https://doi.org/10.56452/2022-07-051

ISSN: 0974-5823 Vol. 7 No. 7 July, 2022)

International Journal of Mechanical Engineering

Analytical Study of Global Domination Number of Kronecker Product Of Wheel Graphs and Petersen Graph

Jitendra Binwal
Professor and HOD, Department of Mathematics, School of Liberal Arts and Sciences, Mody University of Science and Technology, Laxmangarh-332311,Sikar, Rajasthan, India.
Email: dr.jitendrabinwaldkm@gmail.com, Orcid Id: 0000-0001-7183-2494

Bhupendra Singh
Research Scholar, Department of Mathematics, School of Liberal Arts and Sciences, Mody University of Science and Technology, Laxmangarh-332311,Sikar, Rajasthan, India.
Email:bhupendrapoonia003@gmail.com

***Corresponding Author:** Bhupendra Singh
Research Scholar, Department of Mathematics, School of Liberal Arts and Sciences, Mody University of Science and Technology, Laxmangarh-332311,Sikar, Rajasthan, India.
Email:bhupendrapoonia003@gmail.com

ABSTRACT
If G_1 and G_2 are two graphs with their vertex set $V_1 = \{u_1, u_2, u_3, \dots \dots u_n\}$ and $V_2 = \{v_1, v_2, v_3, \dots \dots v_n\}$ Respectively then the Kronecker product of these two graphs is defined to be a graph $K(V_1 \times V_2)$ with its vertex set as $V_{K(V_1 \times V_2)}$ such that $V_{K(V_1 \times V_2)}$ is the Cartesian product of the sets V_1 and V_2 [1]. Where two vertices $(u_p, v_q), (u_r, v_s)$ in Kronecker product graph have an edge if and only if $u_p u_r$ and $v_q v_s$ are edges in graph V_1 and V_2 respectively. The paper is related on the analytical study of global domination number of Kronecker product of Wheel Graph and Petersen Graph. Sampathkumar introduced the global domination in graphs. In this paper, first we have defined the Kronecker product of wheel graph W_n for $n \geq 4$ and Petersen graph. Then, we will find the exact value of global domination number of Kronecker product of Wheel Graph and Petersen Graph in generalize form.

Keyword: Graph, Kronecker Product, Global Domination Number, Wheel Graphs, Petersen Graphs
AMS [2010]:05C30, 05C90, 14G50, 68R10

1. INTRODUCTION:

Graph theory helps us to obtain the solution of many real life problems in engineering and technology, physical science, social science and biological science. In last century, theory of graphs has an unexpected growth in research field due to its extensive applications to inter-disciplinary domains. The notation of global dominating sets in graph theory is one of the important concepts which are very simple but it has lots of important applications. Global dominating sets also play a significant role in the improvement of exact algorithms and parameterized complexity. In 1989, the mathematical study of theory of global domination number was proposed by Sampathkumar [2] then, in 1990 the same concept was revised by Brigham and Dutton [3] in generalized form as factor domination. The mathematical way of global domination in graph theory is a minimization process of operators in any network which has more than one distinct spanning branch. The concept of Kronecker product of two graphs was introduced by Paul Weichsel [1]. S. Maheswari and S. Meenakshi [4] defined the domination and split domination process for Kronecker product of some graphs. In this paper, we generalized the concept of Kronecker product of wheel graph W_n for $n \geq 4$ and Petersen graph and then derived the exact value of global domination number of Kronecker product of Wheel Graph and Petersen Graph in generalize form.

There are lots of applications of global domination number of Kronecker product of two graphs. One of the important applications is that, if we want to find the minimum number of operators to run two systems or two networks at a time with more than one branch. For this the application of global domination number of Kronecker product of two graphs can be applied in such a way that first finds the Kronecker product of both networks then global domination number of that Kronecker product gives the minimum number of operators to run two systems or two networks at a time with more than one branch.

2. RELATED DEFINITIONS AND RESULTS

2.1. Dominating Set and Domination Number: A set of vertices $D \in V$ is called a dominating set of graph G if each vertex in the complement of D in V is in the neighborhood of some vertex in set D and the dominating number $\gamma(G)$ of any G is the least number of members in a dominating set of graph G [5].

2.2. Global Dominating Set and Global Dominating Number: The global dominating set $D_g(G)$ of a graph G with some distinct spanning factors $F_1, F_2, F_3, \dots F_k$ is a set of vertices of graph G such that the members of the set D_g are related with all vertices of all factors $F_1, F_2, F_3, \dots F_k$ and the minimum cardinality of such set is called global domination number [5].

2.3. Kronecker product of two graphs: If G_1 and G_2 are two graphs with their vertex set $V_1 = \{u_1, u_2, u_3, \dots \dots u_n\}$ and $V_2 = \{v_1, v_2, v_3, \dots \dots v_n\}$ respectively then the Kronecker product of these two graphs is defined to be a graph $K(V_1 \times V_2)$ with its vertex set as $V_{V_1 \times V_2}$ such that $V_{V_1 \times V_2}$ is the Cartesian product of the sets V_1 and V_2. Where two vertices $(u_p, v_q), (u_r, v_s)$ in Kronecker product graph have an edge if and only if $u_p u_r$ and $v_q v_s$ are edges in graph V_1 and V_2 respectively [1].

2.3.1. Illustration: Let the two graphs be $G_1 = C_3$ and $G_2 = C_3$ with their vertex set $V_1 = \{u_1, u_2, u_3\}$ and $V_2 = \{v_1, v_2, v_3\}$ respectively.

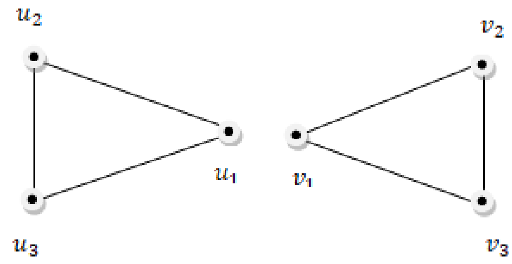

Figure 1: Cycle Graph ($G_1 = C_3$ and $G_2 = C_3$)

The Kronecker product of these two cycle graphs is defined to be a graph $K(C_3 \times C_3)$ such that the set of vertices of Kronecker product graph is $V_{C_3 \times C_3} = \{u_1v_1, u_1v_2, u_1v_3, u_2v_1, u_2v_2, u_2v_3, u_3v_1, u_3v_2, u_3v_3\}$ and the members $(u_p, v_q), (u_r, v_s)$ of the set $V_{C_3 \times C_3}$ have an edge if and only if $u_p u_r$ and $v_q v_s$ are edges in graph G_1 and G_2 respectively. Hence, the graph of the Kronecker product of these two cycle graphs is as follows-

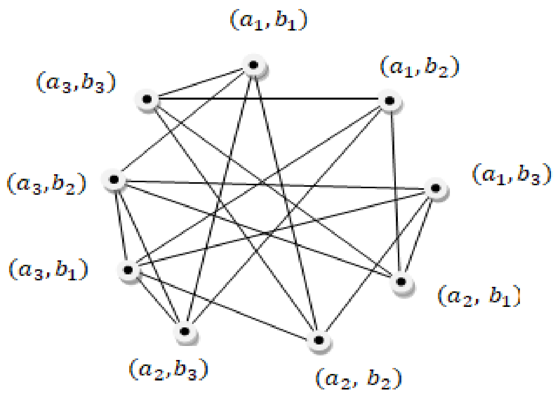

Figure 2: Kronecker Product of two Cycle graphs

2.4. Theorem[6]

Let G be a graph with n number of vertices and having k spanning factors such that highest degree of any vertex is $\Delta(G)$ then global domination number of graph G is always greater than equal to $\frac{nk}{\Delta(G)+k}$.

3. WHEEL GRAPH W_4:

The graph W_4 contains 4 vertices with one central vertex as shown in figure 3. First we will find the Kronecker product of graph W_4 corresponding to its two factors F_1 and F_2-

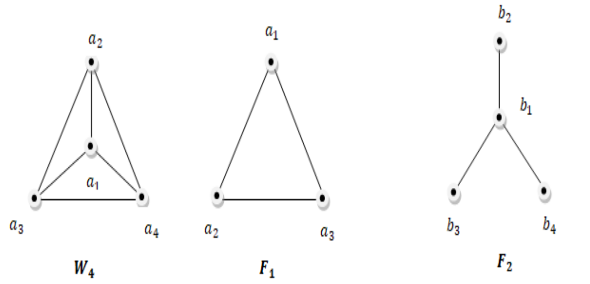

Figure 3: Wheel Graph W_4 and two factors F_1 and F_2

Here factor F_1 contains three vertices and factor F_2 contains four vertices. Hence, the Kronecker product $K(W_4)$ of graph W_4 contains total 12 vertices such that the vertex set of Kronecker product is $V_{K(W_4)} = \{(a_1, b_1), (a_1, b_2), (a_1, b_3), (a_1, b_4), (a_2, b_1), (a_2, b_2), (a_2, b_3), (a_2, b_4), (a_3, b_1), (a_3, b_2), (a_3, b_3), (a_3, b_4)\}$ and the two vertices in Kronecker product $K(W_4)$ have an edge if 1st tuple in both vertices have an edge in factor F_1 and 2nd tuple in both vertices have an edge in factor F_2. Therefore, the graph of Kronecker product $K(W_4)$ is as follows-

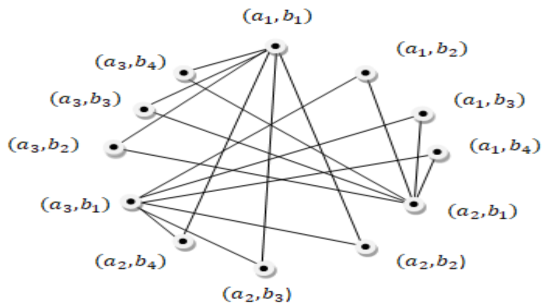

Figure 4: Kronecker Product of Wheel Graph W_4

Now, for finding the global domination number and global dominating set of Kronecker product $K(W_4)$ of wheel graph W_4 corresponding to its two dijoint spanning factors $F_1{}'$ and F_2'.

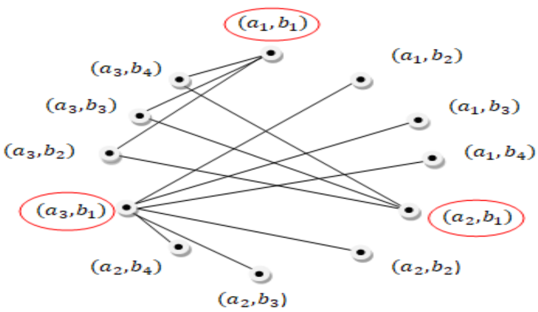

Figure 5: Factors $F_1{}'$ of Kronecker Product of Wheel Graph W_4

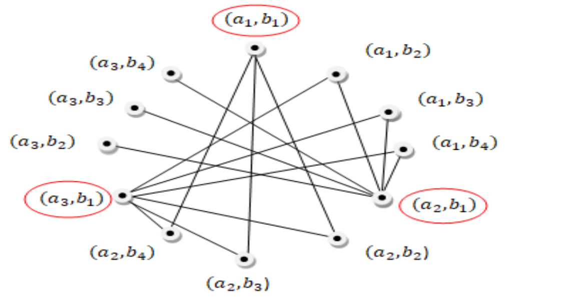

Figure 6: Factors F_2' of Kronecker Product of Wheel Graph W_4

Here, Kronecker product $K(W_4)$ of wheel graph W_4 has order 12 $(i.e. n = 12)$ corresponding to its two dijoint factors $F_1{}'$ and F_2' $(i.e. k = 2)$. the maximum degree of any vertex in kronecker product $K(W_4)$ of wheel graph W_4 is 6 $(i.e. \Delta(K(W_4)) = 6)$. Therefore, by theorem 2.4, we get

$$\gamma_g(K(W_4)) \geq \frac{o(K(W_4)) \times k}{\Delta\big(K(W_4)\big) + k}$$

$$\gamma_g(K(W_4)) \geq \frac{12 \times 2}{6 + 2}$$

$$\gamma_g(K(W_4)) \geq 3$$

The members of the set $S = \{(a_1, b_1), (a_2, b_1), (a_3, b_1)\}$ are related with all the vertices of both factors of kronecker product $K(W_4)$ of wheel graph W_4. Hence, set S dominated both factors F_1' and F_2' of kronecker product $K(W_4)$. Therefore, set S is a global dominating set of Kronecker product $K(W_4)$ of wheel graph W_4. So, $\gamma_g\big(K(W_4)\big) \leq 3$. Hence, from both the results we conclude that global domination number of Kronecker product $K(W_4)$ of wheel graph W_4 is equal to 3 and its global dominating set is $D_g\big(K(W_4)\big) = \{(a_1, b_1), (a_2, b_1), (a_3, b_1)\}$.

4. WHEEL GRAPH W_5:

The graph W_5 contains 5 vertices with one central vertex as shown in figure 7. First we will find the Kronecker product of graph W_5 corresponding to its two factors F_1 and F_2-

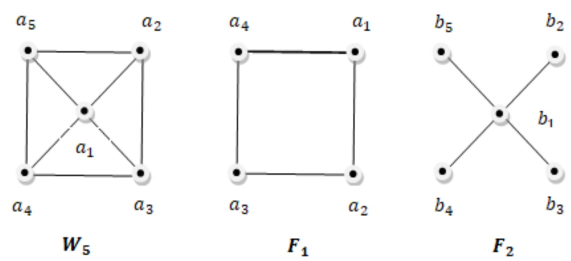

Figure 7: Wheel Graph W_5 and two factors F_1 and F_2

https://doi.org/10.56452/2022-07-051

Here factor F_1 contains 4 vertices and factor F_2 contains 5 vertices. Hence, the Kronecker product $K(W_5)$ of graph W_5 contains total 20 vertices such that the vertex set of Kronecker product is $V_{K(W_5)} =$

$$\{(a_1,b_1),(a_1,b_2),(a_1,b_3),(a_1,b_4),(a_1,b_5)\ (a_2,b_1),(a_2,b_2),(a_2,b_3),(a_2,b_4),(a_2,b_5)\ (a_3,b_1),(a_3,b_2),(a_3,b_3),(a_3,b_4),(a_3,b_5),(a_4,b_1),(a_4,b_2),(a_4,b_3),(a_4,b_4),(a_4,b_5)\}$$

and the two vertices in Kronecker product $K(W_5)$ of graph W_5 have an edge if 1st tuple in both vertices have an edge in factor F_1 and 2nd tuple in both vertices have an edge in factor F_2. Therefore, the graph of Kronecker product $K(W_5)$ is as follows-

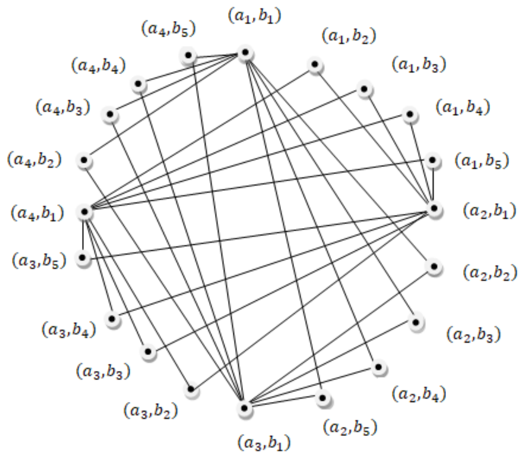

Figure 8: Kronecker Product of Wheel Graph W_5

Now, for finding the global domination number and global dominating set of Kronecker product $K(W_5)$ of wheel graph W_5 corresponding to its two dijoint spanning factors $F_1{}'$ and F_2'.

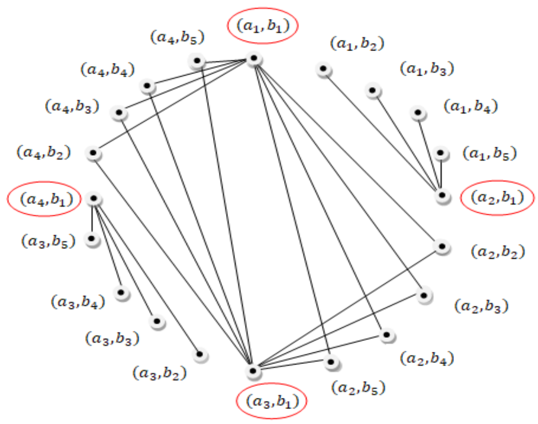

Figure 9: Factors $F_1{}'$ of Kronecker Product of Wheel Graph W_5

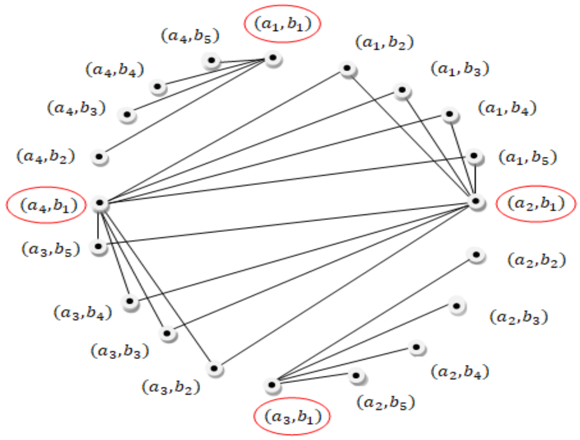

Figure 10: Factors $F_2{}'$ of Kronecker Product of Wheel Graph W_5

Here, Kronecker product $K(W_5)$ of wheel graph W_5 has order 20 $(i.e. n = 20)$ corresponding to its two dijoint factors $F_1{}'$ and F_2' $(i.e. k = 2)$. the maximum degree of any vertex in kronecker product $K(W_5)$ of wheel graph W_5 is 8 $(i.e. \Delta(K(W_4)) = 8)$. Therefore, by theorem 2.4, we get

$$\gamma_g(K(W_5)) \geq \frac{o(K(W_5)) \times k}{\Delta\big(K(W_5)\big) + k}$$

$$\gamma_g(K(W_4)) \geq \frac{20 \times 2}{8 + 2}$$

$$\gamma_g(K(W_4)) \geq 4$$

The members of the set $S = \{(a_1,b_1),(a_2,b_1),(a_3,b_1),(a_4,b_1)\}$ are related with all the vertices of both factors of kronecker product $K(W_5)$ of wheel graph W_5. Hence, set S dominated both factors F_1' and F_2' of Kronecker product $K(W_5)$. Therefore, set S is a global dominating set of Kronecker product $K(W_5)$ of wheel graph W_5. So, $\gamma_g\big(K(W_5)\big) \leq 4$. Hence, from both the results we conclude that global domination number of Kronecker product $K(W_5)$ of wheel graph W_5 is equal to 4 and its global dominating set is $D_g\big(K(W_5)\big) = \{(a_1,b_1),(a_2,b_1),(a_3,b_1),(a_4,b_1)\}$.

5. WHEEL GRAPH W_6:

The graph W_6 contains 6 vertices with one central vertex as shown in figure 11. First we will find the Kronecker product of graph W_6 corresponding to its two factors F_1 and F_2-

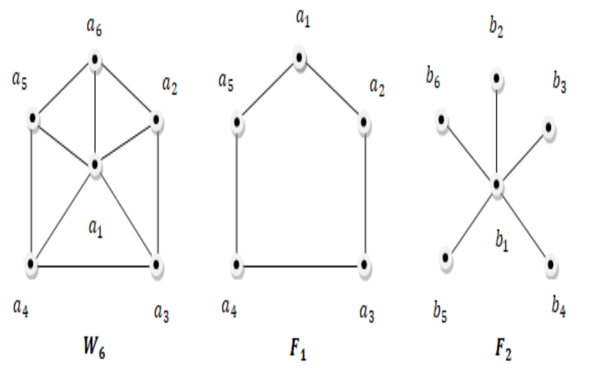

Figure 11: Wheel Graph W_6 and two factors F_1 and F_2

Here factor F_1 contains 5 vertices and factor F_2 contains 6 vertices. Hence, the Kronecker product $K(W_6)$ of graph W_6 contains total 30 vertices such that the vertex set of Kronecker product is $V_{K(W_6)} =$
$\{(a_1,b_1),(a_1,b_2),(a_1,b_3),(a_1,b_4),(a_1,b_5),(a_1,b_6),(a_2,b_1),(a_2,b_2),(a_2,b_3),(a_2,b_4),(a_2,b_5),(a_2,b_6),(a_3,b_1),(a_3,b_2),(a_3,b_3),(a_3,b_4),(a_3,b_5),(a_3,b_6),(a_4,b_1),(a_4,b_2),(a_4,b_3),(a_4,b_4),(a_4,b_5),(a_4,b_6),(a_5,b_1),(a_5,b_2),(a_5,b_3),(a_5,b_4),(a_5,b_5),(a_5,b_6)\}$
and the two vertices in Kronecker product $K(W_6)$ of graph W_6 have an edge if 1st tuple in both vertices have an edge in factor F_1 and 2nd tuple in both vertices have an edge in factor F_2. Therefore, the graph of Kronecker product $K(W_6)$ is as follows-

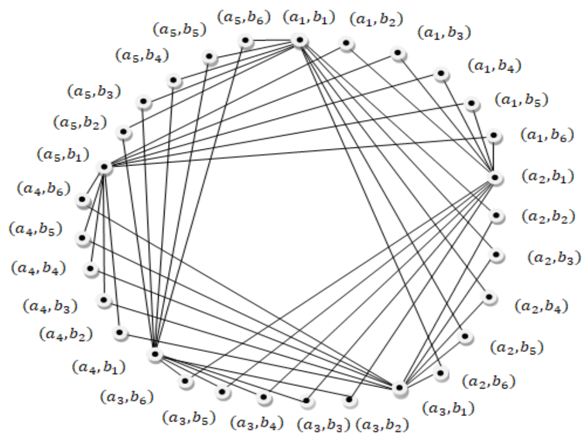

Figure 12: Kronecker Product of Wheel Graph W_6

Now, for finding the global domination number and global dominating set of Kronecker product $K(W_6)$ of wheel graph W_6 corresponding to its two dijoint spanning factors $F_1{}'$ and F_2'.

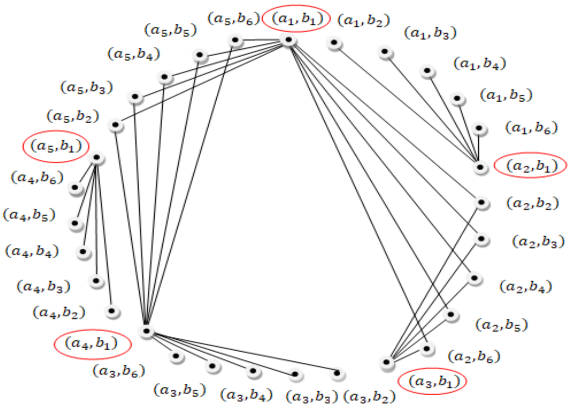

Figure 13: Factors $F_1{}'$ of Kronecker Product of Wheel Graph W_6

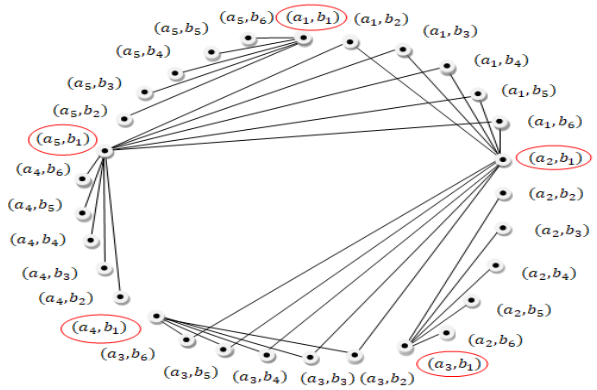

Figure 14: Factors $F_2{}'$ of Kronecker Product of Wheel Graph W_6

Here, Kronecker product $K(W_6)$ of wheel graph W_6 has order 30 $(i.e.\, n = 30)$ corresponding to its two disjoint factors $F_1{}'$ and F_2' $(i.e.\, k = 2)$. the maximum degree of any vertex in kronecker product $K(W_6)$ of wheel graph W_6 is 10 $(i.e.\, \Delta(K(W_6)) = 10)$. Therefore, by theorem 2.4, we get

$$\gamma_g(K(W_6)) \geq \frac{o(K(W_6)) \times k}{\Delta(K(W_6)) + k}$$

$$\gamma_g(K(W_6)) \geq \frac{30 \times 2}{10 + 2}$$

$$\gamma_g(K(W_6)) \geq 5$$

The members of the set $S = \{(a_1,b_1),(a_2,b_1),(a_3,b_1),(a_4,b_1),(a_5,b_1)\}$ are related with all the vertices of both factors of kronecker product $K(W_6)$ of wheel graph W_6. Hence, set S dominated both factors F_1' and F_2' of kronecker product $K(W_6)$. Therefore, set S is a global dominating set of Kronecker product $K(W_6)$ of wheel graph W_6. So, $\gamma_g\big(K(W_6)\big) \leq 5$. Hence, from both the results we conclude that global domination number of Kronecker product $K(W_6)$

https://doi.org/10.56452/2022-07-051

of wheel graph W_6 is equal to 5 and its global dominating set is $D_g(K(W_6)) = \{(a_1,b_1),(a_2,b_1),(a_3,b_1),(a_4,b_1),(a_5,b_1)\}$.

6. WHEEL GRAPH W_n:

Let W_n be a wheel graph having n vertices with two factors F_1 and F_2 such that factor F_1 contains outer cycle of W_n graph with $(n-1)$ vertices and factor F_2 contains inner part of W_n graph with n vertices-

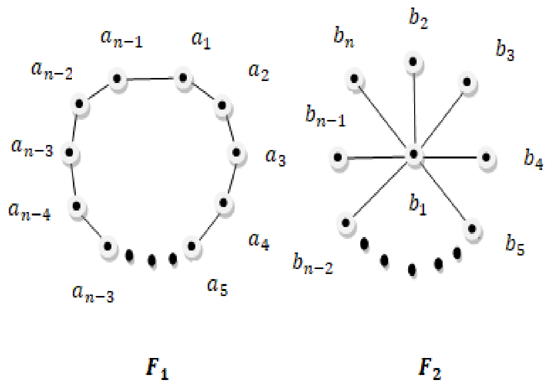

Figure 15: Factors F_1 and F_2 of Wheel Graph W_n

The Kronecker product of graph W_n with factor F_1 and F_2 is a graph $K(W_n)$ of order $n(n-1)$ and its vertex set is $V_{K(W_n)} = \{(a_1,b_1),(a_1,b_2),\dots(a_1,b_{n-1}),(a_1,b_n),(a_2,b_1),\dots(a_2,b_n)$ $\dots\dots,(a_{n-1},b_1),(a_{n-1},b_2),\dots(a_{n-1},b_{n-1}),(a_{n-1},b_n)\}$ and two vertices in Kronecker product $K(W_n)$ have an edge if 1st tuple in both vertices have an edge in factor F_1 and 2nd tuple in both vertices have an edge in factor F_2. Therefore, the graph of Kronecker product $K(W_n)$ is as follows-

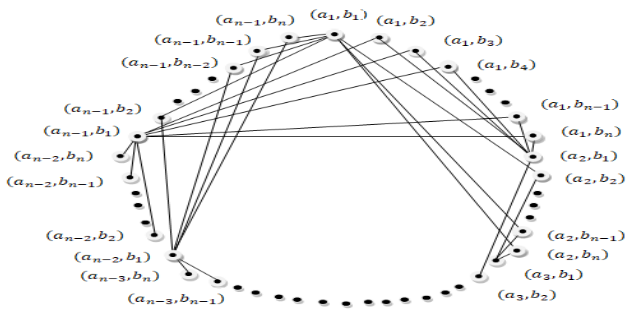

Figure 16: Kronecker Product $K(W_n)$ of Wheel Graph W_n

6.1 Theorem: The Global Domination Number of Kronecker product of wheel graph $W_n, n \geq 4$ is always $(n-1)$ and its minimum global dominating set is $D_g(K(W_n)) = \{(a_1,b_1),(a_2,b_1),\dots\dots,(a_{n-1},b_1)\}$.

Proof: The Kronecker product of graph $W_n, n \geq 4$ is a graph $K(W_n)$ of order $n(n-1)$ and its set of vertices is $V_{K(W_n)} = \{(a_1,b_1),(a_1,b_2),\dots\dots,(a_1,b_{n-1}),(a_1,b_n),(a_2,b_1),(a_2,b_2),\dots\dots(a_1,b_{n-1}),(a_2,b_n),\dots\dots\dots(a_n,b_1),(a_n,b_2),\dots\dots,(a_{n-1},b_{n-1}),(a_{n-1},b_n)\}$. The maximum degree of any vertex in Kronecker product is $2n-2$. that is, $\Delta(K(W_n)) = 2n-2$. Now, we will find global domination number and global dominating set of Kronecker product of graph $W_n, n \geq 4$ corresponding to its two factors. That is, $k = 2$. Therefore, by theorem 2.4, we get

$$\gamma_g(K(W_n)) \geq \frac{o(K(W_n)) \times k}{\Delta(K(W_n)) + k}$$

$$\gamma_g(K(W_n)) \geq \frac{n(n-1) \times 2}{2(n-1) + 2}$$

$$\gamma_g(K(W_n)) \geq (n-1)$$

The members of the set $S = \{(a_1,b_1),(a_2,b_1),\dots\dots,(a_{n-1},b_1)\}$ are related with all the vertices of Kronecker product $K(W_n)$ of wheel graph W_n and the complement of Kronecker product $K(W_n)$ of wheel graph W_n. hence, set S dominate both Kronecker product $K(W_n)$ and its complement graph. Therefore, set S is a global dominating set of Kronecker product $K(W_n)$ of wheel graph W_n. So, $\gamma_g(K(W_n)) \leq (n-1)$. Hence, from both the results we conclude that global domination number of Kronecker product $K(W_n)$ of wheel graph W_n is equal to (n-1) and its global dominating set is $D_g(K(W_n)) = \{(a_1,b_1),(a_2,b_1),\dots\dots,(a_{n-1},b_1)\}$.

7. PETERSEN GRAPH:

Petersen graph is an undirected graph with 10 vertices and 15 edges and it is denoted by $KG(5,2)$. First we will find Kronecker product of Petersen graph corresponding to its two factors F_1 and F_2.

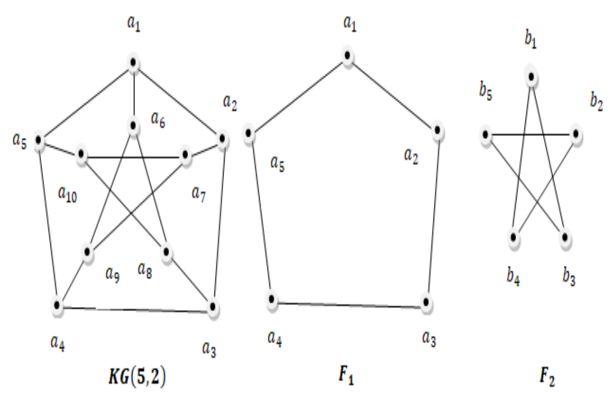

Figure 17: Petersen Graph and its Two Factors F_1 and F_2

Here both factors of Petersen graph contain 5 vertices. Hence, the Kronecker product of Petersen graph contain total 25 vertices such that the vertex set of Kronecker product is $V_{K(KG(5,2))} = \{(a_1,b_1),(a_1,b_2),(a_1,b_3),(a_1,b_4),(a_1,b_5),(a_2,b_1),(a_2,b_2),(a_2,b_3),(a_2,b_4),(a_2,b_5),\dots\dots,(a_5,b_1),(a_5,b_2),(a_5,b_3),(a_5,b_4),(a_5,b_5)\}$ and two vertices in Kronecker product have an edge if 1st tuple in both vertices have an edge in factor F_1 and 2nd tuple in both vertices have an edge in factor F_2. Therefore, the graph of Kronecker product $K(KG(5,2))$ is as follows-

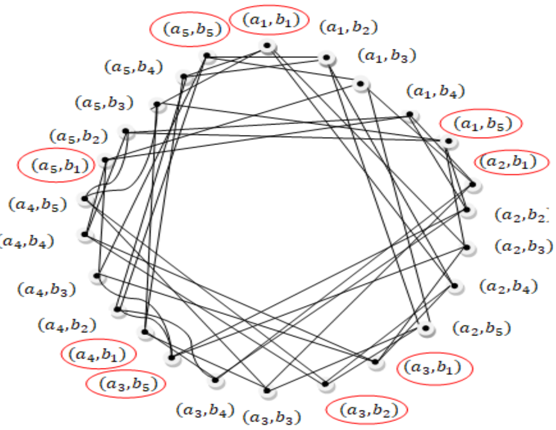

Figure 18: Kronecker Product of Petersen Graph

7.1 Theorem: The Global Domination Number of Kronecker product of Petersen graph is 9.

Proof: let the Kronecker product of Petersen graph $KG(5,2)$ with factor F_1 and F_2 is a graph $K(KG(5,2))$ of order 25 and its vertex set is

$$V_{K(KG(5,2))} = \{(a_1,b_1),(a_1,b_2),(a_1,b_3),(a_1,b_4),(a_1,b_5),(a_2,b_1),(a_2,b_2),(a_2,b_3),(a_2,b_4),(a_2,b_5),\dots\dots,(a_5,b_1),(a_5,b_2),(a_5,b_3),(a_5,b_4),(a_5,b_5)\}.$$

The maximum degree of any vertex in Kronecker product of Petersen graph is 4. That is, $\Delta(K(KG(5,2)))=4$. Now, we will find global domination number and global dominating set of Kronecker product of Petersen graph $KG(5,2)$ corresponding to its two factors. That is, $k=2$.

Therefore, by theorem 2.4, we get

$$\gamma_g(K(KG(5,2))) \geq \frac{o(K(KG(5,2)))\times k}{\Delta\left(K(KG(5,2))\right)+k}$$

$$\gamma_g(K(KG(5,2))) \geq \frac{25\times 2}{4+2}$$

$$\gamma_g(K(KG(5,2))) \geq 8.3$$

The members of the set $S = \{(a_1,b_1),(a_1,b_5),(a_2,b_1),(a_3,b_1),(a_3,b_2),(a_3,b_5),(a_4,b_1),(a_5,b_1),(a_5,b_5)\}$ are related with all the vertices of Kronecker product $K(KG(5,2))$ of Petersen graph $KG(5,2)$ and the complement of Kronecker product $K(KG(5,2))$ of Petersen graph $KG(5,2)$. hence, set S dominated both Kronecker product $K(KG(5,2))$ and its complement graph. Therefore, set S is a global dominating set of Kronecker product $K(KG(5,2))$ of Petersen graph $KG(5,2)$. So, $\gamma_g(K(KG(5,2))) \leq 9$ and we know that global domination number of any graph is always an positive integer. Hence, from both these results we conclude that global domination number of Kronecker product $K(KG(5,2))$ of Petersen graph $KG(5,2)$ is equal to 9 and its global dominating set is $D_g(K(KG(5,2))) = \{(a_1,b_1),(a_1,b_5),(a_2,b_1),(a_3,b_1),(a_3,b_2),(a_3,b_5),(a_4,b_1),(a_5,b_1),(a_5,b_5)\}$.

8. CONCLUSION:

In this paper we have found the global domination number with some interesting results of Kronecker product of wheel graphs and Petersen graph in generalized form. There are lots of applications of global domination number of Kronecker product of two graphs. One of the important application is that, if we want to find the minimum number of operators to run two system or two network at a time with more than one branch's then we will apply the application of global domination number of Kronecker product of two graphs as, first we find the Kronecker product of both networks then global domination number of that Kronecker product gives the minimum number of operators to run two system or two network at a time with more than one branch's.

ACKNOWLEDGMENT:

The author Bhupendra Singh is supported by Junior Research Fellowship (JRF), University Grand Commission, Government of India, New Delhi through the UGC-Ref. No.: 1133/(CSIR-UGC NET JUNE 2019).

REFERENCES

1. P. Weichsel, The Kronecker Product of Graphs, proceeding of the American Mathematical Society, 13(1962) 47-52.
2. E. Sampathkumar, The Global Domination Number of a Graph, J. Math. Phys. Sci. 23 (1989) 377-385.
3. R. C. Brigham and R. D. Dutton, Factor Domination in Graphs, Discrete Math. 86 (1990) 127-136.

4. S. Maheswari and S. Meenakshi, Split Domination Number of Some Special Graphs, International Journal of Pure and Applied Mathematics, 116(2017) 103-117.
5. Haynes, T.W., Hedetniemi, S.T. and Slater, P.J., Fundamentals of domination in Graphs, Marcel Dekkar, Inc-New York (1998).
6. J. Nieminen, Two bounds for the domination number of a graph, J. Inst. Math. Appl. 14 (1974) 183-187.
7. R. Brigham, J. Carrington, R.D. Dutton, Topics in Domination in Graphs, Springer (2020).
8. J. Bondy and U. Murty, Graph Theory with Applications, Elsevier Science Publication (1982).
9. R. D. Dutton and R. C. Brigham, On Global Domination Critical Graphs, Discrete Math. 309 (2009) 5894-5897.
10. R. I. Enciso and R. D. Dutton, Global Domination in Planar Graphs, J. Combin. Math. Combin. Comput. 66 (2008) 273-278.
11. Y. Caro and M. A. Henning, Simultaneous Domination in Graphs, Graphs Combin. 30 (2014) 1399-1416.
12. J. R. Carrington, Global Domination of Factors of a Graph. Ph.D. Dissertation, University of Central Florida (1992).