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Mathematical Model to Study the Effect of PRG4, Hyaluronic Acid and Lubricin on Squeeze Film Characteristics of Diseased Synovial Joint

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Abstract - Synovial fluid functions as a biological lubricant and provides lubrication to articular cartilages to reduce the friction and wear. The principle constituent present in the synovial fluid, which are responsible for its lubricant behavior, are lubricin, hyaluronic acid (HA). Lubricin is a type of proteoglycan more commonly known as PRG-4 and it is encoded by PRG4 gene and synthesized by the chondrocyte cells. In the present study lubrication and other related properties of synovial fluid are studied theoretically. In this article, we have studied the flow of synovial fluid in diseased synovial joint. Further, we have studied the effect of viscosity of synovial fluid, permeability of articular cartilage, thickness of articular cartilage and fluid film thickness on the characteristic of the squeeze film formed between the articular cartilages of the diseased human knee joint. The flow of synovial fluid modeled by considering it as a viscous, incompressible and Newtonian fluid. We have derived the modified Reynolds equation using the principle of hydrodynamic lubrication and continuum mechanics theory and solved it by applying the suitable boundary conditions according to the physical considerations. Subsequently we obtained the expression for pressure distribution in fluid film, load-bearing capacity, squeeze time, and have done the theoretical analysis on these properties for different parameters. Pressure increases with squeeze velocity and viscosity of synovial fluid and decreases with permeability and fluid film thickness. Human knee joint becomes diseased due to excessive pressure and the mobility of the knee joint decreases. The load capacity increases with viscosity and squeeze velocity and decreases with permeability resulting in the reduction of the load carrying capacity of knee joint in diseased condition. Moreover, in the diseased condition of joint the squeeze time also increased.

Index Terms - Synovial Fluid, Reynolds Equation, Hyaluronic Acid, Squeeze Films, Brinkman Equation, Arthritis.

1. Introduction

Synovial joints are the most significant characteristic of the human body since they are the epicenter of the most basic and vital function in humans, which is mobility. A synovial joint is a load-bearing system composed of two mating bones that move in tangential and/or normal directions. A soft sponge-like material known as articular cartilage covers the end of the two mating bones. The joint space filled with a shear-dependent fluid known as synovial fluid, which fills the space between these cartilaginous ends of the bones. The synovial fluid is a transparent yellowish dialysate of blood plasma with hyaluronic acid content. Hyaluronic acid (HA) is a straight, long chain polymer [1]. Synovial fluid provides lubrication to the synovial joint in order to reduce the wear and friction between the mating bones and articular cartilages during the motion and enhances the mechanical functioning of the synovial joint. The thin coating of synovial fluid that covers the surfaces of the joint capsule's inner layer and articular cartilage serves to maintain the joint surfaces lubricated and minimizes friction. Pores of the articular cartilages are very small, hyaluronic acid molecules do not typically pass through the pores of the articular cartilage [2]. Synovial fluid is a viscoelastic polymeric solution, Ogston et al demonstrated this fact and Gibbs et al later confirmed it, they studied the effect of external stimulation on the viscoelastic properties of synovial fluid [3,4]. Despite strong evidences of synovial fluid being viscoelastic there are number of publications, which tells about the viscous nature of synovial fluid. Synovial fluid behaves as viscous fluid as long as it is not under external stimulation.

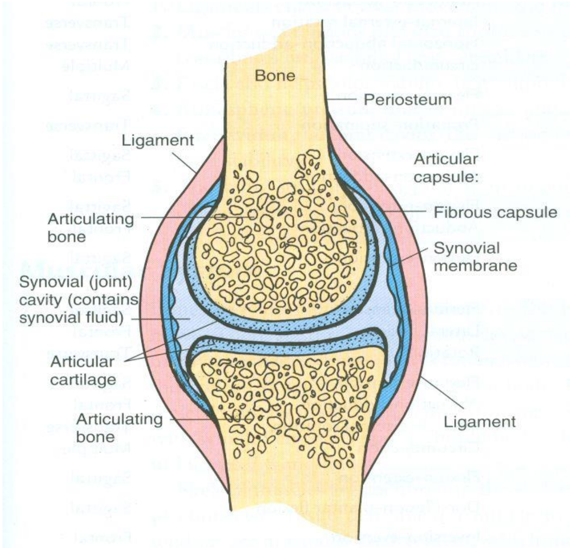

Figure 1: *Synovial Joint*

It is shown that the viscoelastic behavior of the synovial fluid depends on the concentration of the hyaluronic acid [5,6], The synovial fluid behaves as a non-Newtonian pseudo plastic fluid and its viscosity affected by the concentration of hyaluronic acid molecules present in it [7]. It is experimentally established that under certain physical considerations, synovial fluid is viscous like and incompressible and its viscosity depends on shear rate and concentration of hyaluronan [5]. The surface of the cartilage of a sick joint can become rough and fractured and the permeability of articular cartilage increases due to increased permeability these hyaluronic acid molecules enters into cartilage, and the synovial fluid can lose its non-Newtonian behavior. In diseased conditions synovial fluid lose its non-Newtonian behavior and it becomes Newtonian [7]. In the present problem we have consider the synovial fluid as a viscous, Newtonian and incompressible fluid. There are number of inflammatory and degenerative conditions that affects synovial joints. Bursitis and tendonitis, as well as other kinds of arthritis and Lyme disease, are examples of inflammatory joint disorders. There are more than hundred different type of degenerative and inflammatory joint disorders referred as arthritis. Chronic forms of arthritis include osteoarthritis, rheumatoid arthritis, and gouty arthritis. The most prevalent kind of arthritis is osteoarthritis, which develops when the cartilage of the human synovial joint wears away. Osteoarthritis is the most common chronic form of arthritis it is a chronic degenerative condition, often called wear-and-tear arthritis. The bone ends are exposed due to this and allowing them to grind together, because of this issue, synovial joint may have stiffness, discomfort, loss of mobility. Osteoarthritis is a widespread illness that affects the load-bearing joints of the body and it can be caused by a variety of factors, including trauma, heredity, obesity, and others.

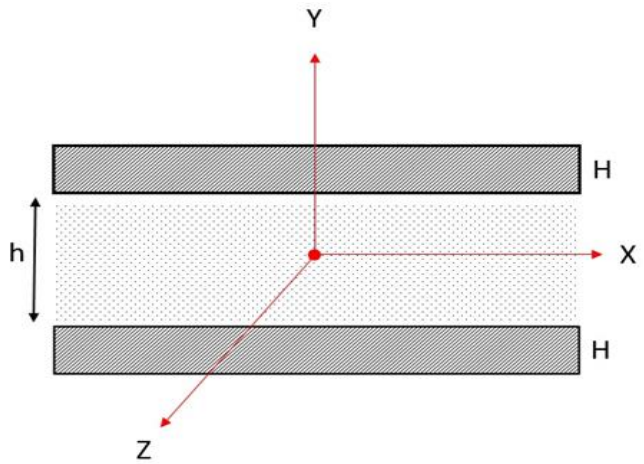

Figure 2: *Simple Geometry (Symmetric Case)*

On the other hand, RA (Rheumatoid Arthritis) is a chronic autoimmune disease and it starts with the swelling of synovial membrane [8]. Synovial fluid supposed to behave as a lubricant along with its two other essential functions, providing the nutrition to the cartilage and removing metabolite from it. Thus, synovial fluid behaves as a lubricant in synovial joints [9]. Further Radin discussed in detail about the lubricating properties of synovial fluid [10] and Dintenfass discussed the lubrication mechanism in synovial joints in different conditions and he arrived on a conclusion about the presence of hydrodynamic and squeeze film lubrication in synovial joints [11]. Scientists and mathematicians have conducted mathematical analyses of squeeze films on occasions. This squeeze film examination covers a study of the pressure distribution and load bearing capability of squeeze films, as well as the approaching times of the lubricating surfaces. Hays investigated the properties of squeeze films between parallel rectangular plates of finite and infinite length and compared the results for both the cases. He also further studied the squeeze films between parallel plates in which one is flat and other having curvature and the effect of curvature of the surface of plates on the formation of squeeze films [12]. Wu investigated the squeeze films formed between parallel rectangular plates, one of which had a porous face. He investigated the effect of porous facing of the plates on squeeze films and discovered that the porosity of the plate has a substantial effect on the squeeze film characteristics [13]. Shukla developed a new theory for lubrication of rough surfaces, derived a generalized form of Reynolds equation, and studied the effect of surface roughness on squeeze films. He shown that the load capacity increases when the roughness of the mating surfaces increases [14]. Naduvinamani et al presented a theoretical investigation of combined impact of lubricant ingredients and the roughness of surface on the characteristics of squeeze film between two rectangular plates of fixed dimensions by driving stochastic Reynolds equation and assuming the lubricant as a couple stress Stokes fluid [15].

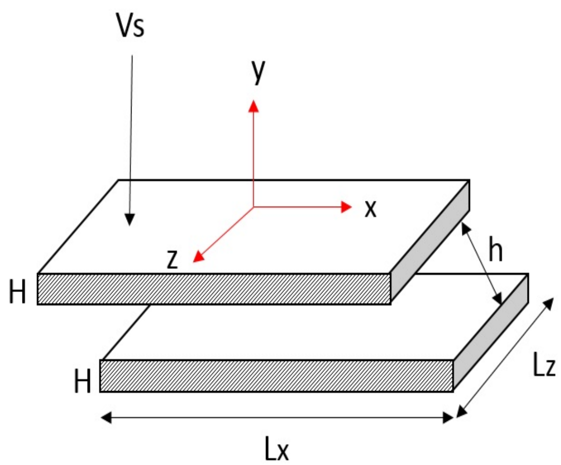

Figure 3: *Parallel Plate Geometry of Synovial Joint*

Deheri et al did the theoretical analysis of squeeze films between rough porous rectangular plates and computed the pressure variation and load capacity of these squeeze films [16]. Naduvinamani et al studied surface roughness and poroelasticity effect on squeeze films of micro polar fluid between rectangular plates and further referred this study to the synovial joint lubrication. The analysis of effect of surface roughness on squeeze film was done in this study and it was shown that the surface roughness has a considerable effect on the lubrication mechanism of synovial joints [17]. Kudennati et al numerically solved the MHD Remolds equation for squeeze film of couple stress fluid and studied the characteristics and effect of surface roughness on this squeeze film [18]. Recently Kumar et al theoretically analyzed the surface roughness effect on the squeeze film formed in spherical bearings based on the deterministic approach of hydrodynamic lubrication and further derived a generalized form of Reynolds equation, studied the significance of squeeze film between parallel plates, and examined the surface roughness effect on this film [19]. In the present problem we have studied, the squeeze film formed in synovial joint and examined the significance of the properties of this squeeze film on the diagnosis of the disease related to synovial joints.

2. Mathematical Model

In the present problem we have consider the flow of synovial fluid in diseased human knee joint under loading conditions. This study is based on the principles of lubrication theory and bio-fluid dynamics. Articular cartilage is considered as a porous material and length of the cartilage is much larger than the width to make the human knee joint geometry similar to the parallel plate model. The human knee joint is divided into two regions, first one is the fluid film and second is cartilage region and to compute the velocity in both the regions the symmetric case of human knee joint has been considered. The governing equations for the flow of synovial fluid in fluid film region are derived from the well-known Navier-Stokes equations for the flows of Newtonian fluids by imposing the restriction according to physical conditions. Darcy’s law for flow in porous medium governs the flow of fluid in porous medium. Synovial fluid flow in cartilage region and the equations governing the flow of synovial fluid in cartilage region are derived from the Brinkman’s equation, which is an extension of Darcy’s law [20]. Further, the modified Reynolds equation is derived for the pressure variation in fluid film and cartilage regions and load capacity and squeeze time is calculated for both the regions. We have considered the synovial fluid as incompressible, non-conducting and non-magnetic, and the flow of synovial fluid is laminar and steady and hence the variation of any property of synovial fluid with time is zero and this flow is due to pressure gradient. In addition, we have considered the variation of pressure along fluid film thickness as zero. Further, we have assumed that there are no external forces are present, while gravitational forces are always present but these forces are small enough so that their effect can be considered as negligible and no slip boundary conditions are assumed at the cartilage surfaces. By applying all these assumptions, the Brinkman’s equation and Navier-Stokes equation reduces to the following equations of synovial fluid, which governs the flow of synovial fluid.

2.1 Governing Equations

Region I: Fluid Film Region $0 \leq y \leq h$

$$\mu \frac{\partial^2 u_1}{\partial y^2} - \frac{\partial p}{\partial x} = 0$$

$$\mu \frac{\partial^2 v_1}{\partial y^2} - \frac{\partial p}{\partial z} = 0 \qquad (1)$$

$$\frac{\partial p}{\partial y} = 0$$

Region II: Articular Cartilage Region $h \leq y \leq h + H$

$$\mu' \frac{\partial^2 u_2}{\partial y^2} - \frac{\mu}{\varphi_1} u_2 - \frac{\partial p}{\partial x} = 0$$

$$\mu' \frac{\partial^2 v_2}{\partial y^2} - \frac{\mu}{\varphi_2} v_2 - \frac{\partial p}{\partial z} = 0 \qquad (2)$$

$$\frac{\partial p}{\partial y} = 0$$

Where u_1 , v_1 are the velocity of synovial fluid in fluid film region in x and z direction and u_1 , v_1 are the velocities of the synovial fluid in articular cartilage in x and z direction respectively.

2.2. Boundary and Matching Conditions

The boundary condition for synovial fluid flowing in fluid film region are given by

$$\frac{\partial u_1}{\partial y} = 0 \qquad at \qquad y = 0$$

$$\frac{\partial v_1}{\partial y} = 0 \qquad at \qquad y = 0 \qquad (3)$$

Boundary conditions for synovial fluid flowing in cartilage are given as follows

$$u_2 = 0 \qquad at \qquad y = h + H$$

$$v_2 = 0 \qquad at \qquad y = h + H \qquad (4)$$

By considering our assumption of no slip boundary condition the velocity of the synovial fluid in fluid film region and articular cartilage region at interface, where cartilage region and fluid film region meets with each other must be the same. Therefore, the matching condition for the velocities at interface can be written as follows,

$$u_1 = u_2 = V_x \ \& \ v_1 = v_2 = V_z \quad at \quad y = h \qquad (5)$$

Where U_x and V_z are the interface velocity in x and z direction respectively.

Shear stresses at the interface is also same, therefore we have the following matching condition for shear stresses,

$$\mu \frac{\partial^2 u_1}{\partial y^2} = \mu' \frac{\partial^2 v_1}{\partial y^2} \quad at \quad y = h$$

$$\mu \frac{\partial^2 u_2}{\partial y^2} = \mu' \frac{\partial^2 v_2}{\partial y^2} \quad at \quad y = h \tag{6}$$

2.3 Solution of the Problem

Now solving the equations governing the flow of synovial fluid in fluid film and articular cartilage given in equation (1) and (2) with the help of boundary conditions and matching conditions given in equation (3)-(5). By solving these equations, we find the velocity of synovial fluid in fluid film region and articular cartilage.

The velocity of synovial fluid in fluid film region in both the directions are given by,

$$u_1 = \frac{1}{\mu}\frac{\partial p}{\partial x}(y^2 - h^2) + V_x$$

$$v_1 = \frac{1}{\mu}\frac{\partial p}{\partial z}(y^2 - h^2) + V_z$$

The velocity of synovial fluid in articular cartilage in both the directions are given by,

$$u_2 = \frac{\varphi_1}{\mu}\frac{\partial p}{\partial x}\frac{1}{sinh\; sinh\;(M_x H)}[sinhM_x(h+H-y) - sinhM_x(h-y) - sinh(M_x H)] + \left[\frac{V_x sinhM_x(h+H-y)}{sinh(M_x H)}\right]$$

$$v_2 = \frac{\varphi_2}{\mu}\frac{\partial p}{\partial z}\frac{1}{sinh(M_z H)}[sinhM_z(h+H-y) - sinhM_z(h-y) - sinh(M_z H)] + \left[\frac{V_z sinhM_z(h+H-y)}{sinh(M_z H)}\right]$$

The velocity at the interface which is the region at which fluid film and articular cartilage meets with each other can be find by using the matching condition for shear stress given in equation (6).

The velocity of the synovial fluid at interface are given by,

$$V_x = \frac{\varphi_1}{\mu}\frac{\partial P}{\partial x} tanh(M_x H)\left(tanh\frac{M_x H}{2} - M_x h\right)$$

$$V_z = \frac{\varphi_2}{\mu}\frac{\partial P}{\partial z} tanh(M_z H)\left(tanh\frac{M_z H}{2} - M_z h\right)$$

Volumetric flow rate of a fluid is the amount of volume of the fluid passing through a surface per unit time and this volumetric flow rate is calculated by integrating the velocity. Therefore, synovial fluid flux in both the directions are calculated by integration of velocity with respect to y starting from initial value as zero to a final value $(h + H)$. Synovial fluid flux Q_x and Q_z are calculated as follows

Synovial fluid flux in x direction is given by

$$Q_x = \int_0^h u_1\, dy + \int_h^{h+H} u_2\, dy$$

$$Q_x = \int_0^h \left(\frac{1}{\mu}\frac{\partial p}{\partial x}(y^2 - h^2) + V_x\right) dy + \int_h^{h+H} \left(\frac{V_x sinh M_x(h+H-y)}{sinh(M_x H)}\right) dy$$
$$+ \int_h^{h+H} \left(\frac{\varphi_1}{\mu}\frac{\partial p}{\partial x}\frac{1}{sinh M_x H}[sinh M_x(h+H-y) - sinh M_x(h-y) - sinh(M_x H)]\right) dy$$

$$Q_x = -\frac{\partial p}{\partial x}\left[\frac{1}{A^3\mu'}\left(tanh M_x H\left(M_x h - tanh\frac{M_x H}{2}\right)^2 + M_x H + 2tanh\frac{M_x H}{2}\right) + \frac{h^3}{3\mu}\right]$$
$$Q_x = -\frac{\partial p}{\partial x}(F_x) \qquad (7)$$

The synovial fluid flux in z direction is given by,

$$Q_z = \int_0^h v_1\, dy + \int_h^{h+H} v_2\, dy$$

$$Q_z = \int_0^h \left(\frac{1}{\mu}\frac{\partial p}{\partial z}(y^2 - h^2) + V_z\right) dy + \int_h^{h+H} \left(\frac{V_z sinh M_z(h+H-y)}{sinh(M_z H)}\right) dy$$
$$+ \int_h^{h+H} \left(\frac{\varphi_2}{\mu}\frac{\partial p}{\partial z}\frac{1}{sinh M_z H}[sinh M_z(h+H-y) - sinh M_z(h-y) - sinh(M_z H)]\right) dy$$

$$Q_z = -\frac{\partial p}{\partial z}\left[\frac{1}{{M_z}^3\mu'}\left(tanh M_z H\left(M_z h - tanh\frac{M_z H}{2}\right)^2 + M_z H + 2tanh\frac{M_z H}{2}\right) + \frac{h^3}{3\mu}\right]$$
$$Q_z = -\frac{\partial p}{\partial z}(F_z) \qquad (8)$$

Where F_x and F_z are given by,

$$F_x = \left[\frac{1}{{M_x}^3\mu'}\left(tanh M_x H\left(M_x h - tanh\frac{M_x H}{2}\right)^2 + M_x H + 2tanh\frac{M_x H}{2}\right) + \frac{h^3}{3\mu}\right]$$

$$F_z = \left[\frac{1}{{M_z}^3\mu'}\left(tanh M_z H\left(M_z h - tanh\frac{M_z H}{2}\right)^2 + M_z H + 2tanh\frac{M_z H}{2}\right) + \frac{h^3}{3\mu}\right]$$

The continuity equation for hydrodynamic zone or fluid film region is

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial z} + \frac{\partial w_1}{\partial y} = 0 \qquad (9)$$

Where, w_1 is the velocity of synovial fluid in fluid film along y direction.

The above continuity equation in integrated form can be written as follows,

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_z}{\partial z} + \frac{\partial w_1}{\partial y} = 0 \qquad (10)$$

Integrating the above equation across the fluid film and using the boundary condition $w_1 = 0$ at $y = 0$ and $w_1 = V_s$ at y = h.

From equation (7) and equation (8) we have

$$Q_x = -\frac{\partial p}{\partial x}(F_x) \quad and \quad Q_z = -\frac{\partial p}{\partial z}(F_z) \qquad (SEQ \backslash * ME$$

Now putting the values of Q_x and Q_z in equation (10) which are given in equation (11) we obtained the following expression

$$\frac{\partial}{\partial x}\left(-F_x\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(-F_z\frac{\partial p}{\partial z}\right) = V_s \qquad (12)$$

This equation is the Reynolds equation for squeeze film between parallel plates to obtain the pressure and other important characteristics of the fluid film. We have considered the flow of synovial fluid in such a way that there is no change in pressure in z direction and hence the rate of change of pressure in this direction will be zero. Therefore, we have $\frac{\partial p}{\partial z} = 0$ and the equation (12) reduces to the following equation

$$\frac{\partial}{\partial x}\left(-F_x\frac{\partial p}{\partial x}\right) = -V_s \qquad (13)$$

The boundary condition for the pressure in x direction are given by

$$p = o \qquad at \qquad x = o$$
$$\frac{\partial p}{\partial x} = o \qquad at \qquad x = L_x \qquad (14)$$

Now we solve equation (13) using simple integration approach and by applying the boundary conditions given in equation (14) we get the expression for the pressure in fluid film and articular cartilage, and the expression for pressure is as follows

$$p(x) = \frac{V_s}{2F_x}(2L_x x - x^2) \qquad (15)$$

Load capacity of the squeeze film is obtained by integrating pressure along the length of the porous medium. Therefore, we integrate the expression of pressure with respect to x from across the length of the articular cartilage to obtain the expression for load capacity in squeeze film formed in between the articular cartilages, Therefore we have

$$W_x = \int_0^{L_x} p(x)\, dx \qquad (16)$$

By substituting the value of $p(x)$ from equation (15) into equation (16), we get the expression for load capacity in squeeze film in between the both articular cartilages, the expression for load capacity given by

$$W_x = \int_0^{L_x} \frac{V_s}{2F_x}(2L_x x - x^2)\, dx$$

$$W_x = \frac{V_s}{3F_x} {L_x}^3 \qquad (17)$$

Squeeze velocity is the rate of change of fluid film thickness with respect to time and given by $V_s = -\left(\frac{dh}{dt}\right)$. By using the expression for squeeze velocity in (17) and integrating from initial film thickness h_1 to the final film thickness h_2 we get the expression for squeeze time

$$T_x = \int_{h_2}^{h_1} \frac{{L_x}^3}{3F_x W_x} dh \qquad (18)$$

To obtain the dimensionless expression for pressure, load capacity and squeeze time we shall use the following dimensionless variables,

$$\underline{x} = \frac{x}{L_x} \qquad \underline{h} = \frac{h}{L_x} \qquad \underline{H} = \frac{H}{L_x}$$

$$\underline{\varphi_x} = \frac{\varphi_x}{{L_x}^2} \qquad \underline{\mu'} = \frac{\mu'}{\mu} \qquad \underline{V_s} = \frac{V_s}{\mu L_x} \qquad (19)$$

$$\underline{F_x} = \frac{F_x}{\left(\frac{{L_x}^3}{\mu}\right)} \quad \underline{W_x} = \frac{W_x}{\left(\frac{{L_x}^3}{\mu}\right)} \quad \underline{T_x} = \frac{T_x}{\left(\frac{\mu L_x}{W_x}\right)}$$

We have used the above dimensionless scheme given in equation (19) to non-dimensionalize the expression of pressure, load capacity and squeeze time given in equation (16)-(18) and obtained the expression for dimensionless pressure, dimensionless load capacity and dimensionless squeeze time, which are as follows

$$\underline{P(x)} = \frac{\underline{V_s}\,\mu^2}{2\underline{F_x}}\left(2\underline{x} - \underline{x}^2\right)$$

$$\underline{W_x} = \frac{\underline{V_s}\,\mu^2}{3\underline{F_x}} L_x$$

$$\underline{T_x} = \int_{h_2}^{h_1} \frac{1}{3\,\underline{F_x}}\, d\underline{h}$$

Where $\underline{F_x}$ and $\underline{M_x}$ are given by

$$\underline{F_x} = \frac{{L_x}^3}{\left(\left(\underline{M_x}\right)^3 \underline{\mu'}\,\mu\right)}\left[tanh\underline{x}\,\underline{H}\left(\underline{M_x}\,\underline{h} - tanh\frac{\underline{M_x}\,\underline{H}}{2}\right)^2 + \underline{M_x}\,\underline{H} + 2tanh\frac{\underline{M_x}\,\underline{H}}{2}\right] + \frac{\underline{h}^3}{3\mu}$$

$$\underline{M_x} = \frac{1}{L_x}\left(\frac{1}{\underline{\mu'}\;\underline{\varphi_1}}\right)^{\frac{1}{2}}$$

3. *Results and Discussion*

In the present problem, we have studied the characteristics of squeeze film formed in between of the articular cartilages of the diseased human knee joints. In order to study the characteristics of the squeeze film, we have derived the well-known Reynolds equation for squeeze films and subsequently obtained the expression for pressure, load carrying capacity and squeeze time. Further, we used the dimensionless variables to get the dimensionless expressions for pressure, load capacity and squeeze time where the squeeze time is the time taken in reducing the film thickness from an initial value to a particular value. In our case we have consider the squeeze time as the time to reduce the film thickness to zero so that the articular cartilages rub each other. We have studied the effect of various parameters on pressure distribution in squeeze film, load capacity and squeeze time for axial directions. The effect of permeability, viscosity, squeeze velocity and fluid film thickness on pressure variation are considered, meanwhile the effect of permeability, squeeze velocity and viscosity on the load capacity of squeeze film is studied. On the other hand, the only parameter, which affects the squeeze time, is viscosity and we have studied this effect in our problem. We have plotted the variation of dimensionless pressure with dimensionless axial distance for different values of fluid film thickness, viscosity of synovial fluid, permeability of articular cartilage and squeeze velocity. Further, we have plotted the variation of dimensionless load capacity with dimensionless fluid film thickness for different values of permeability of articular cartilage and viscosity of synovial fluid. Lastly, we plotted the variation of dimensionless squeeze time with dimensionless articular cartilage length and thickness for different values of viscosity of synovial fluid. Figure 4-7 shows the variation of dimensionless pressure $\underline{P}(x)$ with dimensionless axial distance $\underline{x}$ for these parameters, whereas Figure 8-10 shows the variation of dimensionless load capacity $\underline{W_x}$ with dimensionless fluid film thickness $\underline{h}$ and figure 11-12 shows the variation of dimensionless squeeze time $\underline{T_x}$ with dimensionless cartilage thickness $\underline{H}$. The value of dimensionless pressure $\underline{P}(x)$ increases with axial distance $\underline{x}$ for all the parameters. The pressure $\underline{P}(x)$ in fluid film decreases when the value of dimensionless permeability $\underline{\varphi_1}$ and dimensionless fluid film thickness $\underline{h}$ increases and it increases when the value of dimensionless squeeze velocity $\underline{V_s}$ and the viscosity μ of synovial fluid increases. These results are similar with the results given in [12,21]. The value of dimensionless load capacity $\underline{W_x}$ decreases with an increase in dimensionless fluid film thickness $\underline{h}$ generally, whereas the value of dimensionless load capacity $\underline{W_x}$ increases as the value of viscosity μ and dimensionless squeeze velocity $\underline{V_s}$ increases and this dimensionless load capacity $\underline{W_x}$ increases when the permeability $\underline{\varphi_1}$ of articular cartilage decreases. Our results for load capacity are similar with [12,21–23]. Figure 11 and 12 shows the variation of dimensionless squeeze time $\underline{T_x}$ with dimensionless cartilage length $\underline{L_x}$ and cartilage thickness $\underline{H}$ for different values of viscosity of synovial fluid. Squeeze time $\underline{T_x}$ decreases with cartilage length $\underline{L_x}$ as well as cartilage thickness $\underline{H}$. The value of squeeze time $\underline{T_x}$ decreases as the value of viscosity of synovial fluid increases. The results which we obtained for squeeze time are similar with the results obtained in [18,22,24].

4. Conclusion

In this paper, we have derived the modified Reynolds equation for the flow of synovial fluid in diseased human knee joint. A squeeze film formed between the articular cartilages of the diseased human knee joint and we have considered the flow of synovial fluid in this squeeze film and articular cartilage. We have modelled the articular cartilages as parallel plates to derive this equation. Then further, we have studied the important characteristics of the formed squeeze film the effect of different parameters on the properties of this squeeze film. The effect of viscosity of synovial fluid, permeability of articular cartilage and the thickness of fluid film on the characteristics of squeeze film are studied. These parameters affects the properties of fluid film significantly. We studied the effect of squeeze velocity, fluid film thickness, permeability of articular cartilage and the viscosity of synovial fluid on pressure. We also studied the effect of squeeze velocity, permeability of articular cartilage and viscosity of synovial fluid on the load capacity. It is been observed that the pressure in the squeeze film increases with an increase in the viscosity of the synovial fluid and squeeze velocity of the articular cartilages. On the other hand the pressure decreases when the permeability of the articular cartilages and the thickness of the fluid film in between of the articular cartilages increase. The load carrying capacity (the ability to sustain/bear load) increases with viscosity of synovial fluid and decreases with the increases of permeability of articular cartilages. The variation of dimensionless load carrying capacity with dimensionless fluid film thickness for different values of viscosity, dimensionless permeability and dimensionless squeeze velocity are depicted in the figures. It has been observed by the profile shown in the figures that the load capacity decreases as the dimensionless fluid film thickness increases. Also by observing the same figures it can be seen that the load capacity increases with an increase in viscosity and dimensionless squeeze velocity, however the load capacity decreases when the value of dimensionless permeability increases. It is also observed that the squeeze time does depend on the viscosity of synovial fluid and the thickness of articular cartilages of the knee joint. Squeeze time decreases when the value of the thickness of the articular cartilage increases, while it decreases as the length of the articular cartilage increases. On the bases of the results obtained in this paper, we might get a good insight into the condition of the diseased human knee joint. The diseased human knee joint goes under various types of biochemical and biomechanical changes and due to these conditions the permeability of articular cartilages increases, while the viscosity of synovial fluid decreases. The pressure in knee join is more during exercise and diseased conditions in comparison to the resting and normal conditions. Due to this increased pressure, hypoxic reperfusion free radicals generate in knee joint resulting in the damage of articular cartilage tissue and therefore the increased pressure is a marker of chronic joint disease. We already observed that pressure increases when squeeze velocity and viscosity increases, which clearly shows that during movement and disease pressure increases, on the other hand pressure decreases when film thickness and permeability increases which also, verify the diseased and mobility condition. Therefore we can conclude that the mobility of the human knee joint decrease in the diseased condition due to an increase in pressure. It can also be conclude that the ability of the human knee joint to bear loads decreases in diseased condition and increases during movement, as the permeability is more and viscosity is less in disease conditions while the squeeze velocity is more during movement. Squeeze time increases in diseased joints as it decreases with the thickness of articular cartilage and the viscosity of synovial fluid, and these values decrease in the diseased condition of the human knee joint.

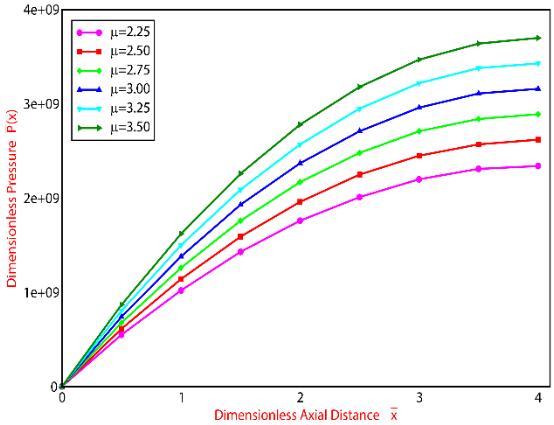

Fig. 4: Variation of Dimensionless Pressure with Dimensionless Axial Distance for Different Viscosity

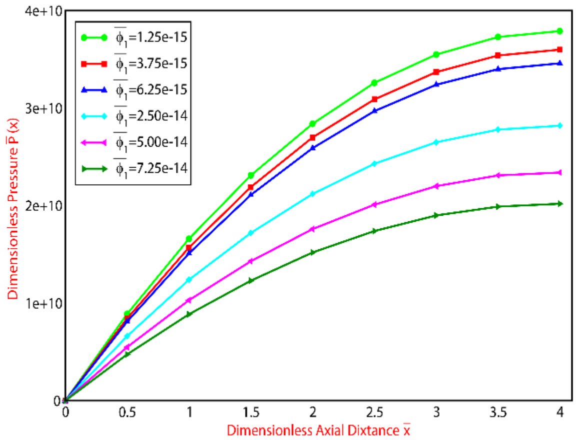

Fig. 5: Variation of Dimensionless Pressure with Dimensionless Axial Distance for Different Permeability

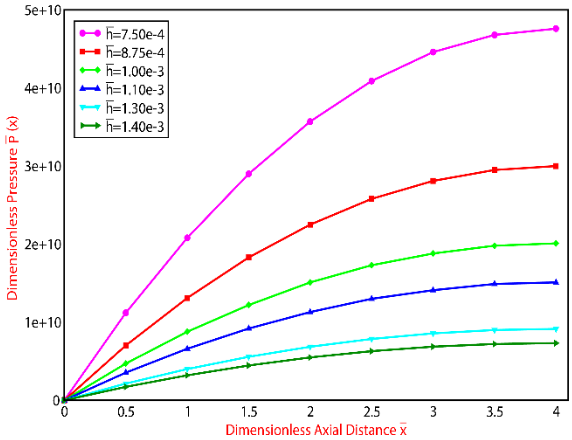

Fig. 6: Variation of Dimensionless Pressure with Dimensionless Axial Distance for Different Film Thickness

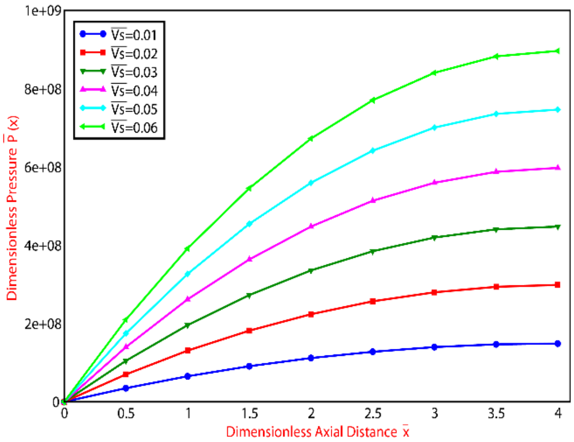

Fig. 7: Variation of Dimensionless Pressure with Dimensionless Axial Distance for Different Squeeze Velocity

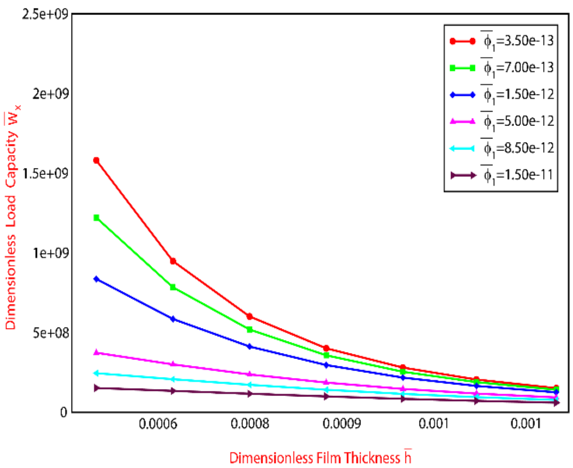

Fig. 8: Variation of Dimensionless Load Capacity with Dimensionless Film Thickness for Different Permeability

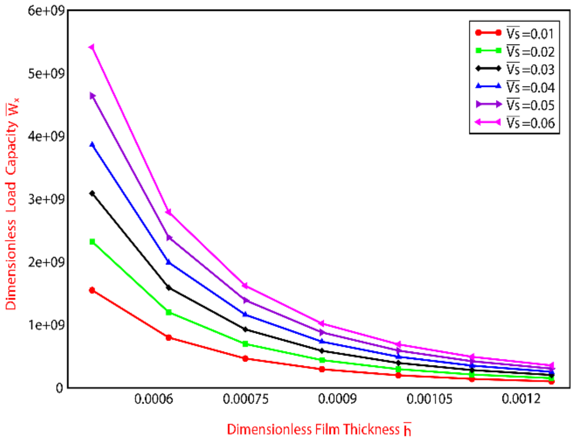

Fig. 9: Variation of Dimensionless Load Capacity with Dimensionless Film Thickness for Different Squeeze Velocity

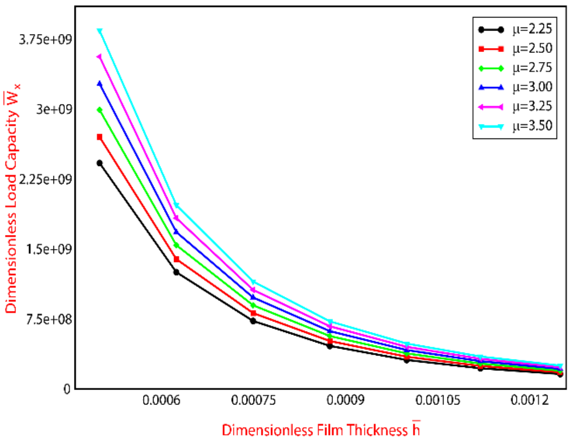

Fig. 10: Variation of Dimensionless Load Capacity with Dimensionless Film Thickness for Different Viscosity

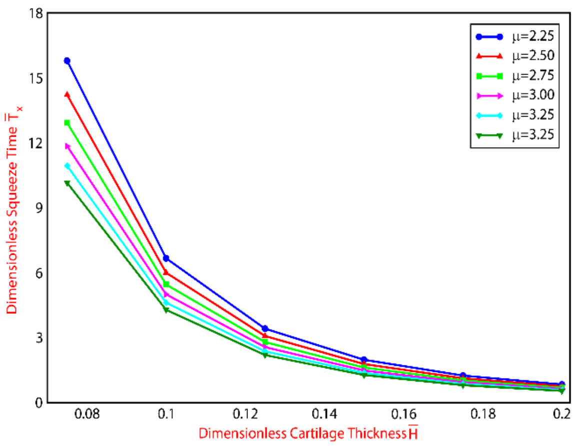

Fig. 11: Variation of Dimensionless Squeeze Time with Dimensionless Cartilage Length for Different Viscosity

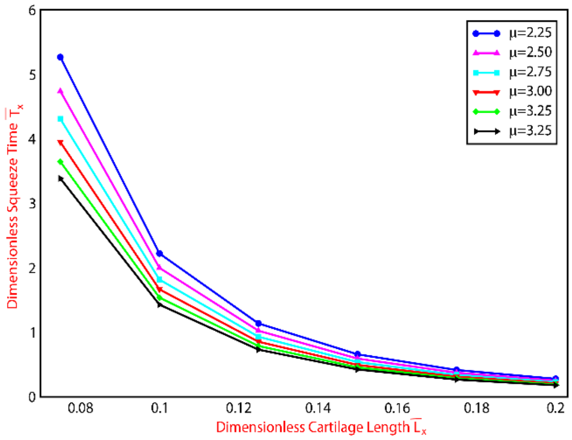

Fig. 12: Variation of Dimensionless Squeeze Time with Dimensionless Cartilage Thickness for Different Viscosity

Nomenclature

$u_1,\ v_1$ → Velocity of Synovial Fluid in Fluid Film

$u_2,\ v_2$ → Velocity of Synovial Fluid in Articular Cartilage

$V_x,\ V_z$ → Velocity of Synovial Fluid at Interface

$\varphi_1,\ \varphi_2$ → Permeability of Articular Cartilage

$Q_x,\ Q_z$ → Synovial Fluid Flux

$x,\ y,\ z$ → Cartesian Co-Ordinate

$M_x = \sqrt{\left(\frac{\mu}{\mu'\varphi_1}\right)},\ and\ M_z = \sqrt{\left(\frac{\mu}{\mu'\varphi_2}\right)}$

$\underline{p}(x)$ → Normal and Dimensionless Pressure

$\underline{W_x}$ → Normal and Dimensionless Load Capacity

$\underline{T_x}$ → Dimensionless Squeeze Time

h → Fluid Film Thickness

H → Articular Cartilage Thickness

w → Velocity of Synovial Fluid along Fluid Film Thickness

V_s → Squeeze Velocity

L_x → Length of Articular Cartilage

References:

[1] J. Damiano, T. Bardin, Synovial fluid, EMC-Rhumatologie-Orthopedie. 1 (2004) 2–16. https://doi.org/10.1016/j.emcrho.2003.11.002.

[2] Mathematically Model for Synovial Joints_Ethesis.Pdf, (n.d.).

[3] B.Y.A.G. Ogston, J.E. Stanier, THE PHYSIOLOGICAL FUNCTION OF HYALURONIC ACID IN SYNOVIAL FLUID ; VISCOUS , ELASTIC AND LUBRICANT PROPERTIES From the Department of Biochemistry , University of Oxford & Bauer (1947). Except for the fact that it contains hyaluronic acid , synovial fluid, 4 (1952) 244–252.

[4] D.A. Gibbs, E.W. Merrill, K.A. Smith, E.A. Balazs, Rheology of hyaluronic acid, Biopolymers. 6 (1968) 777–791. https://doi.org/10.1002/bip.1968.360060603.

[5] A.G. Ogston, J.E. Stanier, The physiological function of hyaluronic acid in synovial fluid; viscous, elastic and lubricant properties, J. Physiol. 119 (1953) 244–252. https://doi.org/10.1113/jphysiol.1953.sp004842.

[6] G.B. Thurston, H. Greiling, Viscoelastic properties of pathological synovial fluids for a wide range of oscillatory shear rates and frequencies, Rheol. Acta. 17 (1978) 433–445. https://doi.org/10.1007/BF01525959.

[7] P. Pustěovská, J. Hron, J. Málek, K.R. Rajagopal, On the modeling of the synovial fluid, Adv. Tribol. 2010 (2010). https://doi.org/10.1155/2010/104957.

[8] B.M. Bazrov, Classification of joints, Russ. Eng. Res. 30 (2010) 399–403. https://doi.org/10.3103/S1068798X10040192.

[9] D. V Davies, Properties of Synovial Fluid, Proc Instn Mech Engrs. 8 (1966) 25–29. https://journals.sagepub.com/doi/pdf/10.1243/PIME_CONF_1966_181_203_02.

[10] J. Coffman, Synovial fluid., Vet. Med. Small Anim. Clin. 75 (1980) 1403–1406. https://doi.org/10.5005/jp/books/12869_13.

[11] L. Dintenfass, Lubrication in synovial joints, Nature. 197 (1963) 496–497. https://doi.org/10.1038/197496b0.

[12] D.F. Hays, Squeeze films for rectangular plates, J. Fluids Eng. Trans. ASME. 85 (1963) 243–246. https://doi.org/10.1115/1.3656568.

[13] H. Wu, An analysis of the squeeze film between porous rectangular plates, J. Tribol. 94 (1972) 64–68. https://doi.org/10.1115/1.3451637.

[14] J.B. Shukla, A new theory of lubrication for rough surfaces, Wear. 49 (1978) 33–42. https://doi.org/10.1016/0043-1648(78)90021-2.

[15] N.B. Naduvinamani, S.T. Fathima, P.S. Hiremath, On the squeeze effect of lubricants with additives between rough porous rectangular plates, ZAMM Zeitschrift Fur Angew. Math. Und Mech. 84 (2004) 825–834. https://doi.org/10.1002/zamm.200310138.

[16] G. Deheri, H. Patel, R. Patel, Load carrying capacity and time height relation for squeeze film between rough porous rectangular plates, Ann. Fac. Eng. Hunedoara. 9 (2011) 33.

[17] N.B. Naduvinamani, G.K. Savitramma, Squeeze Film Lubrication between Rough Poroelastic Rectangular Plates with Micropolar Fluid: A Special Reference to the Study of Synovial Joint Lubrication, ISRN Tribol. 2013 (2013) 1–9. https://doi.org/10.5402/2013/431508.

[18] R.B. Kudenatti, N. Murulidhara, H.P. Patil, Numerical study of squeeze film lubrication between porous and rough rectangular plates, J. Porous Media. 16 (2013) 183–192. https://doi.org/10.1615/JPorMedia.v16.i3.10.

[19] J. Vijaya Kumar, R. Raghavendra Rao, Effects of Surface Roughness in Squeeze Film Lubrication of Spherical Bearings, Procedia Eng. 127 (2015) 955–962. https://doi.org/10.1016/j.proeng.2015.11.442.

[20] H. Verschoor, Experimental data on the viscous force exerted by a flowing fluid on a dense swarm of particles, Appl. Sci. Res. 2 (1951) 155–161. https://doi.org/10.1007/BF00411979.

[21] S.P. Singh, G.C. Chadda, A.K. Sinha, A Model for Micropolar Fluid Film Mechanism with Reference to Human Joints, Indian J. Pure Appl. Math. 19 (1988) 384–394.

[22] P. Sinha, C. Singh, K.R. Prasad, Lubrication of human joints - a microcontinuum approach, Wear. 80 (1982) 159–181. https://doi.org/10.1016/0043-1648(82)90215-0.

[23] P. Sinha, C. Singh, K.R. Prasad, Viscosity variation considering cavitation in a journal bearing lubricant containing additives, Wear. 86 (1983) 43–56. https://doi.org/10.1016/0043-1648(83)90087-X.

[24] U. Srinivasan, Load capacity and time-height relations for squeeze films between double-layered porous plates, Wear. 43 (1977) 211–225. https://doi.org/10.1016/0043-1648(77)90115-6.