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A MATHEMATICAL STUDY FOR HEAT TRANSFER PHENOMENOLOGICAL PROCESSES IN HUMAN SKIN

Vivek Kumar, Sapna Ratan Shah

School of Computational & Integrative Sciences,

Jawaharlal Nehru University, New Delhi-110067(India)

Abstract:

In this present work a mathematical model has been developed to study heat flow characteristic for human skin. Through this study temperature gradient, conduction, convection, and evaporation has been shown in the skin. The model is based on a bioheat equation for thermal energy balance. A study of variation of tissue temperature and vein temperature with distance from the core is done by varying physical parameters such as coefficient of metabolism, metabolic rate, and perfusion rate. Although the results provide insight into the problem and even compare acceptably with some of the scant previous investigations. This study is mathematically useful to predict core temperature rise over time and can estimate water loss from sweat. And also helpful to identitfy the profile of temperature rise in tissues during hyperthermia medication.

Keywords: Heat flow, Skin, Isothermal core, Bioheat, Perfusion, Metabolic rate, Blood vessels, Tissue, Hyperthermia**.**

INTRODUCTION:

Biomedical sciences and mathematics are the key course of study that a person can't perform any study without these two branches. There is an important relationship between these two areas in almost all zones of human life. For instance, scientist and medical practitioner evaluate the amount of medicine by implementing equations of medical. Because of mathematics the medical practitioner are able to determine the patient's percentage of hydration and the quantity of water which patient required to maintain the amount of water for the human body. In the current decade, many researchers and scientists have done a lot of research work in the field of biomathematics where they have concentrated on using distinct hyperthermia perspective to put in different bioheat studies in cancer treatment. Now a days hyperthermia is a extremely interesting subject for study in biomedical engineering and the cancer can be cured by this therapy. A number of research papers have been done for application of transmission of heat for biological tissues. Although, to empathizing the influence of temperature for tissues while hyperthermia medical treatment plays an important role. In current years, several doctors and mathematicians have investigated the use of mathematical models and computational methods for heat transfer in biomedical systems.

Several medical practitioners and scientists also make use of mathematical modelling and engineering techniques to make sure their protection and consider the danger involved in it. The modified Pennes bioheat equation gives an critical outcome for the equation of conduction with a only one response time by including in it the MGT equation. In many recommended experimental research papers the heat transport in biological tissues has been investigated as creating an infinite concentric spherical region through magnetic fluid hyperthermia. Therefore, to analyse thermal reactions triggered by temperature shock, precisely the impact of heat generation during heat remedy on skin tumor, numerical inverse transformation, Laplace transformation techniques are applied. This research work was able to predict the impact of distinct therapeutic methodologies, good examples are metabolism support, cryotherapy sessions, blood perfusion, laser therapy, physical occurrences and transfer. Comparison has been made to conform the validity of suggested model and to check the correctness of numerical results in the literature. There are many phenomenological routes by which the heat transport in tissue is resolved, and that involve the diffusion across microvascular beds, involve blood-tissue convection, advection or blood perfusion, tissue heat exchange and metabolic heat production. Pennes bioheat equation had been investigated for heat transfer process in tissues in nondimensional form and the temperature gradient delay has been taken into consideration for the study by using Maxwell-Cattaneo model. The forces in the environment functional on the outer surface of biological tissue and distinct external energy sources are taken into consideration and has been solved by stochastic perturbations method.

A model for approximating its energy balance has been studied by Pennes in 1948 (Pennes, 1948) and the attention in the thermal behavior of biological tissues has also been grown. His model, often compensates for perfusion of blood influence by implementing a modified Fick relation. This equation has so many uses, including cryosurgical applications (Shih et al., 2007; Shitzer & Kleiner, 1980), evaluating physical characteristics of tissue and arterio-venous counter current heat exchange, blood perfusion rates, (Mitchell & Myers, 1968), and tissue heat transfer using cooling effects of strips in contact with the skin (Steketee & van der Hoek, 1979). Besides thermographic and infrared analyses, the connection between the various processes driving heat transfer in tissue has not been well examined systematically (Coccarelli et al., 2016; Najar et al., 2020). As a result, several researchers may ignore either the metabolic production heat element (Mosleh et al., 2021) or the blood perfusion variable. Many cases involve, one or both of these assumptions may be valid, but it is thought that a more scientific rationale might be presented. The goal of research is to investigate links that exist between convection and conduction through metabolism and blood perfusion, (Subedi, 2021).

The study has performed for uniform-properties, steady-state for the tissue, and is built on comparisons of dimensionless variable that describe everything in this mechanisms. Any biological tissue can experience thermal energy transfer. As a result, every mechanism should be included in the model of this tissue's thermal behavior for the transmission of heat energy through circulating blood (Hristov, 2019; Sun et al., 2017). This fact has been acknowledged, and several researchers use either the convective technique (Nagaraju et al., 2001; Shitzer & Kleiner, 1976) or the technique rely on a advanced Fick's law (Chato & Shitzer, 1971). Conduction, generation, and preservation are three more mechanisms involved in a biological tissue's thermal energy balance. Although due to complicated structure of tissues (Hodson et al., 1986; Krog & Scholander, 1957), several hypotheses must be made to aid the research at this stage. To name a few common assumptions, the tissue is often considered to be identical and isotropic, and the thermophysical parameters are considered as constant.

Some more realistic characterization of thermal compartments of tissues is hampered by lack of thorough data on thermophysical factors and physiological factors, as well as the inherent problems in properly and reliably monitoring in-depth temperatures. Nonetheless, researching models, however basic, may gain some ground in that they may give additional understanding of the systems implicated into thermal balancing. An impact of different geometrical factors and boundary conditions assumption on thermal balance might also be investigated. Very recent work attempted to propose a model to a generic problem relevant to a biological tissues (Shih et al., 2007) This study, however, does not take into consideration heat transmission via blood perfusion. It also lacks pictorial depictions for the analytical results, therefore no physical implications can be derived. The current study aims to propose an outcome of a problem of energy transfer in tissue having general type boundary conditions (Zhitomiskii and Kagna, 1978). The model includes the mechanism of heat transmission through blood perfusion. These systems largely influence a thermoregulatory response in response to a thermal disruption by changing heat loss rates and blood perfusion. As a result, the impact on model behavior may be evaluated indirectly by changing the associated physiologic parameters.

In this work, a mathematical model has been shown to study the heat flow characteristic for human skin. The temperature gradient, conduction, convection, and evaporation has been shown in the skin. A bioheat equation for thermal energy balance has been taken into consideration. The effect of tissue temperature and vein temperature with distance from the core is done by varying physical parameters such as coefficient of metabolism, metabolic rate, and perfusion rate.

FORMULATION OF THE PROBLEM:

The outer layer thickness is very small compared to the radius of curvature of the local surface. The schematic diagram with details of skin has depicted in figure 1. Since the region is very small, therefore, the temperature distribution only in the direction normal to the surface is taken. So, the system can be considered to be onedimensional. The steady-state heat balance equations for tissue and vein are written as follows,

$$k\frac{d^2T}{dx^2} + \left(h.a + C_p.g\right)(T_a - T) + h.a(T_v - T) + M = 0 \quad (1)$$

$$C_p\left[\dot{m}_v + \int_0^x \quad g\,dx\right]\frac{dT_v}{dx} + \left(h.a + C_p.g\right)(T_v - T) \quad (2)$$

where *k* represents thermal conductivity of tissue, *h* represents heat transfer coefficient, a represents the area of blood vessel, C_p is the specific heat capacity, *g* is the tissue perfusion rate in (g/cc-sec), T_a is the temperature of the artery, T_v the temperature of the vein, *M* represents metabolic heat generation rate (cal/cc-sec), ${m_v}^{\circ}$ is the venous flow rate.

It is supposed that M is varying linearly with T, and the following relation is obtained,

$$M = M_0(1 + \alpha T) \quad (3)$$

Where M_0 is the thermal energy generation for resting muscle and skin, and α is the metabolic factor or coefficient of metabolism.

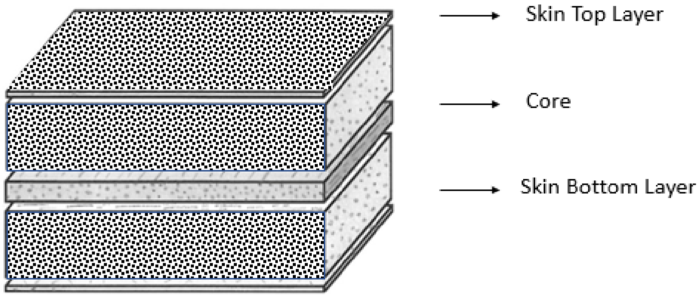

Figure 1: Core-shell-skin structure in human skin

SOLUTION OF THE PROBLEM:

Special Case 1: h.a = 0, (Heat transfer between artery and tissue or vein and tissue has assumed zero) Finding the value of temperature (θ_1) for tissue space in human skin:

The value h.a is considered to be zero because there is no vascular interaction i.e., no heat exchange occurs between artery and tissue or veins and tissue, where *h* is the heat transfer coefficient and a is the area of the blood vessel. With the help of Equations (1), (2), (3) and putting the value of (h.a = 0) in Equation (1), it reduce as follows,

$$k\frac{d^2T}{dx^2} + \left(M_0\alpha + C_p.g\right)T + C_p.gT_a + M_0 = 0. \quad (4)$$

Using the present boundary condition,

At x=0, $T=T_a=T_c$. (5)

At x=L, $T=T_v=T_s$ (6)

With the help of the boundary conditions (4) and (5) the temperature distribution is given as below,

$$T = C_1\cos\cos\left(\sqrt{\lambda_1 x}\right) + C_2\sin\left(\sqrt{\lambda_1 x}\right) - \frac{\lambda_2}{\lambda_1} \quad (7)$$

where the constant C_1 is defined by,

$$C_1 = T_c + \frac{\lambda_2}{\lambda_1} \quad (8)$$

and the coefficient C_2 gives,

$$C_2 = \frac{T_s - \left[T_c + \frac{\lambda_2}{\lambda_1}\right] cos\left(\sqrt{\lambda_1}L\right) + \frac{\lambda_2}{\lambda_1}}{sin\left(\sqrt{\lambda_1}L\right)} \quad (9)$$

And the value of λ_1 is as below,

$$\lambda_1 = \frac{M_0\alpha - C_p g}{k} \quad (10)$$

Similarly, the value of λ_2 is given by,

$$\lambda_2 = \frac{C_p g T_a + M_0}{k} \quad (11)$$

For simplicity, non-dimensional scheme has been used:

$$\theta_1 = \frac{T - T_c}{T_s - T_c} \quad , \quad X = \frac{x}{L} \quad (12)$$

After using non-dimensional scheme, we get,

$$\theta_1 = \left(\frac{C_1}{T_s - T_c}\right) cos\left(\sqrt{\lambda_1}LX\right) + \left(\frac{C_2}{T_s - T_c}\right) sin\left(\sqrt{\lambda_1}LX\right) - \frac{\lambda_2}{\lambda_1(T_s - T_c)} - \frac{T_c}{(T_s - T_c)} \quad (13)$$

Special Case 2: Finding the value of temperature (θ_2) for veins in human skin by assuming h.a=0:

From Equation (2), we obtain,

$$(\dot{m}_v + gx)\frac{dT_v}{dx} + g(T_v - T) = 0 \quad (14)$$

using the present boundary condition,

$$At\ x = 0, T = Ta = Tc \quad (15)$$

$$At\ x = L, T = Tv = Ts \quad (16)$$

Substituting the above value of T from Equation (7) into Equation (14) and solving for T_v, we get,

$$T_v = \frac{g}{(\dot{m}_v + gx)}\left[\frac{C_1 sin\left(\sqrt{\lambda_1}x\right)}{\lambda_1} - \frac{C_2 cos\left(\sqrt{\lambda_1}x\right)}{\lambda_1} - \frac{\lambda_2}{\lambda_1}x\right] + C_4 \quad (17)$$

where,

$$C_4 = T_s - \frac{g}{(\dot{m}_v + gL)}\left[\frac{C_1 sin\left(\sqrt{\lambda_1}L\right)}{\lambda_1} - \frac{C_2 cos\left(\sqrt{\lambda_1}L\right)}{\lambda_1} - \frac{\lambda_2}{\lambda_1}L\right] \quad (18)$$

For simplicity, non-dimensional scheme has been used:

$$\theta_2 = \frac{T_v - T_c}{T_s - T_c} \quad , \quad X = \frac{x}{L} \quad (19)$$

After using non-dimensional variables, we get

$$\theta_2 = \frac{g}{(\dot{m}_v + g.L.X)(T_s - T_c)}\left[\frac{C_1 sin\left(\sqrt{\lambda_1}LX\right)}{\lambda_1} - \frac{C_2 cos\left(\sqrt{\lambda_1}LX\right)}{\lambda_1} - \frac{\lambda_2}{\lambda_1}LX\right] + \frac{C_4}{(T_s - T_c)} - \frac{T_c}{(T_s - T_c)} \quad (20)$$

RESULTS AND DISCUSSION:

All temperature measurements systems, focused on the complicacy of transmission of heat in natural biological tissues systems due to several physiological functions depends upon the spatiotemporal temperature profile in tissues. The transmission of heat in tissues contains several mechanisms that enable transmission of heat in biological tissues and that should be measured, including convection between the tissue and the blood, the process of heat transfer in tissues, delayed blood perfusion and blood perfusion, diffusion through microvascular beds, changes in tissue properties, vascular structure and metabolic heat generation. To introduce some of the aspects in this work, following values of physical and physiological parameters have been taken from the research which has already been done by many researchers (Keller & Seiler, 1971). The values of the parameters thermal conductivity of tissue, specific heat of blood , core temperature, temperature at the surface of skin, artery temperature, and length of tissue are as 0.499 J/m-sec-K, 3799 J/Kg K, 310 K, 302 K, 278 K, and 1 respectively, are used for obtaining the temperature profile in the region under study.

The perfusion rates for blood are investigated from the expression, and tabulated in table (1):

$$\lambda = \sqrt{\frac{C_p g}{K}} \tag{21}$$

TABLE (1)

λ	Perfusion rate (g)
15	0.0295538299552514
16	0.0336256909713082
17	0.0379602526980784
18	0.0425575151355620
19	0.0474174782837589
20	0.0525401421426691

Special Case 1: Heat transfer between artery and tissue or vein and tissue is considered zero (Tissue Temperature Variations (θ_1)):

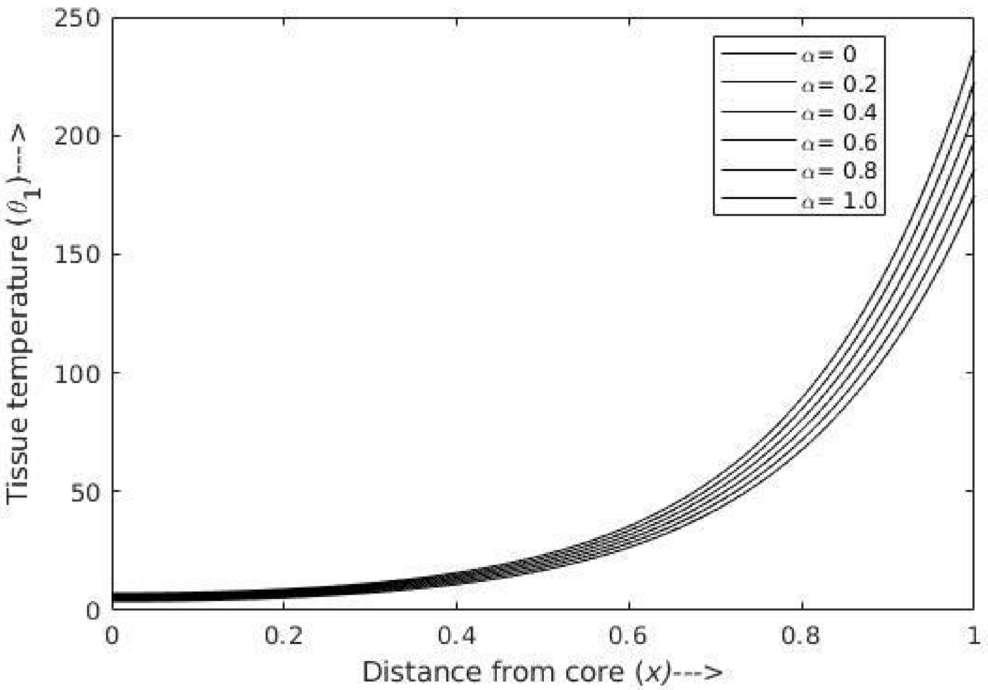

Figure 2: Tissue temperature from core to skin varying metabolic heat coefficient

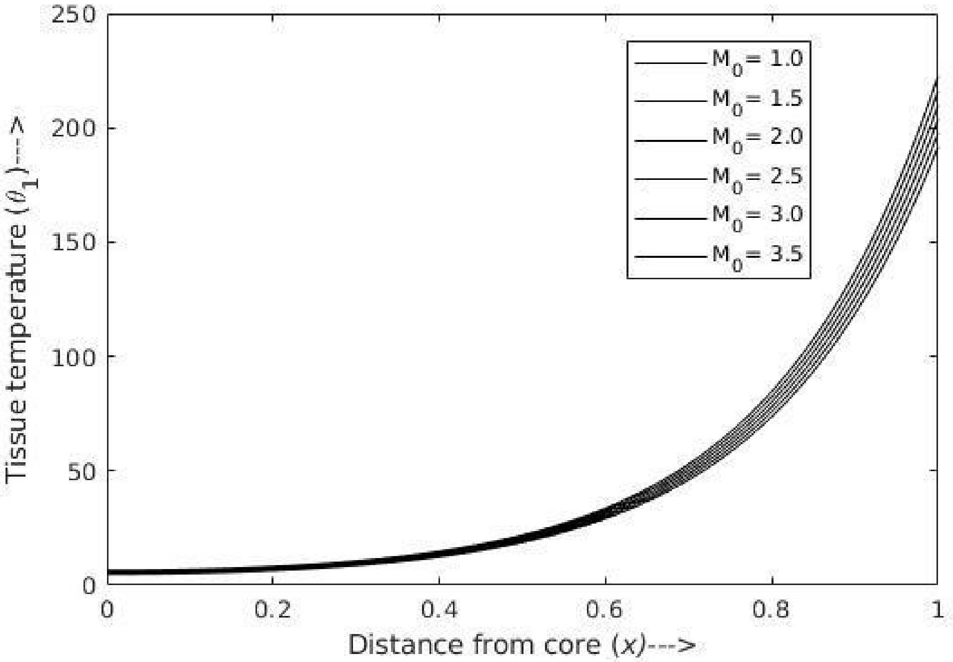

Figure 3: Tissue temperature from core to skin varying metabolic heat generation rate

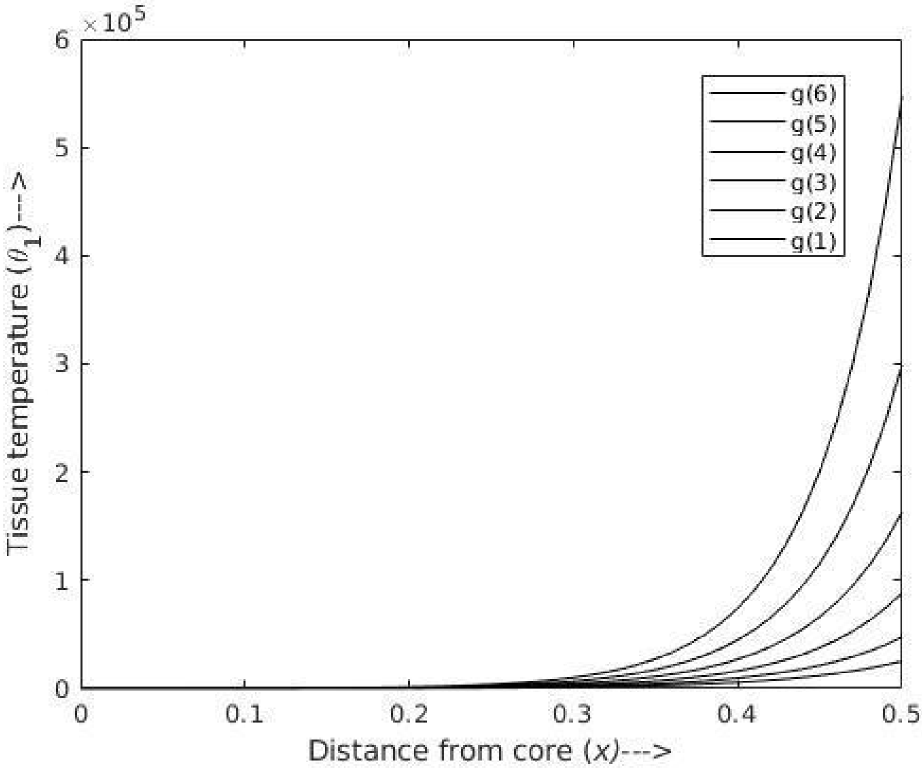

Figure 4: Tissue temperature from core to skin varying perfusion rate

Figure 2 shows the temperature of tissue (θ_1) varying with distance from the core (x). The graph shows an increase in tissue temperature from core to skin. This variation is studied for different values of temperature coefficient of metabolic heat generation (α). Figure 3 also shows the temperature of tissue (θ_1) varying with distance from the core for different values of metabolic heat generation rate (M_0). Figure 4 shows the temperature of tissue (θ_1) varying with distance from the core for different values of perfusion rates (g). The graph is obtained for the condition when tissue is assumed to have a high perfusion rate or tissue is well perfused. The graph shows an increase in tissue temperature from core to skin (Subedi et al., (2021)). The rate of heat transfer from blood to tissue at any point is proportional to the difference between arterial blood and tissue temperature at any point. These results are giving a better understanding of tissue temperature distribution phenomena from core to skin for different perfusion rate.

Special Case 2: Heat transfer between artery and tissue or vein and tissue is considered zero. (Vein Temperature Variations (θ_2)):

Figure 5 shows the temperature of the vein (θ_2) varying with distance from the core (x). The graph shows a decrease in vein temperature from core to skin. This variation is studied for different values of temperature coefficient of metabolic heat generation (α). Figure 6 also shows the temperature of the vein (θ_2) varying with distance from the core. The graph shows a decrease in tissue temperature from core to skin (Mosleh et al., 2021). This variation is for different values of metabolic heat generation rate (M_0). Figure 7 shows the temperature of the vein (θ_2) varying with distance from the core for different values of perfusion rates (g). The graph also shows a decrease in vein temperature from core to skin (Shih et al.,, 2007).

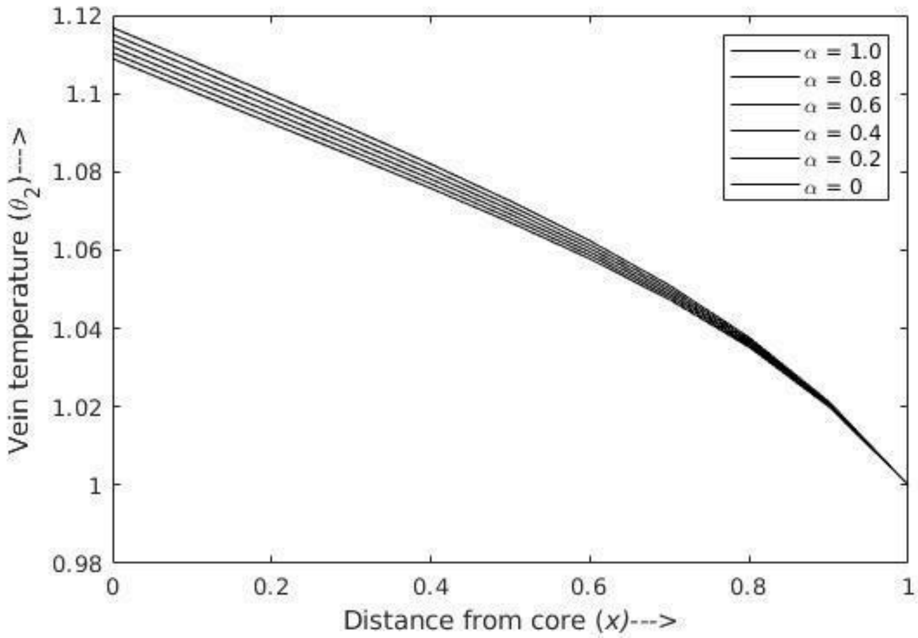

Figure 5: Tissue temperature from core to skin varying metabolic heat coefficient

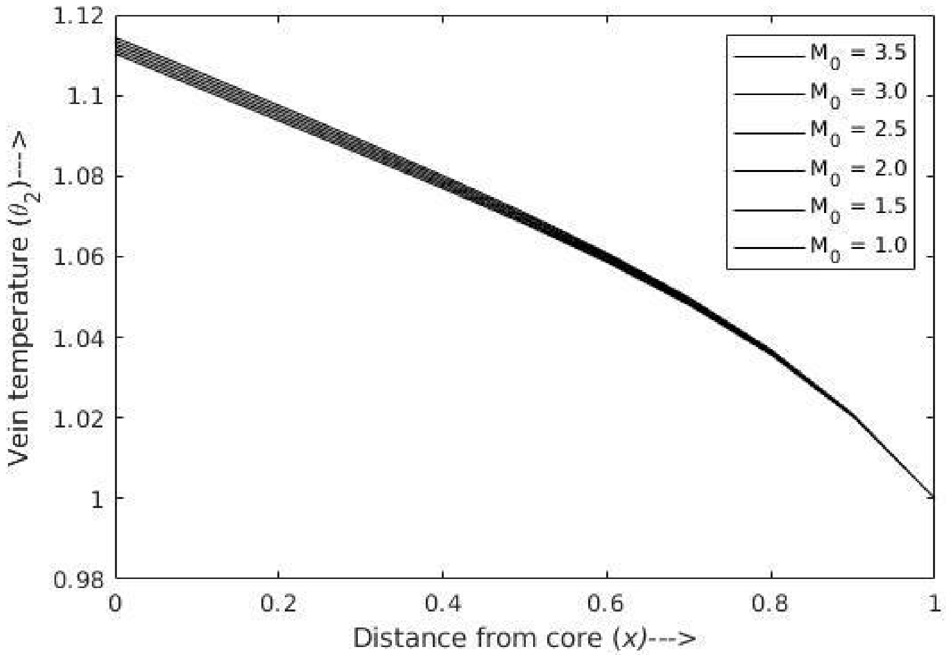

Figure 6: Tissue temperature from core to skin varying metabolic heat generation rate

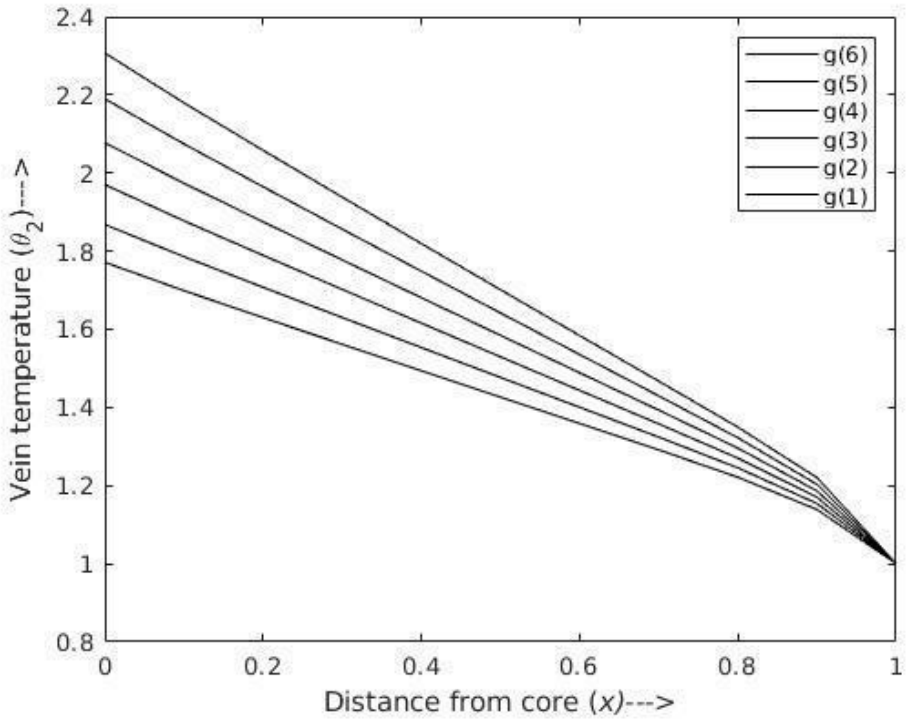

Figure 7: Tissue temperature from core to skin varying perfusion rate

CONCLUSION:

The study concluded that heat flows from core to skin in the tissue region as seen from the results of the first case. The second case gives the conclusion that heat flows from the skin to the core in veins. The two regions have opposite behavior throughout irrespective of the parameters observed in this paper i.e., for the three parameters, coefficient of metabolism (α), metabolic heat generation rate (M_0) and perfusion rate (g), tissue heat profile, and vein heat profile show opposite behavior. A lot of studies are left to do both analytically and experimentally before the numerical outcome can be considered consistent for any medical prediction. This study may also enrich our present interpretation of the temperature distribution in biological tissues during hyperthermia treatment.

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