5 \le x \le a_6 \\ 0, & x \le a_1 \text{ and } x \ge a_6 \end{cases}$$

Note: If $n_1 = n_2 = m_1 = m_2 = 1$, then non linear hexagonal fuzzy number becomes linear hexagonal fuzzy number.

3.5.2 Alpha cut of Non Linear Hexagonal Fuzzy Number with asymmetry:

$$A_\alpha = \begin{cases} A_{1L}(\alpha) = a_1 + \left(\dfrac{\alpha}{h}\right)^{n_1}(a_2 - a_1) \ for \ \alpha \in [0,h] \\ A_{2L}(\alpha) = a_2 + \left(\dfrac{\alpha - h}{1-h}\right)^{n_2}(a_3 - a_2) \ for \ \alpha \in [h,1] \\ A_{2R}(\alpha) = a_5 - \left(\dfrac{\alpha - k}{1-k}\right)^{m_1}(a_5 - a_4) \ for \ \alpha \in [1,k] \\ A_{1R}(\alpha) = a_6 - \left(\dfrac{\alpha}{k}\right)^{m_2}(a_6 - a_5) \ for \ \alpha \in [0,k] \end{cases}$$

Where $A_{1L}(\alpha)$, $A_{2L}(\alpha)$ are increasing functions with respect to α and $A_{2R}(\alpha)$ and $A_{1R}(\alpha)$ are decreasing functions with respect to α.

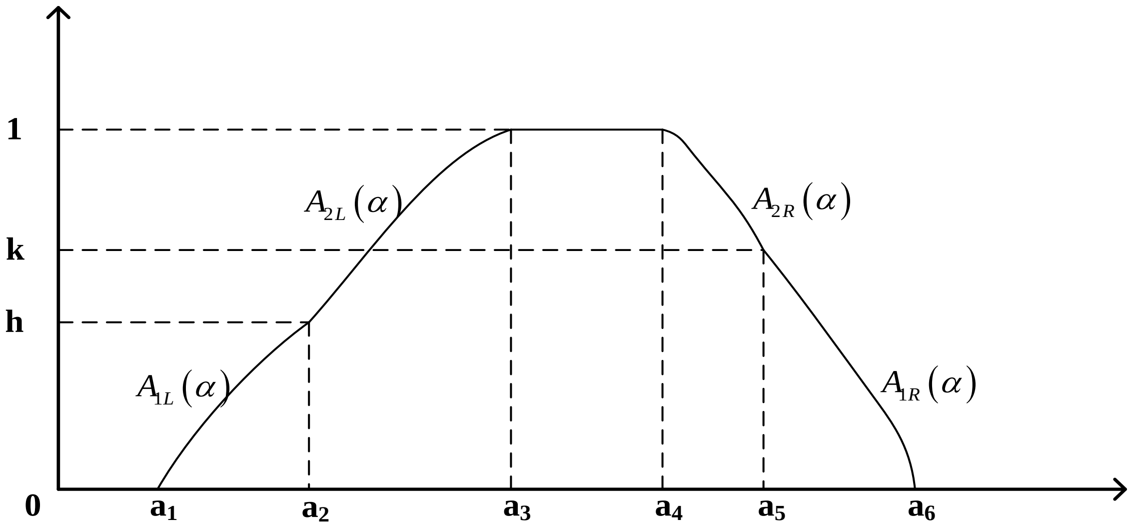

Figure 6: Non Linear Hexagonal Fuzzy Number with Asymmetry

4 Arithmetic operations on linear Hexagonal fuzzy number with symmetry

Let $\tilde{A}_{LS} = (a_1, a_2, a_3, a_4, a_5, a_6; m)$ be a linear hexagonal fuzzy number with symmetry, then

1. Multiplication by a scalar

If c is a positive scalar, then $c\tilde{A}_{LS} = (ca_1, ca_2, ca_3, ca_4, ca_5, ca_6; m)$ and if c is a negative scalar, then $c\tilde{A}_{LS} = (ca_6, ca_5, ca_4, ca_3, ca_2, ca_1; m)$.

2. Addition of two hexagonal fuzzy numbers

Consider two hexagonal fuzzy numbers $\tilde{A}_{LS} = (a_1, a_2, a_3, a_4, a_5, a_6; m_1)$ and $\tilde{B}_{LS} = (b_1, b_2, b_3, b_4, b_5, b_6; m_2)$, then the addition of two hexagonal fuzzy numbers is written as

$$\tilde{C}_{LS} = \tilde{A}_{LS} + \tilde{B}_{LS} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6; m)$$

Where $m = \min\{m_1, m_2\}$

3. Subtraction of two hexagonal fuzzy numbers

Consider two hexagonal fuzzy numbers $\tilde{A}_{LS} = (a_1, a_2, a_3, a_4, a_5, a_6; m_1)$ and $\tilde{B}_{LS} = (b_1, b_2, b_3, b_4, b_5, b_6; m_2)$, then the addition of two hexagonal fuzzy numbers is written as

$$\tilde{D}_{LS} = \tilde{A}_{LS} - \tilde{B}_{LS} = (a_1 - b_6, a_2 - b_5, a_3 - b_4, a_4 - b_3, a_5 - b_2, a_6 - b_1; m)$$

Where $m = \min\{m_1, m_2\}$

5. Haar Ranking method for Hexagonal Fuzzy Number

Let $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6)$ be the Hexagonal fuzzy number. For the convenient of using Haar wavelet ranking, the Hexagonal fuzzy number is rewritten as $\tilde{A}' = (a_1, a_2, a_3, a_3, a_4, a_4, a_5, a_6)$. The average and detailed coefficients namely the scaling and wavelet coefficients of Hexagonal fuzzy number can be calculated as follows.

Step-1: Group the Hexagonal fuzzy numbers in pairs. $[a_1, a_2],[a_3, a_3],[a_4, a_4],[a_5, a_6]$

Step-2: The first four elements of $\tilde{A}'$ with the average of these pairs (approximation coefficients) and replace the last 4 four elements of $\tilde{A}'$ with half of the difference of these pairs (detailed coefficients).

$$\alpha_1 = \left(\frac{a_1 + a_2}{2}\right), \alpha_2 = \left(\frac{a_3 + a_3}{2}\right), \alpha_3 = \left(\frac{a_4 + a_4}{2}\right), \alpha_4 = \left(\frac{a_5 + a_6}{2}\right) ;$$

$$\beta_1 = \left(\frac{a_1 - a_2}{2}\right), \beta_2 = \left(\frac{a_3 - a_3}{2}\right), \beta_3 = \left(\frac{a_4 - a_4}{2}\right), \beta_4 = \left(\frac{a_5 - a_6}{2}\right)$$

The $\tilde{A}'_1$ changed into $\tilde{A}'_1 = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \beta_1, \beta_2, \beta_3, \beta_4)$

Step-3: Group the pair of approximation coefficients of $\tilde{A}'_1$. Then, find the new approximation coefficients and the detailed coefficients for the pair of approximation coefficient of $\tilde{A}'_1 [\alpha_1, \alpha_2],[\alpha_2, \alpha_3]$

$$\gamma_1 = \left(\frac{\alpha_1 + \alpha_2}{2}\right), \gamma_2 = \left(\frac{\alpha_2 + \alpha_3}{2}\right) ; \eta_1 = \left(\frac{\alpha_1 - \alpha_2}{2}\right), \eta_2 = \left(\frac{\alpha_2 - \alpha_3}{2}\right)$$

The $\tilde{A}'_1$ changed into $\tilde{A}'_2 = (\gamma_1, \gamma_2, \eta_1, \eta_2, \beta_1, \beta_2, \beta_3, \beta_4)$

Step-4: Find the pair of approximation coefficient in $\tilde{A}'_2$. Then, find the new approximation and detailed coefficients for the pair of approximation coefficient of $\tilde{A}'_2$. $[\gamma_1, \gamma_2]$

$$\delta_1 = \left(\frac{\gamma_1 + \gamma_2}{2}\right), \delta_2 = \left(\frac{\gamma_1 - \gamma_2}{2}\right)$$

The $\tilde{A}'_2$ changed into $H(\tilde{A}) = (\delta_1, \delta_2, \eta_1, \eta_2, \beta_1, \beta_2, \beta_3, \beta_4)$

Step-5: Determine the Ranking.

- $\tilde{A} \prec \tilde{B}$, if the first element of the ordered tuple of $H(\tilde{A})$ is less than the first element of the ordered tuple of $H(\tilde{B})$.
- $\tilde{A} \succ \tilde{B}$, if the first element of the ordered tuple of $H(\tilde{A})$ is greater than the first element of the ordered tuple of $H(\tilde{B})$.
- $\tilde{A} \approx \tilde{B}$, if and only if all the elements of $H(\tilde{A})$ and $H(\tilde{B})$ are term wise equal.

5.1 Numerical illustration

Let $\tilde{A} = (5,10,15,20,25,30)$ and $\tilde{B} = (2,4,6,8,10,12)$ be two symmetric Hexagonal fuzzy numbers. For the convenient of using Haar wavelet ranking, the Hexagonal fuzzy numbers are rewritten as $\tilde{A}' = (5,10,15,15,20,20,25,30)$ and $\tilde{B}' = (2,4,6,6,8,8,10,12)$. The average and detailed coefficients namely the scaling and wavelet coefficients of Hexagonal fuzzy numbers can be calculated as follows,

Step 1: Group the Hexagonal fuzzy numbers in pairs.

For $\tilde{A}'$, the pairs will be $[5,10],[15,15],[20,20],[25,30]$

For $\tilde{B}$, the pairs will be $[2,4],[6,6],[8,8],[10,12]$.

Step 2: The first four elements of $\tilde{A}$ with the average of these pairs (approximation coefficients) and replace the last 4 four elements of $\tilde{A}$ with half of the difference of these pairs (detailed coefficients).

$$\alpha_1 = \left(\frac{5+10}{2}\right), \alpha_2 = \left(\frac{15+15}{2}\right), \alpha_3 = \left(\frac{20+20}{2}\right), \alpha_4 = \left(\frac{25+30}{2}\right).$$

$$\beta_1 = \left(\frac{5-10}{2}\right), \beta_2 = \left(\frac{15-15}{2}\right), \beta_3 = \left(\frac{20-20}{2}\right), \beta_4 = \left(\frac{25-30}{2}\right).$$

Then $\tilde{A}$ changed into $\tilde{A}_1 = (7.5, 15, 20, 27.5, -2.5, 0, 0, -2.5)$

Similarly $\tilde{B}$ changed into $\tilde{B}_1 = (3, 6, 8, 11, -1, 0, 0, -1)$

Step-3: Group the pair of approximation coefficients of $\tilde{A}_1$. Then, find the new approximation coefficients and the detailed coefficients for the pair of approximation coefficient of $\tilde{A}_1$

$[7.5, 15]$ and $[20, 27.5]$.

$$\gamma_1 = \left(\frac{7.5+15}{2}\right), \gamma_2 = \left(\frac{20+27.5}{2}\right), \eta_1 = \left(\frac{7.5-15}{2}\right), \eta_2 = \left(\frac{20-27.5}{2}\right)$$

Then $\tilde{A}_1$ changed into $\tilde{A}_2 = (11.25, 23.75, -3.75, -3.75, -2.5, 0, 0, -2.5)$

Similarly $\tilde{B}_1$ changed into $\tilde{B}_2 = (4.5, 9.5, -1.5, -1.5, -1, 0, 0 -1)$

Step-4: Find the pair of approximation coefficient in $\tilde{A}_2$. Then, find the new approximation and detailed coefficients for the pair of approximation coefficient of $\tilde{A}_2$. $[11.25, 23.75]$

$$\delta_1 = \left(\frac{11.25+23.75}{2}\right), \delta_2 = \left(\frac{11.25-23.75}{2}\right)$$

Then $\tilde{A}_2$ changed into $H(\tilde{A}) = (17.5, -6.25, -3.75, -3.75, -2.5, 0, 0, -2.5)$

Similarly $\tilde{B}_2$ changed into $H(\tilde{B}) = (7, -2.5, -1.5, -1.5, -1, 0, 0, -1)$

Step-5: Determine the Ranking

$\tilde{A} \succ \tilde{B}$, since the first element of the ordered tuple of $H(\tilde{A})$ is greater than the first element of the ordered tuple of $H(\tilde{B})$.

6 Fuzzy Assignment Problem

Consider there are *t* jobs which are to be executed and *t* persons are available for executing these jobs. Assume that each person can do each job at a time.

Let $\tilde{A}_{mn}$ be a fuzzy cost of assigning the m^{th} person to the n^{th} job.

Let the decision parameter C_{mn} denote the allotment of the m^{th} person to the n^{th} job.

The objective is to determine an optimal assignment of jobs to persons on one- one basis (which job should be given to which person on one- one basis) so that the total cost of executing all jobs is optimum. These kinds of problems are known as assignment problem. Mathematically, the problem is defined as

$$\min Z = \sum_{m=1}^{i}\sum_{n=1}^{j} \tilde{A} C_{mn}$$

Subject to

$$\sum_{m=1}^{i} C_{mn} = 1 \quad for\ m = 1, 2, ..., i.$$

$$\sum_{n=1}^{j} C_{mn} = 1 \quad for\ \mathrm{n} = 1, 2, ..., j.$$

Where,

$$C_{mn} = \begin{cases} 1, \text{ if the m}^{th} \text{ person is alloted to the n}^{th} \text{ job} \\ 0, \text{ if the m}^{th} \text{ person is not alloted to the n}^{th} \text{ job} \end{cases}$$

$$\tilde{A}^{K}_{mn} = (\tilde{A}^{1}_{mn}, \tilde{A}^{2}_{mn}, \tilde{A}^{3}_{mn}, \tilde{A}^{4}_{mn}, \tilde{A}^{5}_{mn}, \tilde{A}^{6}_{mn}, \tilde{A}^{7}_{mn})$$

6.1 Fuzzy Assignment Problem in Solid Waste Management

Let us consider the following fuzzy assignment problem. In which, our aim is to dispose five types wastages in each of five different locations. Rows represent different locations and columns represent different solid wastages. Proper disposal of solid wastages are needed for clean environment. Let $[\tilde{A}_{mn}]$ be the production matrix whose elements are given by Hexagonal fuzzy numbers. The objective is to get the optimal assignment of solid wastages to locations in a way that the total transportation cost becomes minimized.

	W_1	W_2	W_3
L_1	(10,13,15,18,20,23)	(14,17,20,23,26,29)	(12,15,17,20,22,25)
L_2	(9,12,14,17,19,22)	(18,21,24,27,30,33)	(15,18,20,23,25,28)
L_3	(11,14,16,19,21,24)	(20,23,26,29,32,35)	(20,23,25,28,30,33)
L_4	(15,18,20,23,25,28)	(24,27,30,33,36,39)	(30,33,35,38,40,43)
L_5	(20,23,25,28,30,33)	(28,31,34,37,40,43)	(40,43,45,48,50,53)

	W_4	W_5
L_1	(12,15,17,20,22,25)	(8,11,13,15,17,20)
L_2	(16,19,21,24,26,29)	(18,21,23,25,27,30)
L_3	(19,22,24,27,29,32)	(28,31,33,35,37,40)
L_4	(21,24,26,28,31,34)	(38,41,43,45,47,50)
L_5	(23,26,28,30,33,36)	(48,51,53,55,57,60)

Table 1: Fuzzy Hexagonal Matrix

Then, Hexagonal fuzzy matrix is transformed into Haar Fuzzy Hexagonal Matrix.

	W_1	W_2	W_3	W_4	W_5
L_1	(16.5, -3.25, -1.75, -1.75, -1.5, 0, 0, -1.5)	(21.5, -3.75, -2.25, -2.25, -1.5, 0, 0, -1.5)	(18.5, -3.25, -1.75, -1.75, -1.5, 0, 0, -1.5)	(18.5, -3.25, -1.75, -1.75, -1.5, 0, 0, -1.5)	(14, -2.75, -1.75, -1.75, -1.5, 0, 0, -1.5)
L_2	(15.5, -3.25, -1.75, -1.75, -1.5, 0, 0, -1.5)	(25.5, -3.75, -2.25, -2.25, -1.5, 0, 0, -1.5)	(21.5, -3.25, -1.75, -1.75, -1.5, 0, 0, -1.5)	(22.5, -3.25, -1.75, -1.75, -1.5, 0, 0, -1.5)	(24, -2.75, -1.75, -1.75, -1.5, 0, 0, -1.5)
L_3	(17.5, -3.25, -1.75, -1.75, -1.5, 0, 0, -1.5)	(27.5, -3.75, -2.25, -2.25, -1.5, 0, 0, -1.5)	(26.5, -3.25, -1.75, -1.75, -1.5, 0, 0, -1.5)	(25.5, -3.25, -1.75, -1.75, -1.5, 0, 0, -1.5)	(34, -2.75, -1.75, -1.75, -1.5, 0, 0, -1.5)
L_4	(21.5, -3.25, -1.75, -1.75, -1.5, 0, 0, -1.5)	(31.5, -3.75, -2.25, -2.25, -1.5, 0, 0, -1.5)	(36.5, -3.25, -1.75, -1.75, -1.5, 0, 0, -1.5)	(27.25, -3, -1.75, -2.25, -1.5, 0, 0, -1.5)	(44, -2.75, -1.75, -1.75, -1.5, 0, 0, -1.5)
L_5	(26.5, -3.25, -1.75, -1.75, -1.5, 0, 0, -1.5)	(35.75, -4, -2.25, -2.75, -1.5, 0, 0, -2.5)	(46.5, -3.25, -1.75, -1.75, -1.5, 0, 0, -1.5)	(29.25, -3, -1.75, -2.25, -1.5, 0, 0, -1.5)	(54, -2.75, -1.75, -1.75, -1.5, 0, 0, -1.5)

Table 2: Haar Hexagonal fuzzy Matrix

Next, the optimal solution is obtained through the following Normalized accuracy function of Haar Hexagonal fuzzy number.

$$M\left(H(\tilde{A})\right)=\left(\frac{2a_1+3a_2+4a_3+5a_4+5a_5+4a_6+3a_7+2a_8}{28}\right)$$

	W_1	W_2	W_3	W_4	W_5
L_1	0.107	3.892	3.892	3.892	3.607
L_2	2.107	11.89	9.892	11.89	16.39
L_3	1.892	15.89	19.89	17.89	36.39
L_4	9.892	23.89	39.89	19.64	56.39
L_5	19.89	29.07	59.89	23.75	76.39

Table 3: Normalized Haar Fuzzy Hexagonal Matrix

To get optimal allocation, we solve the above assignment problem using the Hungarian method. After solving, we get the optimal solution 63.031 and the optimal allocation of low cost transportation for dumping the solid waste is (1, 5), (2, 3), (3, 2), (4, 1) and (5, 4). It means that the Location L_1 is assigned to the solid waste W_5, the Location L_2 is assigned to the solid waste W_3, the location L_3 is assigned to the Solid waste W_2, the Location L_4 is assigned to the Solid waste W_1, and the Location L_5 is assigned to the Solid waste W_4.

7 Conclusion

In this paper Hexagonal Fuzzy number has been newly introduced and the linear and non-linear hexagonal fuzzy numbers are formulated under uncertain circumstances and also the Haar ranking for the hexagonal fuzzy number is proposed. To validate the proposed ranking method, Assignment problem is taken to determine the minimum transportation cost for proper disposal of solid wastages in locations that protects environment.

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