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On (N_{τ_i}, N_{τ_j}) Neutrosophic Baire Spaces and (N_{τ_i}, N_{τ_j}) Neutrosophic Semi - Baire Spaces

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Abstract. The concept of baire spaces and semi - baire spaces in neutrosophic bitopological spaces are introduced and its properties are studied. The concepts are explained using examples.

Key Words:(N_{τ_i},N_{τ_j}) Neutrosophic nowhere dense set, (N_{τ_i},N_{τ_j}) Neutrosophic first category set, (N_{τ_i},N_{τ_j}) Neutrosophic Baire spaces, (N_{τ_i},N_{τ_j}) Neutrosophic semi nowhere dense set, (N_{τ_i},N_{τ_j}) Neutrosophic semi first category set, (N_{τ_i},N_{τ_j}) Neutrosophic semi Baire spaces.

1. Introduction and Preliminaries Neutrosophy is a branch of Philosophy introduced by Smarandache in [1980]. In the neutrosophic logic, each proposition is estimated to have the percentage of truth in a subset T, the percentage of indeterminancy in a subset I, and the percentage of falsity in a subset F. The idea of baire space in fuzzy set was introduced by Thangaraj and Anjalmose [7]. The idea of baire spaces in fuzzy bitopological spaces was introduced by Thangaraj. G and Sethuraman.S [8]. The idea of neutrosophic bitopological spaces was defined Taha Yasin Ozturk and Alkan Ozkan [10]. Its properties are studied by Dimacha Dwibrang Mwchahary and Bhimraj Basumatary [2].

The following definitions are taken from [2], [10] " *Let the two different Neutrosophic topologies on H be (H,N_{τ_i})and (H,N_{τ_j}) . Then (H, N_{τ_i}, N_{τ_j}) is called a Neutrosophic Bitopological Space (Neut- B- T - Space). The indices i, j takes the value* $\in \{1,2\}$*and* $i \neq j$. *Let (H, N_{τ_i}, N_{τ_j}) be the Neut-B-T-Space. Then for a set* $L = \{<, \mu_{ij}, \sigma_{ij}, \gamma_{ij} >: h \in H\}$, *neutrosophic ($N_{\tau_i}$,$N_{\tau_j}$) Neut-interior of K is the union of all (N_{τ_i},N_{τ_j}) Neu-open sets of H contained in K and defined as follows:(N_{τ_i},N_{τ_j}) Neut-int(K)=* $< h, \cap_{N_{\tau_i}} \cap_{N_{\tau_j}} \mu_{ij}, \cap_{N_{\tau_i}} \cap_{N_{\tau_j}} \sigma_{ij}, \cup_{N_{\tau_i}} \cup_{N_{\tau_j}} \gamma_{ij} >: h \in H\}$. Here μ_{ij} represents degree of membership function ,σ_{ij} represents the degree of indeterminancy,γ_{ij} represents the degree of non-membership function of a neutrosophic set and *i* is related with neutrosophic topologyN_{τ_i}, *j* is related with neutrosophictopology N_{τ_j}. *Let (H, N_{τ_i}, N_{τ_j}) be the Neut-B-T-Space. Then for a set K=* $\{<, \mu_{ij}, \sigma_{ij}, \tau_{ij} >: h \in H\}$, *neutrosophic ($N_{\tau_i}$,$N_{\tau_j}$) Neut-closure of K is the intersection of all (N_{τ_i},N_{τ_j}) Neut-closed sets of H contained in K and defined as follows:(N_{τ_i},N_{τ_j}) Neut-cl(K)=* $< h, \cap_{N_{\tau_i}} \cap_{N_{\tau_j}} \mu_{ij}, \cap_{N_{\tau_i}} \cap_{N_{\tau_j}} \sigma_{ij}, \cup_{N_{\tau_i}} \cup_{N_{\tau_j}} \gamma_{ij} >: h \in H\}$".

The following results are taken from[2] "Let (H, N_{τ_i}, N_{τ_j}) be the Neut-B-T-Space, then

i)(N_{τ_i},N_{τ_j}) Neut-int(0_N)=0_N, (N_{τ_i},N_{τ_j}) Neut-int(1_N)=1_N,

ii) (N_{τ_i},N_{τ_j}) Neut-int(K)$\leq$ K.

iii) K is neutrosophic open iff K= (N_{τ_i}, N_{τ_j})Neut-int(K).

iv)(N_{τ_i}, N_{τ_j}) Neu-int[(N_{τ_i}, N_{τ_j}) Neut-int(K)] = K.

v) K ≤ L implies (N_{τ_i}, N_{τ_j}) Neut-int(K)≤ (N_{τ_i}, N_{τ_j}) Neut-int(L).

vi) (N_{τ_i}, N_{τ_j}) Neut-int(K)∪ (N_{τ_i}, N_{τ_j}) Neut-int(KL)≤ (N_{τ_i}, N_{τ_j}) Neut-int(K∪ L).

vii)$N_{\tau_i}, N_{\tau_j})$ Neut-int(K)∩ (N_{τ_i}, N_{τ_j})Neut-int(L)= (N_{τ_i}, N_{τ_j}) Neut-int(K∩ L).

Let (H, N_{τ_i}, N_{τ_j}) be the Neut-B-T-Space, then

i)(N_{τ_i}, N_{τ_j}) Neut-cl(0_N)=0_N, (N_{τ_i}, N_{τ_j}) Neut-cl (1_N)=1_N,

ii) (N_{τ_i}, N_{τ_j}) Neut-cl(K)≥ K.

iii) K is neutrosophic closed iff K= (N_{τ_i}, N_{τ_j}) Neut-cl(K).

iv) (N_{τ_i}, N_{τ_j}) Neut-cl[(N_{τ_i}, N_{τ_j}) Neut-cl(K)] = K.

v) K≤ L implies (N_{τ_i}, N_{τ_j}) Neut-cl(K)≤ (N_{τ_i}, N_{τ_j}) Neut-cl(K).

vi) ((N_{τ_i}, N_{τ_j}) Neut-cl(K))∪ ((N_{τ_i}, N_{τ_j}) Neut-cl(L))= (N_{τ_i}, N_{τ_j}) Neut-cl(K∪ L).

vii) ((N_{τ_i}, N_{τ_j}) Neut-cl(K)) ∩ ((N_{τ_i}, N_{τ_j}) Neut-cl(L))≥ (N_{τ_i}, N_{τ_j}) Neut-cl(K∩ L).

Let (H,N_{τ_i}, N_{τ_j}) be the Neut-B-T-Space, then

i) (N_{τ_i}, N_{τ_j}) Neut-int(K^c) = $[(N_{\tau_i}, N_{\tau_j})\ Neut-cl(K)]^c$

ii) (N_{τ_i}, N_{τ_j}) Neut- cl(K^c)=$[(N_{\tau_i}, N_{\tau_j})\ Neut-int(K)]^c$

iii) (N_{τ_i}, N_{τ_j}) Neut-int(K) = $\left[\left(N_{\tau_i}, N_{\tau_j}\right) Neut-cl(K)\right]^c$

iv) (N_{τ_i}, N_{τ_j}) Neut-cl(K) = $[(N_{\tau_i}, N_{\tau_j})\ Neut-int(K^c)]^c$".

2. (N_{τ_i}, N_{τ_j}) Neutrosophic nowhere dense sets

Definition 2.1.*A Neut- Set K in a Neut- B- T- Space (H ,N_{τ_1}, N_{τ_2}) is called (N_{τ_i}, N_{τ_j}) neutrosophic denseset(Neut – D - Set) if there exists no Neut-CSet L in (X,N_{τ_1}, N_{τ_2}) such that* $Neut-cl_{\tau_1}\left(Neut-cl_{\tau_2}(K)\right) = Neut-cl_{\tau_2}\left(Neut-cl_{\tau_1}(K)\right) = 1_N$.

Example 2.1. Let H={k, l}and K= { < k, 0.6, 0.6, 0.3 >, < l, 0.6, 0.6 0.5 > } L= { < k, 0.6, 0.6, 0.4 >, < l, 0.5, 0.5 0.5 > }, M = { < k, 0.7, 0.6, 0.3 >, < l, 0.6, 0.5 0.5 >}. Then, N_{τ_1}= $\{0_N, 1_N, K\}$ and N_{τ_2}= $\{0_N, 1_N, L, M\}$. Then (H,N_{τ_1}, N_{τ_2}) is a Neut – B – T - Space. Here K, L, M,$\overline{K}$, $\overline{L}$ are (N_{τ_i}, N_{τ_j}) Neut – D - Setin (H,N_{τ_1}, N_{τ_2}).

Definition 2.2.*A Neut-Set K in a Neut- B- T- Space (H , N_{τ_1}, N_{τ_2}) is called (N_{τ_i}, N_{τ_j}) neutrosophic nowhere dense(Neut-N-D-Set) if there exists no Neut-OSet L in (H , N_{τ_1}, N_{τ_2}) such that* $Neut-int_{\tau_1}\left(Neut-cl_{\tau_2}(K)\right) = Neut-int_{\tau_2}\left(Neut-cl_{\tau_1}(K)\right) = 0_N$.

Example 2.2. If H={k, l}and K= { < k, 0.6, 0.6, 0.3 >, < l, 0.6, 0.6 0.5 > }, L={< k, 0.6, 0.6, 0.4 >,

<l, 0.5, 0.5, 0.5 > },M ={< k, 0.7, 0.6, 0.3 >, < l, 0.6, 0.5, 0.5 > }.Then, $N_{\tau_1}= \{0_N, 1_N, K\}$ and $N_{\tau_2}= \{0_N, 1_N, L, M\}$. Then (H,$N_{\tau_1}, N_{\tau_2}$) is a Neut – B – T - Space. $Neut - int_{\tau_1}\left(Neut - cl_{\tau_2}(\overline{M})\right) = Neut - int_{\tau_1}(\overline{M}) = 0_N \; and \; Neut - int_{\tau_2}\left(Neut - cl_{\tau_1}(\overline{M})\right) = Neut - int_{\tau_2}(\overline{K}) = 0_N$. Therefore, we have $Neut - int_{\tau_1}\left(Neut - cl_{\tau_2}(\overline{M})\right) = Neut - int_{\tau_2}\left(Neut - cl_{\tau_1}(\overline{M})\right)$. Hence $\overline{M}$ is a (N_{τ_i},N_{τ_j}) Neut - N - D - Set in (H,N_{τ_1}, N_{τ_2}).

Proposition 2.1.*If K has a Neut - CSet with* $Neut - int_{\tau_i}(K) = 0_{N,}$ *(i=1, 2) in (H,*N_{τ_1}, N_{τ_2}*), then K has a (*N_{τ_i}*,*N_{τ_j}*) Neut -N- D- Set in (H,*N_{τ_1}, N_{τ_2}*).*

*Proof***.** Let K be the Neut-CSet with $Neut - int_{\tau_i}(K) = 0_{N,}$ (i=1, 2) in (H,N_{τ_1}, N_{τ_2}). Then$Neut - cl_{\tau_1}(K) = K$ and $Neut - cl_{\tau_2}(K) = K$. Also, we have $Neut - int_{\tau_1}(K) = 0_N$ and $Neutint_{\tau_2}(K) = 0_N$. Then, $Neut - int_{\tau_1}\left(Neut - cl_{\tau_2}(K)\right) = Neut - int_{\tau_1}(K) = 0_N$ and $Neut - int_{\tau_2}\left(Neut - cl_{\tau_1}(K)\right) = Neut - int_{\tau_2}(K) = 0_N$ implies that $Neut - int_{\tau_1}\left(Neut - cl_{\tau_2}(K)\right) = Neut - int_{\tau_2}\left(Neut - cl_{\tau_1}(K)\right) = 0_N$. Therefore, K is a ($N_{\tau_i}$,$N_{\tau_j}$) Neut-N-D-Set in (H,$N_{\tau_1}, N_{\tau_2}$). ■

Proposition 2.2.*If K has a (*N_{τ_i}*,*N_{τ_j}*) Neut – N -D - Set in (H,*N_{τ_1}, N_{τ_2}*), then* $Neut - int_{\tau_i}(K) = 0_{N,}$ *(i=1, 2).*

*Proof.*Let K be the (N_{τ_i},N_{τ_j})Neut – N – D - Set in (H,N_{τ_1}, N_{τ_2}), then we have $Neut - int_{\tau_1}\left(Neut - cl_{\tau_2}(K)\right) = Neut - int_{\tau_2}\left(Neut - cl_{\tau_1}(K)\right) = 0_N$.Now, $K \subseteq Neut - cl_{\tau_2}(K)$ implies that $Neut - int_{\tau_1}(K) \subseteq Neut - int_{\tau_1}\left(Neut - cl_{\tau_2}(K)\right)$. Then, we have $Neut - int_{\tau_1}(K) = 0_N$. Also, $K \subseteq Neut - cl_{\tau_1}(K)$ implies that $Neut - int_{\tau_2}(K) \subseteq Neut - int_{\tau_2}\left(Neut - cl_{\tau_1}(K)\right)$. Then, we have $Neut - int_{\tau_2}(K) = 0_N$. Therefore, $Neut - int_{\tau_i}(K) = 0_{N,}$ (i = 1, 2). ■

Proposition 2.3.*If K is a (*N_{τ_i}*,*N_{τ_j}*) Neut – N – D - Set in (H,* N_{τ_1}, N_{τ_2}*), then* $\overline{K}$ *is a (*N_{τ_i}*,*N_{τ_j}*) Neut – D - Set in (H,* N_{τ_1}, N_{τ_2}*).*

*Proof.*Let K be the (N_{τ_i},N_{τ_j}) Neut – N – D - Set in (H,N_{τ_1}, N_{τ_2}), then we have $Neut - int_{\tau_1}\left(Neut - cl_{\tau_2}(K)\right) = Neut - int_{\tau_2}\left(Neut - cl_{\tau_1}(K)\right) = 0_N$. Then, $1 - Neut - int_{\tau_1}\left(Neut - cl_{\tau_2}(K)\right) = 1 - 0 = 1_N$. Then, $Neut - cl_{\tau_1}\left(1 - Neut - cl_{\tau_2}(K)\right) = 1_N$, Which implies that $Neut - cl_{\tau_1}\left(Neut - int_{\tau_2}(1 - K)\right) = 1_N$. But $Neut - cl_{\tau_1}\left(Neut - int_{\tau_2}(1 - K)\right) \subseteq Neut - cl_{\tau_1}\left(Neut - cl_{\tau_2}(1 - K)\right)$. Hence, $1_N \subseteq Neut - cl_{\tau_1}\left(Ncl_{\tau_2}(1 - K)\right)$. That is, $Neut - cl_{\tau_1}\left(Neut - cl_{\tau_2}(1 - K)\right) = 1_N$. Also, $1 - Neut - int_{\tau_2}\left(Neut - cl_{\tau_1}(K)\right) = 1 - 0 = 1_N$. Then, $Neut - cl_{\tau_2}\left(1 - Neut - cl_{\tau_1}(K)\right) = 1_N$, Which implies that $Neut - cl_{\tau_2}\left(Neut - int_{\tau_1}(1 - K)\right) = 1_N$. But Neut $- cl_{\tau_2}\left(Neut - int_{\tau_1}(1 - K)\right) \subseteq Neut - cl_{\tau_2}\left(Neut - cl_{\tau_1}(1 - K)\right)$. Hence, $1_N \subseteq Ncl_{\tau_2}\left(Ncl_{\tau_1}(1 - K)\right)$. That is $Ncl_{\tau_2}\left(Ncl_{\tau_1}(1 - K)\right) = 1_N$. Therefore, $\overline{K}$ is a (N_{τ_i},N_{τ_j}) Neut – D - Set in (H,N_{τ_1}, N_{τ_2}). ■

Remark. If K is a (N_{τ_i},N_{τ_j}) Neut – D - Set in (H,N_{τ_1}, N_{τ_2}). Then $\overline{K}$**need not** be a (N_{τ_i},N_{τ_j}) Neut – N - D - Set in (X,N_{τ_1}, N_{τ_2}). For this consider the following example:

Let X = {k, l}and K= { < k, 0.2, 0.6, 0.8 >, < l, 0.5, 0.3 0.5 > }

L= { < k, 0.7, 0.7, 0.3 >, < l, 0.4, 0.5 0.5 > }.

Then, $N_{\tau_1}= \{0_N, 1_N, K\}$ and $N_{\tau_2}= \{0_N, 1_N, L\}$. Then (H,$N_{\tau_1}, N_{\tau_2}$) is a (Neut-B-T-Space) .Now $Neut - cl_{\tau_1}\left(Neut - cl_{\tau_2}(K)\right) = Neut - cl_{\tau_1}(\overline{L}) = 1_N$and $Neut - cl_{\tau_2}\left(Neut - cl_{\tau_1}(K)\right) = Neut - cl_{\tau_2}(\overline{K}) = 1_N$ and also, $Neut - cl_{\tau_1}\left(Neut - cl_{\tau_2}(L)\right) = Neut - cl_{\tau_1}(1) = 1_N$and $Neut - \left(Neut - cl_{\tau_1}(L)\right) = Neut - cl_{\tau_2}(1) = 1_N$and therefore K and L are (N_{τ_i},N_{τ_j}) Neut – D - Set in (H, N_{τ_1}, N_{τ_2}).

Also we have, $Neut-int_{\tau_1}\left(Neut-cl_{\tau_2}(\bar{K})\right)=Neut-int_{\tau_1}(1)=1\ \neq 0_N\ and\ Neut-int_{\tau_2}\left(Neut-cl_{\tau_1}(\bar{K})\right)=Neut-int_{\tau_2}(\bar{K})=0_N$ And $Neut-int_{\tau_1}\left(Neut-cl_{\tau_2}(\bar{L})\right)=Nint_{\tau_1}(\bar{L})=K\ and Nint_{\tau_2}\left(Ncl_{\tau_1}(\bar{L})\right)=Nint_{\tau_2}(1)=1_N\neq 0_N$. Therefore, we have $Neut-int_{\tau_1}\left(Neut-cl_{\tau_2}(\bar{K})\right)\neq Neut-int_{\tau_2}\left(Neut-cl_{\tau_1}(\bar{K})\right)$

And have $Neut-int_{\tau_1}\left(Neut-(\bar{L})\right)\neq\ Neut-int_{\tau_2}\left(Neut-cl_{\tau_1}(\bar{L})\right)$. Therefore $\bar{K}$ and $\bar{L}$ are not (N_{τ_i},N_{τ_j}) Neut – N – D - Set in (H, N_{τ_1}, N_{τ_2}).

Proposition 2.4. *If L is a (N_{τ_i},N_{τ_j}) Neut – N – D- Set in (H, N_{τ_1}, N_{τ_2}) and if K ⊆ L for a Neut - Set K in (H, N_{τ_1}, N_{τ_2}), then K is a (N_{τ_i},N_{τ_j}) Neut – N – D - Set in (H, N_{τ_1}, N_{τ_2}).*

Proof. Let L be the (N_{τ_i},N_{τ_j})Neut - N – D - Set in (H, N_{τ_1}, N_{τ_2}). Then, $Neut-int_{\tau_1}\left(Neut-cl_{\tau_2}(L)\right)=Neut-int_{\tau_2}\left(Neut-cl_{\tau_1}(L)\right)=0_N$. Now, $K\subseteq L$ implies that $Neut-int_{\tau_1}\left(Neut-cl_{\tau_2}(K)\right)\subseteq Neut-int_{\tau_1}\left(Neut-cl_{\tau_2}(L)\right)$ and $Neut-int_{\tau_2}\left(Neut-cl_{\tau_1}(K)\right)\subseteq Neut-int_{\tau_2}\left(Neut-cl_{\tau_1}(L)\right)$. Hence, $Neut-int_{\tau_1}\left(Neut-cl_{\tau_2}(K)\right)\subseteq 0_N$ and $Neut-int_{\tau_2}\left(Neut-cl_{\tau_1}(K)\right)\subseteq 0_N$. That is, $Neut-int_{\tau_1}\left(Neut-cl_{\tau_2}(K)\right)=Neut-int_{\tau_2}\left(Neut-cl_{\tau_1}(K)\right)=0_N$. Therefore, K is a ($N_{\tau_i}$,$N_{\tau_j}$) Neut – N – D - Set in (H, N_{τ_1}, N_{τ_2}).

3. (N_{τ_i},N_{τ_j}) Neutrosophic Baire Spaces

Definition 3.1. *Let (H, N_{τ_1}, N_{τ_2}) be a Neut – B – T - Space. A Neut - Set K in (H, N_{τ_1}, N_{τ_2}) is a (N_{τ_i},N_{τ_j}) neutrosophic first category set(Neut – F – C - Set)if $K=\cup_{n=1}^{\infty}K_n$, where K_n's are (N_{τ_i}, N_{τ_j}) Neut – N –D - Set in (H, N_{τ_1}, N_{τ_2}). Anyother Neut – Set in (H, N_{τ_1}, N_{τ_2})is a(N_{τ_i},N_{τ_j}) neutrosophic second category set (Neut – S - C- Set) in (H, N_{τ_1}, N_{τ_2}).*

Example 3.1. Let H = {k, l}and K= { < k, 0.6, 0.6, 0.3 >, < l, 0.6, 0.5 0.5 > }

L= { < k, 0.6, 0.6, 0.4 >, < l, 0.5, 0.5 0.5 > } M= { < k, 0.7, 0.6, 0.3 >, < l, 0.6, 0.5 0.5 >

}.Then, $N_{\tau_1}=\{0_N,1_N,K,L\}$ and $N_{\tau_2}=\{0_N,1_N,L,M\}$. Then (H, N_{τ_1}, N_{τ_2}) is a

Neut – B – T - Space. $\bar{K}$, $\bar{L}$, $\bar{M}$ are (N_{τ_i},N_{τ_j}) Neut – N – D – Set in (H, N_{τ_1}, N_{τ_2}). $\bar{K}\cup\bar{L}\cup\bar{M}=\bar{L}$ is a (N_{τ_i},N_{τ_j}) Neut – F – C - Set in (H, N_{τ_1}, N_{τ_2}).

Definition 3.2. *If K has a (N_{τ_i},N_{τ_j}) Neut – F – C - Set in (H, N_{τ_1}, N_{τ_2}), then (1- K) is a (N_{τ_i},N_{τ_j}) neutrosophic residual set (Neut – R - Set) in (H, N_{τ_1}, N_{τ_2}).*

Definition 3.3. *A Neut – B – T - Space (H, N_{τ_1}, N_{τ_2}) is a (N_{τ_i},N_{τ_j}) neutrosophic first category space(Neut – F – C – Space) if Neut - Set 1_N is a Neut – F – C- Set in (H, N_{τ_1}, N_{τ_2}). That is, $1_N=\cup_{n=1}^{\infty}K_n$, where K_n's are (N_{τ_i},N_{τ_j}) Neut – N – D - Set in (H, N_{τ_1}, N_{τ_2}). Otherwise, (H, N_{τ_1}, N_{τ_2}) is a (N_{τ_i},N_{τ_j}) neutrosophic second category space(Neut – S – C - Space)*

Proposition 3.1. *If K has a (N_{τ_i}, N_{τ_j}) Neut – F – C – Set in (H, N_{τ_1}, N_{τ_2}), then $1-K=\cap_{n=1}^{\infty}L_n$, where $Ncl_{\tau_i}(L_n)=1_N$, (i=1, 2).*

Proof. Let K be the (N_{τ_i},N_{τ_j})Neut – F – C – Set in (H, N_{τ_1}, N_{τ_2}). Then, $K=\cup_{n=1}^{\infty}K_n$, where K_n's are (N_{τ_i},N_{τ_j}) Neut – N – D – Set in (H, N_{τ_1}, N_{τ_2}). Now, $1-K=1-\cup_{n=1}^{\infty}K_n=\cap_{n=1}^{\infty}(1-K_n)$. Since, K_n's are Neut – N – D - Set in (H, N_{τ_1}, N_{τ_2}). By proposition 2.3, $\overline{K_n}$'s are Neut – D - Set in (H, N_{τ_1}, N_{τ_2}). Let us substitute $L_n=1-\ K_n$. Then, $1-K=\cap_{n=1}^{\infty}(L_n)$, where $Ncl_{\tau_i}(L_n)=1_N$, (i=1,2)

Proposition 3.2. *If L has a (N_{τ_i},N_{τ_j}) Neut – F – C - Set in (H, N_{τ_1}, N_{τ_2}) and if K ⊆ L for a Neut - Set K in (H, N_{τ_1}, N_{τ_2}), then K has a (N_{τ_i},N_{τ_j}) Neut – F – C - Set in (H, N_{τ_1}, N_{τ_2}).*

Proof. Let L be the (N_{τ_i}, N_{τ_j}) Neut – F – C - Set in (H, N_{τ_1}, N_{τ_2}). Then, $L = \bigcup_{n=1}^{\infty} L_n$, where L_n's are (N_{τ_i},N_{τ_j}) Neut – N – D - Set in (H, N_{τ_1}, N_{τ_2}). Now, $K \cap L = K \cap (\bigcup_{n=1}^{\infty} L_n)$= $\bigcup_{n=1}^{\infty}(K \cap L_n)$. Since, $K \cap L_n \subseteq L_n$and L_n's are Neut – N – D - Set in (H, N_{τ_1}, N_{τ_2}). Therefore, $(K \cap L_n)$'s are Neut – N –D - Set in (H, N_{τ_1}, N_{τ_2}). Hence $K = \bigcup_{n=1}^{\infty}(K \cap L_n)$, where $(K \cap L_n)$'s are Neut – N – D - Set in (H, N_{τ_1}, N_{τ_2}), implies that K has a (N_{τ_i}, N_{τ_j}) Neut – F – C - Set in (H, N_{τ_1}, N_{τ_2}). ∎

Definition 3.4.*A Neut – B - T - Space (H, N_{τ_1}, N_{τ_2}) is a (N_{τ_i}, N_{τ_j}) neutrosophic baire space(Neut – B - Space) if $Neut - int_{\tau_i}(\bigcup_{n=1}^{\infty} K_n) = 0_N$, (i=1,2) where K_n's are (N_{τ_i},N_{τ_j}) Neut – N – D – Set in (H, N_{τ_1}, N_{τ_2}).*

Example 3.2. Let H={k, l}and K= { < k, 0.6, 0.6, 0.3 >, < l, 0.6, 0.5 0.5 > }

L= { < k, 0.6, 0.6, 0.4 >, < l, 0.5, 0.5 0.5 > } M= { < k, 0.7, 0.6, 0.3 >, < l, 0.6, 0.5 0.5 > }.

Then, $N_{\tau_1}= \{0_N, 1_N, K, L\}$ and $N_{\tau_2}= \{0_N, 1_N, L, M\}$. Then (H, N_{τ_1}, N_{τ_2}) is Neut – B – T - Space. Here $\bar{L}$ is a (N_{τ_i}, N_{τ_j}) Neut – N – D – Set in (H, N_{τ_1}, N_{τ_2}). which is a (N_{τ_i},N_{τ_j}) Neut – F – C - Set in (H, N_{τ_1}, N_{τ_2}). Hence, $Neut - int_{\tau_1}(\bar{L}) = 0_N$ and also $Neut - int_{\tau_2}(\bar{L}) = 0_N$. Therefore, (H, N_{τ_1}, N_{τ_2}) is a (N_{τ_i}, N_{τ_j}) Neut – B – Space.

Proposition 3.3.*Let (H, N_{τ_1}, N_{τ_2}) be a Neut – B – T - Space. Then the following are equivalent.*

(1) (H, N_{τ_1}, N_{τ_2}) is a (N_{τ_i}, N_{τ_j}) Neut – B - Space.

(2) $Neut - \ int_{\tau_i}(K) = 0_N$, (i = 1,2), for every (N_{τ_i}, N_{τ_j}) Neut – F – C - Set K in (H, N_{τ_1}, N_{τ_2}).

(3) $Neut - \ cl_{\tau_i}(L) = 1_N$, (i = 1,2), for every (N_{τ_i}, N_{τ_j}) Neut – R - Set L in (H, N_{τ_1}, N_{τ_2}).

*Proof***.**(1) ⟹(2) Let K be the (N_{τ_i}, N_{τ_j}) Neut – F – C - Set in (H, N_{τ_1}, N_{τ_2}). Then, $K = \bigcup_{n=1}^{\infty} K_n$, where K_n's are (N_{τ_i}, N_{τ_j}) Neut – F – C - Set in (H, N_{τ_1}, N_{τ_2}). Now, $Neut - \ int_{\tau_i}(K) = \ Neut - int_{\tau_i}(\bigcup_{n=1}^{\infty} K_n)$ = 0_N, (i = 1,2) [since (H, N_{τ_1}, N_{τ_2}) is a (N_{τ_i},N_{τ_j}) Neut – B – T - Space]. Therefore, $Nint_{\tau_i}(K) = 0_N$, where K_n's are (N_{τ_i},N_{τ_j}) Neut – B - Space in (H, N_{τ_1}, N_{τ_2}).

(2) ⟹(3) Let L be the (N_{τ_i}, N_{τ_j}) Neut – R - Set in (H, N_{τ_1}, N_{τ_2}). Then, $\bar{L}$ is a (N_{τ_i},N_{τ_j}) Ne-F-C-Set in (H, N_{τ_1}, N_{τ_2}). By hypothesis, $Nint_{\tau_i}(\bar{L}) = 0_N$, (i = 1, 2), implies that $1 - Ncl_{\tau_i}(L)$= 0_N. Hence, $Ncl_{\tau_i}(L) = 1_N$, (i = 1, 2).

(3) ⟹(1) Let K be the (N_{τ_i}, N_{τ_j}) Neut – F – C - Set in (H, N_{τ_1}, N_{τ_2}) . Then, $K = \bigcup_{n=1}^{\infty} K_n$, where K_n's are (N_{τ_i}, N_{τ_j})Neut – N – D - Set in (H, N_{τ_1}, N_{τ_2}). Now, K is a (N_{τ_i},N_{τ_j}) Neut – F – C - Set in (X, N_{τ_1}, N_{τ_2}) implies that $\bar{K}$ is a (N_{τ_i},N_{τ_j})Neut – R - Set in (H, N_{τ_1}, N_{τ_2}). By hypothesis, we have $Ncl_{\tau_i}(\bar{K}) = 1_N$, which implies that $1 - Nint_{\tau_i}(K) = 1_N$, (i = 1, 2). Then, $Nint_{\tau_i}(K) = 0_N$. That is, $Nint_{\tau_i}(\bigcup_{n=1}^{\infty} K_n)$ = 0_N, (i = 1,2), where K_n's are (N_{τ_i},N_{τ_j}) Neut – N – D - Set in (H, N_{τ_1}, N_{τ_2}). Hence, (H, N_{τ_1}, N_{τ_2}) is a (N_{τ_i}, N_{τ_j}) Neut – B – Bi - Space. ∎

Proposition 3.4.*If the (N_{τ_i},N_{τ_j}) Neut – F – C - Set K is a Neut - CSet in a (N_{τ_i},N_{τ_j}) Neut – B -Bi - Space (H, N_{τ_1}, N_{τ_2}), then K is a (N_{τ_i},N_{τ_j}) Neut – N – D - Set in (H, N_{τ_1}, N_{τ_2}).*

*Proof.*Let K be the (N_{τ_i},N_{τ_j}) Neut – F – C - Set in (H, N_{τ_1}, N_{τ_2}) and $Ncl_{T_i}(K) = K$, (i = 1, 2) --- (1). By proposition 3.3, $Nint_{\tau_i}(K) = 0_N$,(i = 1, 2) --- (2) for the (N_{τ_i},N_{τ_j}) Ne-F-C-Set K in (H, N_{τ_1}, N_{τ_2}). Then, from (1) and (2) we have N$int_{\tau_1}\left(Ncl_{\tau_2}(K)\right) = Nint_{\tau_2}\left(Ncl_{\tau_1}(K)\right) = 0_N$. Hence, R is a ($N_{\tau_i}$,$N_{\tau_j}$) Neut – N – D - Set in (H, N_{τ_1}, N_{τ_2}).

Proposition 3.5.*If L is a (N_{τ_i},N_{τ_j}) Neut – F – C - Set in (H, N_{τ_1}, N_{τ_2}) and if $K \subseteq L$ for a Neut - Set R in (H, N_{τ_1}, N_{τ_2}), then K is a (N_{τ_i},N_{τ_j}) Neut – F – C - Set in (H, N_{τ_1}, N_{τ_2}).*

Proof. Let L be the (N_{τ_i},N_{τ_j}) Neut – F – C - Set in (H, N_{τ_1}, N_{τ_2}). Then, $L = \bigcup_{n=1}^{\infty} L_n$, where L_n's are (N_{τ_i},N_{τ_j}) Neut – N – D - Set in (H, N_{τ_1}, N_{τ_2}). Now, $K \cap L = K \cap (\bigcup_{n=1}^{\infty} L_n)$= $\bigcup_{n=1}^{\infty}(K \cap L_n)$. Since, $K \cap$

$L_n \subseteq L_n$and L_n's are$(N_{\tau_i}$,$N_{\tau_j})$ Neut – N – D - Set in (H, N_{τ_1}, N_{τ_2}). Therefore, $(K \cap L_n)$'s are $(N_{\tau_i}$,$N_{\tau_j})$ Neut – N – D - Set in (H, N_{τ_1}, N_{τ_2}). Hence $K = \bigcup_{n=1}^{\infty}(K \cap L_n)$, where $(K \cap L_n)$'s are $(N_{\tau_i}$,$N_{\tau_j})$ Neut – N – D - Set in (H, N_{τ_1}, N_{τ_2}), implies that K is a $(N_{\tau_i}$,$N_{\tau_j})$ Neut – F – C - Set in (H, N_{τ_1}, N_{τ_2}).

Proposition 3.6.*If K is a $(N_{\tau_i}$,$N_{\tau_j})$ Neut – R - Set in (H, N_{τ_1}, N_{τ_2}) and if $K \subseteq L$ for a Neut - Set L in (X, N_{τ_1}, N_{τ_2}), then L is a $(N_{\tau_i}$,$N_{\tau_j})$Neut – R - Set in (H, N_{τ_1}, N_{τ_2}).*

Proof. Let K be the (N_{τ_i}, N_{τ_j}) Neut – R - Set in (H, N_{τ_1}, N_{τ_2}). Then, $\overline{K}$ is a $(N_{\tau_i}$,$N_{\tau_j})$ Neut – F – C - Set in (H, N_{τ_1}, N_{τ_2}). Now, $K \subseteq L$ for a Neut - Set L in (H, N_{τ_1}, N_{τ_2}), implies that $\overline{K} \supseteq \overline{L}$. Then $\overline{L}$ is a (N_{τ_i}, N_{τ_j}) Neut – F – C - Set in (H, N_{τ_1}, N_{τ_2}). Hence, L is a$(N_{\tau_i}$,$N_{\tau_j})$Neut – R - Set in (H, N_{τ_1}, N_{τ_2}). ■

Proposition 3.7.*If the Neut – B – T - Space (H, N_{τ_1}, N_{τ_2}) is a(N_{τ_i}, N_{τ_j}) Neut – B - Space, then no non-zero Neut - OSet is a (N_{τ_i}, N_{τ_j}) Neut – F – C - Set in (H, N_{τ_1}, N_{τ_2}).*

*Proof.*Let K be a non-zero Neut - OSet in (H, N_{τ_1}, N_{τ_2}). Then, $Nint_{\tau_i}(K) = K$, (i = 1,2). Suppose that K is a (N_{τ_i}, N_{τ_j}) Neut – F – C - Set in (H, N_{τ_1}, N_{τ_2}).Since, (X, N_{τ_1}, N_{τ_2}) is a (N_{τ_i}, N_{τ_j}) Neut – B -Space by proposition 3.3, $Neut - int_{\tau_i}(K) = 0_N$, (i = 1,2). This implies K= 0_N, is a contradiction. Therefore, no non-zero Neut - OSet is a (N_{τ_i}, N_{τ_j}) Neut – F – C - Set in (H, N_{τ_1}, N_{τ_2}). ■

Proposition 3.8.*If (H, N_{τ_1}, N_{τ_2}) is a$(N_{\tau_i}$,$N_{\tau_j})$, Neut – B - Space then each $(N_{\tau_i}$,$N_{\tau_j})$ Neut – R - Set is a $(N_{\tau_i}$,$N_{\tau_j})$ Neut –D - Set in (H, N_{τ_1}, N_{τ_2}).*

Proof. Let K be the $(N_{\tau_i}$,$N_{\tau_j})$ Neut – R - Set in the $(N_{\tau_i}$,$N_{\tau_j})$ Neut – B – Space (H, N_{τ_1}, N_{τ_2}). Then, by proposition 3.3, $Neut - cl_{\tau_i}(K) = 1_N$, (i = 1,2) in (H, N_{τ_1}, N_{τ_2}). Hence, $Neut - cl_{\tau_1}\left(Neut - cl_{\tau_2}(K)\right) = Neut - cl_{\tau_2}\left(Neut - cl_{\tau_1}(K)\right) = 1_N$. Therefore, K is a $(N_{\tau_i}$,$N_{\tau_j})$ Neut – D - Set in (H, N_{τ_1}, N_{τ_2}). ■

Proposition 3.9.*If the(N_{τ_i}, N_{τ_j}) Neut – F – C - Set K is a Neut - CSet in a $(N_{\tau_i}$,$N_{\tau_j})$ Neut – B - Space (H, N_{τ_1}, N_{τ_2}), then K is a $(N_{\tau_i}$,$N_{\tau_j})$ Neut – N – D - Set in (H, N_{τ_1}, N_{τ_2}).*

Proof. Let K be a (N_{τ_i}, N_{τ_j}) Neut – F – C - Set in the $(N_{\tau_i}$,$N_{\tau_j})$ Neut – B - Space (H, N_{τ_1}, N_{τ_2}) and $Neut - cl_{\tau_i}(K) = K$,(i = 1,2) --- (1). By proposition 3.3, $Neut - int_{\tau_i}(K) = 0_N$, (i = 1,2) --- (2) for the $(N_{\tau_i}$,$N_{\tau_j})$ Neut – F - C - Set K in (H, N_{τ_1}, N_{τ_2}). Then, from (1) and (2) we have $Neut - int_{\tau_1}\left(Neut - cl_{\tau_2}(K)\right) = Neut - int_{\tau_2}\left(Neut - cl_{\tau_1}(K)\right) = 0_N$. Hence, K is a $(N_{\tau_i}$,$N_{\tau_j})$ Neut –N – D - Set in (H, N_{τ_1}, N_{τ_2}). ■

Proposition 3.10.*If the Neut – B – T - Space (H, N_{τ_1}, N_{τ_2}) is a $(N_{\tau_i}$,$N_{\tau_j})$ Neut – B - Space , then (H, N_{τ_1}, N_{τ_2}) is a $(N_{\tau_i}$,$N_{\tau_j})$ Neut – S – C - Space.*

Proof. Let(X, N_{τ_1}, N_{τ_2}) be a $(N_{\tau_i}$,$N_{\tau_j})$ Neut – B - Space . Then, we have $Neut - int_{\tau_i}(\bigcup_{n=1}^{\infty} K_n) = 0_N$, (i =1,2), where $K_n's$ are $(N_{\tau_i}$,$N_{\tau_j})$ Neut – N – D - Set in (H, N_{τ_1}, N_{τ_2}). Now, we claim that $1_N \neq \bigcup_{n=1}^{\infty} K_n$. Suppose that $1_N = \bigcup_{n=1}^{\infty} K_n$, Then $Neut - int_{\tau_i}(\bigcup_{n=1}^{\infty} K_n)$ =$Neut - int_{T_i}(1_N)$ =1_N, (i = 1, 2) which implies that 0 = 1, a contradiction. Hence, we must have $1_N \neq \bigcup_{n=1}^{\infty} K_n$. Therefore, the $(N_{\tau_i}$,$N_{\tau_j})$ Neut - B - Space (H, N_{τ_1}, N_{τ_2}) is a $(N_{\tau_i}$,$N_{\tau_j})$ Neut – S – C - Space.. ■

Proposition 3.11.*If $Neut - int_{\tau_i}(\bigcup_{n=1}^{\infty} K_n) = 0_N$, (i = 1, 2), where $Neut - int_{\tau_i}(K_n) = 0_N$, (i = 1, 2) and K_n's are Neut - CSets in (H, N_{τ_1}, N_{τ_2}). Then (H, N_{τ_1}, N_{τ_2}) is a $(N_{\tau_i}$,$N_{\tau_j})$ Neut – B - Space.*

Proof. Now, $\overline{K_n} \in N_{\tau_i}$, (i=1, 2 and n ≥1) implies $Neut - int_{\tau_i}(\overline{K_n}) = \overline{K_n}$ and hence $Neut - cl_{\tau_i}(K_n) = K_n$, (i = 1, 2 and n ≥1). Now,$Neut - int_{\tau_i}(K_n) = 0_N$ and $Neut - cl_{\tau_i}(K_n) = K_n$ implies that $Neut - int_{\tau_i}(Neut - cl_{\tau_i}(K_n)) = Neut - int_{\tau_i}(K_n) = 0_N$(i.e) $Neut - int_{\tau_i}(Neut - cl_{\tau_i}(K_n)) = 0_N$, (i = 1,2 and n ≥1). In particular, $Neut - int_{\tau_1}(Neut - cl_{\tau_2}(K_n)) = 0_N$ and $Neut - int_{\tau_2}(Neut - cl_{\tau_1}(K_n)) = 0_N$ and for n ≥1. Hence, K_n's are $(N_{\tau_i}$,$N_{\tau_j})$ Neut – N – D - Sets in (H, N_{τ_1}, N_{τ_2}). Therefore, $Neut -$

$int_{\tau_i}(\bigcup_{n=1}^{\infty} K_n) = 0_N$, (i=1, 2), where K_n's are (N_{τ_i}, N_{τ_j}) Neut – N – D - Sets in (H, N_{τ_1}, N_{τ_2}). Hence, (H, N_{τ_1}, N_{τ_2}) is a (N_{τ_i}, N_{τ_j}) Neut – B - Space. ■

Proposition 3.12.*If $Neut - cl_{\tau_i}(\bigcap_{n=1}^{\infty} K_n) = 1_N$, (i=1, 2), where K_n's are (N_{τ_i}, N_{τ_j}) Neut - D and Neut - OSets in (H, N_{τ_1}, N_{τ_2}), then (H, N_{τ_1}, N_{τ_2}) is a (N_{τ_i}, N_{τ_j}) Neut – B - Space.*

*Proof.*Let K_n's be(N_{τ_i}, N_{τ_j}) Neut - D and Neut - OSets in (H, N_{τ_1}, N_{τ_2}). Now, $Neut - cl_{\tau_i}(\bigcap_{n=1}^{\infty} K_n) = 1_N$ implies that $1 - Neut - cl_{\tau_i}(\bigcap_{n=1}^{\infty} K_n) = 1 - 1_N = 0_N$, for (i=1, 2). Then, $Neut - int_{\tau_i}(1 - \bigcap_{n=1}^{\infty} K_n) = 0_N$, (i=1, 2) and hence $Neut - int_{\tau_i}((\bigcup_{n=1}^{\infty}(1 - K_n)) = 0_N$ ---(1). Since, K_n's are (N_{τ_i}, N_{τ_j}) Neut – D - Sets in (H, N_{τ_1}, N_{τ_2}), $Neut - cl_{\tau_i}(K_n) = 1_N$, ((i=1, 2 and n ≥1). Then $1 - Neut - cl_{\tau_i}(K_n) = 1 - 1_N = 0_N$ which implies $Neut - int_{\tau_i}(1 - K_n) = 0_N$, (i=1, 2). Hence by proposition 3.3, (H, N_{τ_1}, N_{τ_2}) is a (N_{τ_i}, N_{τ_j}) Neut – B - Space.

■

4. (N_{τ_i}, N_{τ_j}) Neutrosophic semi nowhere dense set

Proposition 4.1[3].*Let K be a Neut - Set in (H, N_τ). Then $Neut - int(K) \subseteq Neut - Se - -int(K) \subseteq K \subseteq Neut - Se - cl(K) \subseteq Neut - cl(K)$.* ■

Definition 4.1[6].*Let (H, N_{τ_1}, N_{τ_2}) be a neutrosophic bitopological space(Neut – B –T - Space).A Neut - Set K in a Neut – B - T - Space (H,N_{τ_1}, N_{τ_2}) is a (N_{τ_i}, N_{τ_j}) Neutrosophic semi open set (Neut – S -OSet)if $K \subseteq Neut - cl_{\tau_2}(Neu.int_{\tau_1}(K))$ and $K \subseteq Neu.cl_{\tau_1}(Neu.int_{\tau_2}(K))$.*

Definition 4.2.*Let (H, N_{τ_1}, N_{τ_2}) be a Neut – B – T - Space. A Neut - Set K in (H, N_{τ_1}, N_{τ_2}) is a (N_{τ_i}, N_{τ_j}) Neutrosophic semi closed set (Neut – S - CSet) if $Neut - int_{\tau_2}\left(Neut - cl_{\tau_1}(K)\right) \subseteq K$ and $Neut - int_{\tau_1}(Neut - cl_{\tau_2}(K)) \subseteq K$.*

Definition 4.3.*Let (X, N_{τ_1}, N_{τ_2}) be a Neut – B – T - Space. A Neu - Set K in (H,N_{τ_1}, N_{τ_2}) is a (N_{τ_i}, N_{τ_j}) Neut – Se - CSet if $K = Neut - Se - cl(K)$and (N_{τ_i}, N_{τ_j}) Neut – Se - CSet if $K = Neu - Se - int(K)$.*

Definition 4.4[2].*Let (H, N_{τ_i}, N_{τ_j}) be the Neut –B – T - Space. Then for a set $K = \{< h, \mu_{ij}, \sigma_{ij}, \gamma_{ij} >: h \in H\}$, neutrosophic (N_{τ_i}, N_{τ_j}) N- semi interior of K is the union of all (N_{τ_i}, N_{τ_j}) Neut –Se - OSets of H contained in K and defined as follows:*

(N_{τ_i}, N_{τ_j}) Neut – Se - (int(K)) = $\{< h, \bigcup_{N_{\tau_i}} \bigcup_{N_{\tau_j}} \mu_{ij}, \bigcup_{N_{\tau_i}} \bigcup_{N_{\tau_j}} \sigma_{ij}, \bigcap_{N_{\tau_i}} \bigcap_{N_{\tau_j}} \gamma_{ij} >: h \in H\}$

Definition 4.5[2].*Let (H,N_{τ_i}, N_{τ_j}) be a Neut – B – T - Space. Then for a set $K = \{< h, \mu_{ij}, \sigma_{ij}, \gamma_{ij} >: h \in H\}$, neutrosophic (N_{τ_i}, N_{τ_j}) N- semi closure of K is the intersection of all (N_{τ_i}, N_{τ_j})Neut – Se-Csets of H containing K and defined as follows:*

(N_{τ_i}, N_{τ_j}) Neut – Se-(cl(K)) = $\{< h, \bigcap_{N_{\tau_i}} \bigcap_{N_{\tau_j}} \mu_{ij}, \bigcap_{N_{\tau_i}} \bigcap_{N_{\tau_j}} \sigma_{ij}, \bigcup_{N_{\tau_i}} \bigcup_{N_{\tau_j}} \gamma_{ij} >: h \in H\}$

Definition 4.6.*A Neut - Set K in (H,N_{τ_1}, N_{τ_2}) is called (N_{τ_i}, N_{τ_j}) neutrosophic semi dense (Neut – Se – D - Set) if there exists no (N_{τ_i}, N_{τ_j}) neutrosophic semi closed set (Neut – Se – CSet)L in (H,N_{τ_1}, N_{τ_2})suchthat $Neut - Se - cl_{\tau_1}\left(Neu - Se - cl_{\tau_2}(K)\right) = Neu - Se - cl_{\tau_2}\left(Neu - Se - cl_{\tau_1}(K)\right) = 1_N$.*

Definition 4.7.*A Neu-Set K in (H , N_{τ_1}, N_{τ_2}) is a (N_{τ_i}, N_{τ_j}) neutrosophic semi nowhere dense set (Neut – Se - N – D - Set) if there exists no (N_{τ_i}, N_{τ_j}) Neut – Se - OSet L in (H , N_{τ_1}, N_{τ_2}) such that $Neut - Se - int_{\tau_1}\left(Neut - Se - cl_{\tau_2}(K)\right) = Neut - Se - int_{\tau_2}\left(Neut - Se - cl_{\tau_1}(K)\right) = 0_N$.*

Proposition 4.2.*If K has a (N_{τ_i}, N_{τ_j}) Neut – Se - CSet with $Neut - S - int_{\tau_i}(K) = 0_N$, (i = 1, 2)in (H, N_{τ_1}, N_{τ_2}), then K has a (N_{τ_i}, N_{τ_j}) Neut – Se – N – D -Set in (H, N_{τ_1}, N_{τ_2}).*

Proof. Let K be the (N_{τ_i}, N_{τ_j}) Neut – Se – CSet in $(X, N_{\tau_1}, N_{\tau_2})$. The $Neut - Se - cl_{\tau_1}(K) = K$ and $Neut - Se - cl_{\tau_2}(K) = K$. Also, we have $Neut - Se - int_{\tau_1}(K) = 0_N$and $Neut - Se - int_{\tau_2}(K) = 0_N$. Then,$Neut - Se - int_{\tau_1}\left(Neut - Se - cl_{\tau_2}(K)\right) = Neut - Se - int_{\tau_1}(K) = 0_N$and $Neut - Se - int_{\tau_2}\left(Neut - Se - cl_{\tau_1}(K)\right) = Neut - Se - int_{\tau_2}(K) = 0_N$. Then, $Neut - Se - int_{\tau_1}\left(Neut - Se - cl_{\tau_2}(K)\right) = Neut - Se - int_{\tau_2}\left(Neut - Se - cl_{\tau_1}(K)\right) = 0_N$. Therefore, K is a (N_{τ_i}, N_{τ_j}) Neut – Se – N – D -Set in $(X, N_{\tau_1}, N_{\tau_2})$. ■

Proposition 4.3.*If K has a (N_{τ_i}, N_{τ_j}) Neut – Se – N - D Set in (H, N_{τ_1}, N_{τ_2}), then $Neut - Se - int_{\tau_i}(K) = 0_N$.*

Proof. Let K be the (N_{τ_i}, N_{τ_j})Neut – Se – N – D - Set in (H, N_{τ_1}, N_{τ_2}). Then, $Neut - Se - int_{\tau_1}\left(Neut - Se - cl_{\tau_2}(K)\right) = Neut - Se - int_{\tau_2}\left(Neut - Se - cl_{\tau_1}(K)\right) = 0_N$.

Now, K$\subseteq Neut - Se - cl_{\tau_2}(K)$implies that $Neu - Se - int_{\tau_1}(K) \subseteq Neut - Se - int_{\tau_1}(Neut - Se - cl_{\tau_2}(K))$. Then $Neut - Se - int_{\tau_1}(K) \subseteq 0_N$, implies that $Neut - Se - int_{\tau_1}(K) = 0_N$. Also,$K \subseteq Neut - Se - cl_{\tau_1}(K)$ implies Neut $- Se - int_{\tau_2}(K) \subseteq Neut - Se - int_{\tau_2}(Neu.S.cl_{\tau_1}(K))$.

Then $Neut - Se - int_{\tau_2}(K) \subseteq 0_N$, implies that $Neut - Se - int_{\tau_2}(K) = 0_N$. Hence, we have $Neut - Se - int_{\tau_i}(K) = 0_{N,}$ (i = 1, 2). ■

Proposition 4.4.*If K has a (N_{τ_i}, N_{τ_j}) Neut - Se- N - D - Set in (H, N_{τ_1}, N_{τ_2}), then $1 - K$ has a (N_{τ_i}, N_{τ_j}) Neut – S e - D - Set in (H, N_{τ_1}, N_{τ_2}).*

*Proof.*Let K be the (N_{τ_i}, N_{τ_j})Neut – Se - N - D - Set in (H, N_{τ_1}, N_{τ_2}), then $Neut - Se - int_{\tau_1}\left(Neut - Se-.cl_{\tau_2}(K)\right) = Neut - Se - int_{\tau_2}\left(Neut - Se - cl_{\tau_1}(K)\right) = 0_N$. Then, $1 - Neut - Se - int_{\tau_1}\left(Neut - Se - cl_{\tau_2}(K)\right) = 1 - 0 = 1_N$. Then, $Neut - Se - cl_{\tau_1}\left(1 - Neut - Se - cl_{\tau_2}(K)\right) = 1_N$, Which implies that $Neut - Se-.cl_{\tau_1}\left(Neut - Se - int_{\tau_2}(1 - K)\right) = 1_N$. But $Neut - Se - cl_{\tau_1}\left(Neut - Se - int_{\tau_2}(1 - K)\right) \subseteq Neut - Se - cl_{\tau_1}\left(Neut - Se - cl_{\tau_2}(1 - K)\right)$. Hence, $1_N \subseteq Neut - Se - cl_{\tau_1}\left(Neut - Se - cl_{\tau_2}(1 - K)\right)$. That is $Neut - Se - cl_{\tau_1}\left(Neut - Se - cl_{\tau_2}(1 - K)\right) = 1_N$. Also, 1$-Neut - Se - int_{\tau_2}\left(Neut - Se - cl_{\tau_1}(K)\right) = 1 - 0 = 1_N$. Then, $Neut - Se - cl_{\tau_2}\left(1 - Neut - Se - cl_{\tau_1}(K)\right) = 1_N$, Which implies that $Neut - Se - cl_{\tau_2}\left(Neut - Se - int_{\tau_1}(1 - K)\right) = 1_N$. But $Neut - Se - cl_{\tau_2}\left(Neut - Se - int_{\tau_1}(1 - K)\right) \subseteq Neut - Se - cl_{\tau_2}\left(Neut - Se - cl_{\tau_1}(1 - K)\right)$. Hence, $1_N \subseteq Neut - Se - cl_{\tau_2}\left(Neut - Se - cl_{\tau_1}(1 - K)\right)$. That is $Neut - Se - cl_{\tau_2}\left(Neut - Se - cl_{\tau_1}(1 - K)\right) = 1_N$. Therefore, $1 - K$ is a (N_{τ_i}, N_{τ_j})Neut – S - D - Set in (H, N_{τ_1}, N_{τ_2}). ■

Proposition 4.5.*If L has a(N_{τ_i},N_{τ_j}) Neut – Se – N – D - Set in (H, N_{τ_1}, N_{τ_2}) and $K \subseteq L$ for a Neut - Set K in (H, N_{τ_1}, N_{τ_2}), then K also has a (N_{τ_i},N_{τ_j}) Neut – S – N – D - Set in (H, N_{τ_1}, N_{τ_2}).*

*Proof.*Let L be a (N_{τ_i}, N_{τ_j})Neut – Se – N – D - Set in (H, N_{τ_1}, N_{τ_2}). Then, $Neut - Se - int_{\tau_1}\left(Neut - Se - cl_{\tau_2}(L)\right) = Neut - Se - int_{\tau_2}\left(Neut - Se - cl_{\tau_1}(L)\right) = 0_N$. Now, $K \subseteq L$ implies that $Neut - Se - int_{\tau_1}\left(Neut - Se - cl_{\tau_2}(K)\right) \subseteq Neut - Se - \left(Neut - Se - cl_{\tau_2}(L)\right)$. Then, $Neut - Se - int_{\tau_1}\left(Neut - Se - cl_{\tau_2}(K)\right) \subseteq 0_N$. Hence, $Neut - Se - int_{\tau_1}\left(Neut - Se - cl_{\tau_2}(K)\right) = 0_N$. Now, $K \subseteq L$ implies that $Neut - Se - int_{\tau_2}\left(Neut - Se - cl_{\tau_1}(K)\right) \subseteq Neut - Se - int_{\tau_2}\left(Neut - Se - cl_{\tau_1}(L)\right)$. Then, $Neut - Se - int_{\tau_2}(Neut - Se - cl_1(K)) \subseteq 0_N$. Hence, $Neut - Se - int_{\tau_2}\left(Neut - Se - cl_{\tau_1}(K)\right) = 0_N$.Hence,

$Neut - Se - int_{\tau_1}\left(Neut - Se - cl_{\tau_2}(K)\right) = Neut - Se - int_{\tau_2}\left(Neut - Se - cl_{\tau_1}(K)\right) = 0_N$. Therefore, K also has a (N_{τ_i},N_{τ_j})Neut – Se – N – D - Set in (H, N_{τ_1}, N_{τ_2}). ∎

Proposition 4.6. *Every (N_{τ_i}, N_{τ_j}) Neut – Se – N – D - Set in (H, N_{τ_1},N_{τ_2}) is a (N_{τ_i},N_{τ_j}) Neut – N – D - Set in (H, N_{τ_1}, N_{τ_2}).*

Proof. Let K be the (N_{τ_i},N_{τ_j})Neut – Se – N – D - Set in a Neut – B – T - Space (H, N_{τ_1}, N_{τ_2}), then we have $Neut - Se - int_{\tau_1}\left(Neut - Se - cl_{\tau_2}(K)\right) = Neut - Se - int_{\tau_2}\left(Neut - Se - cl_{\tau_1}(K)\right) = 0_N$. By proposition(4.1)$Neut - int_{\tau_1}\left(Neut - cl_{\tau_2}(K)\right) \subseteq Neut - Se - int_{\tau_1}\left(Neut - Se - cl_{\tau_2}(K)\right)$ and $Neut - int_{\tau_2}\left(Neut - cl_{\tau_1}(K)\right) \subseteq Neut - Se - int_{\tau_2}\left(Neut - Se - cl_{\tau_1}(K)\right)$. Hence, $Neut - int_{\tau_1}\left(Neut - cl_{\tau_2}(K)\right) \subseteq 0_N$ and $Neut - int_{\tau_2}\left(Neut - cl_{\tau_1}(K)\right) \subseteq 0_N$. Therefore, $Neut - int_{\tau_1}\left(Neut - cl_{\tau_2}(K)\right) = 0_N$and $Neut - int_{\tau_2}\left(Neut - cl_{\tau_1}(K)\right) = 0_N$. Hence, K is a ($N_{\tau_i}$,$N_{\tau_j}$) Neut- N – D - Set in (H, N_{τ_1}, N_{τ_2}). ∎

Proposition 4.7. *Every (N_{τ_i}, N_{τ_j}) Neut – Se - D - Set in (H, N_{τ_1},N_{τ_2}) is a (N_{τ_i}, N_{τ_j}) Neut – D - Set in (H, N_{τ_1}, N_{τ_2}).*

Proof. Let K be the (N_{τ_i},N_{τ_j}) Neut - Se - D -Set in (H, N_{τ_1}, N_{τ_2}), then $Neut - Se - cl_{\tau_1}\left(Neut - Se - .cl_{\tau_2}(K)\right) = Neu.S.cl_{\tau_2}\left(Neu.S.cl_{\tau_1}(K)\right) = 1_N$. By proposition (4.1)$Neu.S.cl_{\tau_1}(K) \subseteq Neu.cl_{\tau_1}(K)$ and $Neu.S.cl_{\tau_2}(K) \subseteq Neu.cl_{\tau_2}(K)$. Hence, $Neu.S.cl_{\tau_2}(Neu.S.cl_{\tau_1}(K) \subseteq Neu.S.cl_{\tau_2}(Neu.cl_{\tau_1}(K))$and $Neu.S.cl_{\tau_1}(Neu.S.cl_{\tau_2}(K) \subseteq Neu.S.cl_{\tau_1}(Neu.cl_{\tau_2}(K))$.

By proposition (4.1)$Neu.S.cl_{\tau_2}(Neu.S.cl_{\tau_1}(K) \subseteq Neu.cl_{\tau_2}(Neu.cl_{\tau_1}(K))$and $Neu.S.cl_{\tau_1}(Neu.S.cl_{\tau_2}(K) \subseteq Neu.cl_{\tau_1}(Neu.cl_{\tau_2}(K))$. But, $Neu.S.cl_{\tau_1}\left(Neu.S.cl_{\tau_2}(K)\right) = Neu.S.cl_{\tau_2}\left(Neu.S.cl_{\tau_1}(K)\right) = 1_N$. Hence, $1_N \subseteq Neu.cl_{\tau_2}(Neu.cl_{\tau_1}(K))$and $1_N \subseteq Neu.cl_{\tau_1}(Neu.cl_{\tau_2}(K))$. Therefore, $Neu.cl_{\tau_1}\left(Neu.cl_{\tau_2}(K)\right) = Neu.cl_{\tau_2}\left(Neu.cl_{\tau_1}(K)\right) = 1_N$ and so K is a (N_{τ_i},N_{τ_j})Ne-D-Set in (H, N_{τ_1}, N_{τ_2}). ∎

Proposition 4.8. *Let (H, N_{τ_1},N_{τ_2}) be a Neut – B – T -Space. If a non-zero Neut - Set K in (H, N_{τ_1},N_{τ_2}) is a (N_{τ_i}, N_{τ_j}) Neut – N – D - Set with $Neut - Se - int_{\tau_i}(K) = 0_N$, (i = 1, 2), then K is a (N_{τ_i}, N_{τ_j}) Neut – Se – N –D - Set in (H, N_{τ_1}, N_{τ_2}).*

Proof. Let K be the (N_{τ_i},N_{τ_j})Neut – N – D - Set in (H, N_{τ_1},N_{τ_2}), then $Neut - int_{\tau_1}\left(Neut - cl_{\tau_2}(K)\right) = Neut - int_{\tau_2}\left(Neut - cl_{\tau_1}(K)\right) = 0_N$ and we have $Neut - int_{\tau_1}(Neut - cl_{\tau_2}(K)) \subseteq K$ and $Neut - int_{\tau_2}\left(Neut - (K)\right) \subseteq K$. Hence, K is a ($N_{\tau_i}$,$N_{\tau_j}$) Neut – S - CSet with $Neut - Se - int_{\tau_i}(K) = 0_N$, (i = 1, 2), in (H, N_{τ_1},N_{τ_2}). Hence by proposition 4.2, K is a (N_{τ_i}, N_{τ_j})Neut – Se - N-D - Set in (H, N_{τ_1},N_{τ_2}).

Proposition 4.9. *Let (H, N_{τ_1},N_{τ_2}) be a Neut – B – T - Space. If K has a (N_{τ_i}, N_{τ_j}) Neut - OSet and (N_{τ_i}, N_{τ_j}) Neut – D - Set in (H, N_{τ_1},N_{τ_2}) and $L \subseteq 1 - K$ with $Neut - Se - int_{\tau_i}(K) = 0_N$,(i = 1, 2), then L is a (N_{τ_i}, N_{τ_j}) Neut – Se – N – D - Set in (H, N_{τ_1}, N_{τ_2}).* ∎

Proof. Let K be a (N_{τ_i}, N_{τ_j})Neut - OSet and (N_{τ_i}, N_{τ_j})Neut - D – Set in (H, N_{τ_1},N_{τ_2}). Then, 1- K is a (N_{τ_i}, N_{τ_j})Neut – N – D - Set in (H, N_{τ_1},N_{τ_2}) and $Neut - int_{\tau_1}\left(Neut - cl_{\tau_2}(1 - K)\right) = Neut - int_{\tau_2}\left(Neut - cl_{\tau_1}(1 - K)\right) = 0_N$. Now, $L \subseteq 1 - K$ implies that $Neut - int_{\tau_1}\left(Neut - cl_{\tau_2}(L)\right) \subseteq Neut - int_{\tau_1}\left(Neut - cl_{\tau_2}(1 - K)\right)$ and $Neut - int_{\tau_2}\left(Neut - cl_{\tau_1}(L)\right) \subseteq Neut - int_{\tau_2}\left(Neut - cl_{\tau_1}(1 - K)\right)$. But $Neut - int_{\tau_1}\left(Neut - cl_{\tau_2}(1 - K)\right) = Neut - int_{\tau_2}\left(Neut - cl_{\tau_1}(1 - K)\right) = 0_N$.

Hence, $Neut - int_{\tau_1}\left(Neut - cl_{\tau_2}(L)\right) \subseteq 0_N$and $Neut - int_{\tau_2}\left(Neut - cl_{\tau_1}(L)\right) \subseteq 0_N$. Therefore, $Neut - int_{\tau_1}\left(Neut - cl_{\tau_2}(L)\right) = 0_N$ and $Neut - int_{\tau_2}\left(Neut - cl_{\tau_1}(L)\right) = 0_N$. Hence,L is a ($N_{\tau_i}$,$N_{\tau_j}$)Neut – N – D - Set with $Neut - Se - int_{\tau_i}(K) = 0_{N,}$ (i = 1, 2) in (H, N_{τ_1}, N_{τ_2}). Therefore, by proposition 4.8, L is a (N_{τ_i},N_{τ_j})Neut – Se – N – D - Set in (H, N_{τ_1}, N_{τ_2}). ■

Proposition 4.10.*Let (H, N_{τ_1}, N_{τ_2}) be a Neut – B – T - Space. If L is a (N_{τ_i},N_{τ_j}) Neut – N – D - Set in (X, N_{τ_1}, N_{τ_2}) and if K ⊆ L for a Neut – Set K with $Neut - Se - int_{\tau_i}(K) = 0_{N,}$ (i = 1, 2), then K is a (N_{τ_i}, N_{τ_j}) Neut – S – N – D - Set in (H, N_{τ_1}, N_{τ_2}).*

Proof. Let L be the (N_{τ_i}, N_{τ_j})Neut – N – D – Set in(H, N_{τ_1}, N_{τ_2}). Then, $Neut - int_{\tau_1}\left(Neut - cl_{\tau_2}(L)\right) = Neut - int_{\tau_2}\left(Neut - cl_{\tau_1}(L)\right) = 0_N$. Now, K ⊆ L implies $Neut - int_{\tau_1}\left(Neut - cl_{\tau_2}(K)\right) \subseteq Neut - int_{\tau_1}\left(Neut - cl_{\tau_2}(L)\right)andNeut - int_{\tau_2}\left(Neut - cl_{\tau_1}(K)\right) \subseteq Neut - int_{\tau_2}\left(Neu.cl_{\tau_1}(L)\right)$. But $Neu.int_{\tau_1}\left(Neut - cl_{\tau_2}(L)\right) = Neut - int_{\tau_2}\left(Neut - cl_{\tau_1}(L)\right) = 0_N$. Hence, $Neut - int_{\tau_1}\left(Neut - cl_{\tau_2}(K)\right) \subseteq 0_N$ and $Neut - int_{\tau_2}\left(Neut - cl_{\tau_1}(K)\right) \subseteq 0_N$. Therefore, $Neut - int_{\tau_1}\left(Neut - cl_{\tau_2}(K)\right) = 0_N$and $Neut - int_{\tau_2}\left(Neut - cl_{\tau_1}(K)\right) = 0_N$. Hence, K is a ($N_{\tau_i}$,$N_{\tau_j}$) Neut – N – D - Set with $Neut - Se - int_{\tau_i}(K) = 0_{N,}$ (i = 1, 2), and by proposition 4.7, K is a (N_{τ_i}, N_{τ_j})Neut – Se – N – D - Set in (H, N_{τ_1}, N_{τ_2}). ■

Definition 4.8.*Let (H, N_{τ_1}, N_{τ_2}) be a Neut – B – T – Space. A Neut - Set K in (H, N_{τ_1}, N_{τ_2}) is a (N_{τ_i}, N_{τ_j}) neutrosophic semi first category set (Neut – Se – F - C -Set) if $K = \bigcup_{n=1}^{\infty} K_n$, where K_n's are (N_{τ_i}, N_{τ_j}) Neut – Se – N- D - Sets in (H, N_{τ_1}, N_{τ_2}). Anyother neutrosophic set is a (N_{τ_i}, N_{τ_j}) neutrosophic semi second category set (Neut – Se – S – C - Set).*

Definition 4.9.*If K has a (N_{τ_i},N_{τ_j}) Neut – Se – F – C – Set in (H, N_{τ_1}, N_{τ_2}), then Neut - Set (1- K)has a (N_{τ_i},N_{τ_j}) neutrosophic semi residual set (Neut – S – R - Set) in (H, N_{τ_1}, N_{τ_2}).*

Definition 4.10.*A Neut –B – T - Space (H, N_{τ_1}, N_{τ_2}) is a (N_{τ_i},N_{τ_j}) neutrosophic semi first category space(Neut – Se – F – C - Space), if the neutrsophic set 1_N is a Neut – Se - F – C – Set in (H, N_{τ_1}, N_{τ_2}). That is, $1_N = \bigcup_{n=1}^{\infty} K_n$, where K_n's are (N_{τ_i},N_{τ_j}) Neut – Se – N – D – Set in (H, N_{τ_1}, N_{τ_2}). Otherwise, (H, N_{τ_1}, N_{τ_2}) is a(N_{τ_i}, N_{τ_j}) neutrosophic semi second category space. (Neut – Se – S – C - Space).* ■

Proposition 4.11.*If K has a (N_{τ_i}, N_{τ_j}) Neut – Se - F – C - Set in (H, N_{τ_1}, N_{τ_2}), then $1 - K = \bigcap_{n=1}^{\infty} L_n$, where L_n' s are (N_{τ_i},N_{τ_j}) Neut – Se – D -Set in (H, N_{τ_1}, N_{τ_2}).*

Proof. Let K be the (N_{τ_i}, N_{τ_j})Neut – F – C - Set in (H, N_{τ_1}, N_{τ_2}). Then, $K = \bigcup_{n=1}^{\infty} K_n$, where K_n's are (N_{τ_i},N_{τ_j}) Neut – Se – N – D - Set in (H, N_{τ_1}, N_{τ_2}). Now, $1 - K = 1 - \bigcup_{n=1}^{\infty} K_n = \bigcap_{n=1}^{\infty} 1 - K_n$. Since, K_n is a (N_{τ_i},N_{τ_j}) Neut – S –N – D – Set in (H, N_{τ_1}, N_{τ_2}). By proposition 4.4, $1 - K_n$ is a (N_{τ_i},N_{τ_j})Neut – Se – D - Setin (H, N_{τ_1}, N_{τ_2}). Let us take $L_n = 1 - K_n$. Then, $1 - K = \bigcap_{n=1}^{\infty} L_n$, where L_n is a (N_{τ_i},N_{τ_j})Neut – Se – D - Set in (H, N_{τ_1}, N_{τ_2}). ■

Proposition 4.12.*If L has a (N_{τ_i}, N_{τ_j}) Neut –Se - F – C - Set in (H, N_{τ_1}, N_{τ_2}) and if K ⊆ L for a Neut - set K in (H, N_{τ_1}, N_{τ_2}), then K also has a (N_{τ_i}, N_{τ_j}) Neut - F – C – Set in (H, N_{τ_1}, N_{τ_2}).*

Proof. Let L be the (N_{τ_i}, N_{τ_j}) Neut –Se - F – C - Set in (H, N_{τ_1}, N_{τ_2}). Then, we have $L = \bigcup_{n=1}^{\infty} L_n$, where L_n's are (N_{τ_i}, N_{τ_j}) Neut – Se – N – D -Set in (H, N_{τ_1}, N_{τ_2}). Now, $K \cap L = K \cap (\bigcup_{n=1}^{\infty} L_n) = \bigcup_{n=1}^{\infty}(K \cap L_n)$. Also, $K \subseteq L$ implies that $K \cap L = K$. Therefore, $K = \bigcup_{n=1}^{\infty}(K \cap L_n)$. Since, $(K \cap L_n) \subseteq L_n$and L_n' s are (N_{τ_i}, N_{τ_j}) Neut – Se – N – D - Set in (H, N_{τ_1}, N_{τ_2}). Hence, $K = \bigcup_{n=1}^{\infty}(K \cap L_n)$,where $(K \cap L_n)$'s are (N_{τ_i},N_{τ_j})Neut – Se – N – D - Set in (H, N_{τ_1}, N_{τ_2}), implies that K has a (N_{τ_i}, N_{τ_j}) set Neut – Se - F- C -Set in (H, N_{τ_1}, N_{τ_2}). ■

Proposition 4.13. *If K has a (N_{τ_i}, N_{τ_j}) Neut – Se – R -Set in e (H, N_{τ_1}, N_{τ_2}) and if $K \subseteq L$ for a Neut - Set L in (H, N_{τ_1}, N_{τ_2}), then L also has a (N_{τ_i}, N_{τ_j}) Neut – Se - R-Set in (H, N_{τ_1}, N_{τ_2}).*

Proof. Let K be the (N_{τ_i}, N_{τ_j}) Neut – Se – R -Set in (H, N_{τ_1}, N_{τ_2}). Then, $1 - K$ is a (N_{τ_i}, N_{τ_j}) Neut – Se – F – C - Set in (H, N_{τ_1}, N_{τ_2}). Now, $K \subseteq L$ implies that $1 - K \supseteq 1 - L$. Then, by proposition 4.12, $1 - L$ is a (N_{τ_i}, N_{τ_j}) Neut – Se – F – C -Set in (H, N_{τ_1}, N_{τ_2}). Hence, L is a (N_{τ_i}, N_{τ_j}) Neut – Se – R - Set in (H, N_{τ_1}, N_{τ_2}). ■

Proposition 4.14. *If K has a (N_{τ_i}, N_{τ_j}) Neut – Se – F – C - Set in a Neut – B – T - Space (H, N_{τ_1}, N_{τ_2}), then K has a (N_{τ_i}, N_{τ_j}) Neut – F – C -Set in (H, N_{τ_1}, N_{τ_2}).*

Proof. Let K be the (N_{τ_i}, N_{τ_j}) Neut – Se – F – C -Set in (H, N_{τ_1}, N_{τ_2}). Then, we have $K = \bigcup_{n=1}^{\infty} K_n$, where K_n's are (N_{τ_i}, N_{τ_j}) Neut – Se – N – D - Sets in (H, N_{τ_1}, N_{τ_2}). By proposition 2.5, K_n's are (N_{τ_i}, N_{τ_j}) Neut – N – D - Sets in (H, N_{τ_1}, N_{τ_2}) and hence $K = \bigcup_{n=1}^{\infty} K_n$, where K_n's are (N_{τ_i}, N_{τ_j}) Neut – N - D- Sets in (H, N_{τ_1}, N_{τ_2}). Therefore, K is a (N_{τ_i}, N_{τ_j}) Neut – F – C -Set in (H, N_{τ_1}, N_{τ_2}). ■

Proposition 4.15. *If (H, N_{τ_1}, N_{τ_2}) is a (N_{τ_i}, N_{τ_j}) Neut – Se – B - Space, then (H, N_{τ_1}, N_{τ_2}) is a (N_{τ_i}, N_{τ_j}) Neut – B -Space.*

Proof. Let K be the (N_{τ_i}, N_{τ_j}) Neut – S – F – C -Set in a (N_{τ_i}, N_{τ_j}) Neut – Se – B - Space (H, N_{τ_1}, N_{τ_2}). Then, $Neut - Se - int_{\tau_i}(K) = 0_N$, (i = 1, 2). By proposition 4.14, the (N_{τ_i}, N_{τ_j}) Neut – S – F – C – Set is a (N_{τ_i}, N_{τ_j}) Neut – F – C - Set in (H, N_{τ_1}, N_{τ_2}). By proposition (4.1), $Neut - int_{\tau_i}(K) \subseteq Neut - Se - int_{\tau_i}(K)$, (i = 1, 2). Since, $Neut - Se - int_{\tau_i}(K) = 0_N$, (i = 1, 2) we have $Neut - int_{\tau_i}(K) \subseteq 0_N$, (i = 1, 2). Therefore, $Neut - int_{\tau_i}(K) = 0_N$, (i = 1, 2) and (H, N_{τ_1}, N_{τ_2}) is a (N_{τ_i}, N_{τ_j}) Neut–B-Space. ■

Proposition 4.16. *If $K = \bigcup_{n=1}^{\infty} K_n$, where K_n's are (N_{τ_i}, N_{τ_j}) Neut – N – D - Sets with $Neut - Se - int_{\tau_i}(K) = 0_N$, $(i = 1, 2)$, is a (N_{τ_i}, N_{τ_j}) Neut – F – C - Set in (H, N_{τ_1}, N_{τ_2}), then K is a (N_{τ_i}, N_{τ_j}) Neut – S – F – C -Set in (H, N_{τ_1}, N_{τ_2}).*

Proof. Let K be the (N_{τ_i}, N_{τ_j}) Neut – F – C -Set in (H, N_{τ_1}, N_{τ_2}). Then, $K = \bigcup_{n=1}^{\infty} K_n$, where K_n's are (N_{τ_i}, N_{τ_j}) Neut – N – D -Sets By proposition 4.8, the (N_{τ_i}, N_{τ_j}) Neut – N – D - Sets are (N_{τ_i}, N_{τ_j}) Neut – Se – N- D- Sets in (H, N_{τ_1}, N_{τ_2}) and hence $K = \bigcup_{n=1}^{\infty} K_n$, where K_n's are (N_{τ_i}, N_{τ_j}) Neut – Se – N – D - Sets and hence K is a (N_{τ_i}, N_{τ_j}) Neut – S – F - C - Set in (H, N_{τ_1}, N_{τ_2}).

5. (N_{τ_i}, N_{τ_j}) Neutrosophic Semi Baire Space

Definition 5.1. *A Neut – B – T - Space (H, N_{τ_1}, N_{τ_2}) is a (N_{τ_i}, N_{τ_j}) neutrosophic semi baire space (Neut – Se – B - Space) if $Neu - Se - int_{\tau_i}(\bigcup_{n=1}^{\infty} K_n) = 0_N$, (i = 1, 2) where K_n's are (N_{τ_i}, N_{τ_j}) Neut - Se – N – D - Sets in (H, N_{τ_1}, N_{τ_2}).*

Example 5.1. Let H={k,l}and K= { < l, 0.6, 0.6,0.3 >, < m, 0.6, 0.6,0.5 > }

L= { < k, 0.6, 0.6, 0.4 >, < l, 0.5, 0.6, 0.5 > } N= { < k, 0.6, 0.6, 0.2 >, < l, 0.7, 0.6, 0.5 > }.

Then, $N_{\tau_1} = \{0_N, K, L, 1_N\}$ and $N_{\tau_2} = \{0_N, L, M, 1_N, \}$. Then (H, N_{τ_1}, N_{τ_2}) is a Neut – B – T - Space . Here $\overline{L}$ is a (N_{τ_i}, N_{τ_j}) Neut – Se – N –D -Set in (H, N_{τ_1}, N_{τ_2}). which is a (N_{τ_i}, N_{τ_j}) Neut – Se – F -C-Set in (H, N_{τ_1}, N_{τ_2}). Hence, $Neut - Se - int_{\tau_1}(\overline{K}) = 0_N$ and also $Neut - Se - int_{\tau_2}(\overline{K}) = 0_N$. Therefore, (H, N_{τ_1}, N_{τ_2}) is called a (N_{τ_i}, N_{τ_j}) Neut – Se – B - Space

Proposition 5.1. *Let (H, N_{τ_1}, N_{τ_2}) be a Neut – B – T - Space. Then the following conditions are equivalent.*

(1) (H, N_{τ_1}, N_{τ_2}) is a (N_{τ_i}, N_{τ_j}) Neut - Se – B - Space.

(2) $Neut - Se - int_{\tau_i}(K) = 0_N$, (i = 1, 2), for every (N_{τ_i}, N_{τ_j}) Neut – Se – F – C - Set K in (H, N_{τ_1}, N_{τ_2}).

(3) $Neut - Se - cl_{\tau_i}(L) = 1_N$*, (i = 1, 2), for every* (N_{τ_i}, N_{τ_j}) *Neut – Se – R - Set L in (H,* N_{τ_1}, N_{τ_2}*).*

Proof.(1) ⟹(2) Let K be the (N_{τ_i}, N_{τ_j}) Neut – Se – F – C - Set in (H, N_{τ_1}, N_{τ_2}). Then, K= $\bigcup_{n=1}^{\infty} K_n$, where K_n's are (N_{τ_i}, N_{τ_j}) Neut – Se – N – D - Set in (H, N_{τ_1}, N_{τ_2}). Now, $Neut - Se - int_{\tau_i}(K) = Neut - Se - int_{\tau_i}(\bigcup_{n=1}^{\infty} K_n) = 0_N$, (i = 1, 2) [since (H, N_{τ_1}, N_{τ_2}) is a (N_{τ_i}, N_{τ_j}) neutrosophic semi baire bitopological space]. $Neut - Se - int_{\tau_i}(K) = 0_N$, where K_n's are (N_{τ_i}, N_{τ_j}) Neut – Se – N – D - Sets in (H, N_{τ_1}, N_{τ_2}).

(2) ⟹(3) Let L be the (N_{τ_i}, N_{τ_j}) Neut – Se – R - Set in (H, N_{τ_1}, N_{τ_2}). Then, $\bar{L}$ is a (N_{τ_i}, N_{τ_j}) Neut –Se – F - C - Set in (H, N_{τ_1}, N_{τ_2}). By hypothesis, $Neut - Se - int_{\tau_i}(\bar{L}) = 0_N$, (i = 1, 2), which implies $1 - Neut - Se - cl_{\tau_i}(L) = 0_N$. Hence, $Neut - Se - cl_{\tau_i}(L) = 1_N$, (i = 1, 2).

(3) ⟹(1) Let K be a (N_{τ_i}, N_{τ_j}) Neut – Se – F – C - Set in (H, N_{τ_1}, N_{τ_2}). Then, $K = \bigcup_{n=1}^{\infty} K_n$, where K_n's are (N_{τ_i}, N_{τ_j}) Neut – Se – N – D - Sets in (H, N_{τ_1}, N_{τ_2}). Now, K is a (N_{τ_i}, N_{τ_j}) Neut – Se – F - C-Set in (H, N_{τ_1}, N_{τ_2}) implies that $\bar{K}$ is a (N_{τ_i}, N_{τ_j})Neut – Se – R - Set in (H, N_{τ_1}, N_{τ_2}). By hypothesis, we have $Neut - Se - cl_{\tau_i}(\bar{K}) = 1_N$, (i = 1, 2) which implies that $1 - Neut - Se - int_{\tau_i}(K) = 1_N$, (i = 1, 2). Then, $Neut - Se - int_{\tau_i}(K) = 0_N$. That is, $Neut - Se - int_{\tau_i}(\bigcup_{n=1}^{\infty} K_n) = 0_N$, (i = 1, 2), where K_n's are (N_{τ_i}, N_{τ_j}) Neut – Se – N – D - Set in (H, N_{τ_1}, N_{τ_2}). Hence, (H, N_{τ_1}, N_{τ_2}) is a (N_{τ_i}, N_{τ_j}) neutrosophic semi baire bitopological space. ■

Proposition 5.2.*If (H,* N_{τ_1}, N_{τ_2}*)has a* (N_{τ_i}, N_{τ_j}) *Neut – Se - B- Space, then (H,* N_{τ_1}, N_{τ_2}*)has a* (N_{τ_i}, N_{τ_j}) *Neut - Se - S- C - Space.*

*Proof.*Let (H, N_{τ_1}, N_{τ_2}) be the (N_{τ_i}, N_{τ_j}) Neut – Se – B - Space. Then, $Neut - Se - int_{\tau_i}(\bigcup_{n=1}^{\infty} K_n) = 0_N$, (i = 1, 2) where K_n's are (N_{τ_i}, N_{τ_j}) Neut – Se – N – D - Sets in (H, N_{τ_1}, N_{τ_2}). Now, we claim that $\bigcup_{n=1}^{\infty} K_n \neq 1_N$, (i=1,2). Suppose that,$\bigcup_{n=1}^{\infty} K_n = 1_N$, (i = 1, 2). Then, $Neut - Se - int(\bigcup_{n=1}^{\infty} K_n) = Neut - Se - int(1_N) = 1_N$, (i = 1, 2), which implies that $0_N = 1_N$ a contradiction. This contradiction shows that $\bigcup_{n=1}^{\infty} K_n \neq 1_N$. Therefore, (H, N_{τ_1}, N_{τ_2}) is a (N_{τ_i}, N_{τ_j}) Neut - Se – S - C - Space..

Proposition 5.3.*If the Neut- B - T - Space (H,* N_{τ_1}, N_{τ_2}*) is a* (N_{τ_i}, N_{τ_j}) *Neu - Se – B - Space, then no non-zero* ■ (N_{τ_i}, N_{τ_j}) *Neut – Se - OSet is a* (N_{τ_i}, N_{τ_j}) *Neut - Se - F- C- Set in (H,* N_{τ_1}, N_{τ_2}*).*

*Proof.*Let K be the non-zero Neut – Se - OSet in (H, N_{τ_1}, N_{τ_2}). Then, $Neut - Se - int_{\tau_i}(K) = K$, (i = 1, 2). Suppose K is a (N_{τ_i}, N_{τ_j}) Neut - Se - F - C- Set in (H, N_{τ_1}, N_{τ_2}). Since, (H, N_{τ_1}, N_{τ_2}) is a (N_{τ_i}, N_{τ_j}) Neut – Se – B - Space, by proposition 5.1, $Neut - Se - int_{\tau_i}(K) = 0_N$, (i = 1, 2). This implies K= 0_N, a contradiction. Hence, no non-zero Neut – Se - OSet is a (N_{τ_i}, N_{τ_j}) Neut – Se - F – C-Set in(H, N_{τ_1}, N_{τ_2}). ■

Proposition 5.4.*If the Neut – B – T - Space (H,* N_{τ_1}, N_{τ_2}*) is a* (N_{τ_i}, N_{τ_j}) *Neut – Se – B - Space, then each* (N_{τ_i}, N_{τ_j}) *Neut – Se - R - Set is a* (N_{τ_i}, N_{τ_j}) *Neut – Se - D - Set in (H,* N_{τ_1}, N_{τ_2}*).*

Proof. Let K be the (N_{τ_i}, N_{τ_j}) Neut- Se- R - Set in the (N_{τ_i}, N_{τ_j}) Neut – Se – B - Space (H, N_{τ_1}, N_{τ_2}). Then, by proposition 5.1, $Neut - Se - cl_{\tau_i}(K) = 1_N$, (i = 1, 2) in (H, N_{τ_1}, N_{τ_2}).Hence,we have $Neut - Se - cl_{\tau_1}\left(Neut - Se - cl_{\tau_2}(K)\right) = Neut - Se - cl_{\tau_2}\left(Neut - Se - cl_{\tau_1}(K)\right) = 1_N$. Therefore, K is a (N_{τ_i}, N_{τ_j}) Neut – Se - D- Set in (H, N_{τ_1}, N_{τ_2}). ■

Proposition 5.5.*If the* (N_{τ_i}, N_{τ_j}) *Neut – Se – F – C - Set K is a* (N_{τ_i}, N_{τ_j}) *Neut – Se - CSet in* (N_{τ_i}, N_{τ_j}) *Neut – Se – B - Space (H,* N_{τ_1}, N_{τ_2}*), then K is a* (N_{τ_i}, N_{τ_j}) *Neut – Se – N - D - Set in (H,* N_{τ_1}, N_{τ_2}*).*

Proof. Let K be the (N_{τ_i}, N_{τ_j}) Neut – Se - F- C- Set in (H, N_{τ_1}, N_{τ_2}). Since K is a Neut – Se - CSet then $Neut - Se - cl_{\tau_i}(K) = L$, (i = 1, 2) ---(1) $Neut - Se - int_{\tau_i}(K) = 0_N$, (i = 1, 2) --- (2) for the (N_{τ_i}, N_{τ_j}) Neut – Se - F- C - Set K in (Z, N_{τ_1}, N_{τ_2}) . Then, from (1) and (2) we have $Neut - Se - int_{\tau_1}\left(Neut - Se - cl_{\tau_2}(K)\right) = Neut - Se - int_{\tau_2}\left(Neut - Se - cl_{\tau_1}(K)\right) = 0_N$. Hence, K is a (N_{τ_i}, N_{τ_j}) Neut – Se – N – D - Set in (H, N_{τ_1}, N_{τ_2}). ■

Proposition 5.6.*If* $Neut - Se - int_{\tau_i}(\bigcup_{n=1}^{\infty} K_n) = 0_N$*, (i = 1, 2), where* $Neut - Se - int_{\tau_i}(K_n) = 0_N$*, (i = 1, 2) and* K_n*'s are Neu-Se- CSets in (H,* N_{τ_1}, N_{τ_2}*). Then (H,* N_{τ_1}, N_{τ_2}*) is a (*N_{τ_i}*,*N_{τ_j}*) Neut – Se -B- Space.*

*Proof.*Now, $\overline{K_n} \in \tau_i$, (i = 1, 2 and n ≥ 1) implies that $Neut - Se - int_{\tau_i}(\overline{K_n}) = \overline{K_n}$ and hence $Neut - Se - cl_{\tau_i}(K_n) = K_n$, (i = 1, 2 and n ≥ 1). Now,$Neut - Se - int_{\tau_i}(K_n) = 0_N$ and $Neut - Se - cl_{\tau_i}(K_n) = K_n$ implies that $Neut - Se - int_{\tau_i}(Neu - Se - cl_{\tau_i}(K_n)) = Neu - Se - int_{\tau_i}(K_n) = 0_N$. (i.e) $Neut - Se - int_{\tau_i}(Neut - Se - cl_{\tau_i}(K_n)) = 0_N$, (i=1,2 and n ≥ 1). In particular, $Neut - Se - int_{\tau_1}(Neut - Se - cl_{\tau_2}(K_n)) = 0_N$ and $Neut - Se - int_{\tau_2}(Neut - Se - cl_{\tau_1}(K_n)) = 0_N$ and for n ≥ 1. Hence, K_n's are (N_{τ_i},N_{τ_j}) Neut – Se – N - D- Sets in (H, N_{τ_1}, N_{τ_2}). Therefore, we have $Neut - Se - int_{\tau_i}(\bigcup_{n=1}^{\infty} K_n) = 0_N$, (i = 1, 2), where K_n's are (N_{τ_i}, N_{τ_j}) Neut - Se- N - D- Set in (H, N_{τ_1}, N_{τ_2}). Hence, (H, N_{τ_1}, N_{τ_2}) is a (N_{τ_i}, N_{τ_j}) Neut - Se- B - Space. ■

Proposition 5.7.*If* $Neut - Se - cl_{\tau_i}(\bigcap_{n=1}^{\infty} K_n) = 1_N$*, (i = 1, 2), where* K_n*'s are (*N_{τ_i}*,* N_{τ_j}*) Neut-Se-D-Set and Neut-Se-OSets in (H,* N_{τ_1}, N_{τ_2}*), then (H,* N_{τ_1}, N_{τ_2}*) is a (*N_{τ_i}*,* N_{τ_j}*) Neut – Se – B - Space.*

*Proof.*Let K_n's be the (N_{τ_i}, N_{τ_j}) Neut – Se - D-Set and Neut – Se - OSets in (H, N_{τ_1}, N_{τ_2}). Now, $Neut - S - cl_{\tau_i}(\bigcap_{n=1}^{\infty} K_n) = 1_N$ implies that $1 - Neut - Se - cl_{\tau_i}(\bigcap_{n=1}^{\infty} K_n) = 1 - 1_N = 0_N$, for (i = 1, 2). Then, we have $Neut - Se - int_{\tau_i}(1 - \bigcap_{n=1}^{\infty} K_n) = 0_N$, (i = 1, 2) and hence $Neut - Se - int_{\tau_i}((\bigcup_{n=1}^{\infty}(1 - K_n)) = 0_N$ -- - (1). Since, K_n's are (N_{τ_i},N_{τ_j}) Neut – Se - D-Sets in (H, N_{τ_1}, N_{τ_2}), $Neut - Se - cl_{\tau_i}(K_n) = 1_N$, (i = 1, 2 and n ≥ 1). Then $1 - Neut - Se - cl_{\tau_i}(K_n) = 1_N - 1_N = 0_N$which implies that $Neut - Se - int_{\tau_i}(1 - K_n) = 0_N$, (i = 1, 2), where K_n's are (N_{τ_i}, N_{τ_j}) Neut –Se - OSets in (H, N_{τ_1}, N_{τ_2}). Hence by proposition 5.1, we have (H, N_{τ_1}, N_{τ_2}) is a (N_{τ_i}, N_{τ_j}) Neut - Se – B - Space. ■

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