ISSN: 0974-5823 Vol. 7 No. 6 June, 2022

International Journal of Mechanical Engineering

A Study on Square Difference labelling of Some Graphs and its Energy Power

[*1] C. Sankar, [2*] V.E. Sasikala

[*1] M.Phil Scholar, [2*] Corresponding Author, Research Supervisor,

Department of Mathematics, Vels Institute of Science,

Technology & Advanced Studies (VISTAS), Chennai, India.

Abstract

A graph G = (V, E) with p vertices and q edges is said to admit square difference labelling, if there exists a bijection f : V(G) →{ 0,1, …., p-1} such that the induced function f*: E(G)→N given by f*(uv) = | $[f(u)]^2$ - $[f(v)]^2$ | for every uv∈ E(G) are all distinct. A graph which admits square difference labelling is called square difference graph. In this paper we prove that some classes of graph like crown graph C_noK_1, One point union of six copies of p_4 admits square difference labelling and also find energy of the crown graph C_noK_1.

Keywords: Square difference labelling, Cycle Graph, Comb Graph.

Introduction:

This project deals with graph labelling. All the graphs considered here are finite, simple and undirected. A graph labelling is an assignment of integers to the vertices or edges or both the subject to certain conditions. If the domain of the mappings is the set of vertices (or edges), then the labelling is called a vertex labelling (or an edge labelling). The concept of graph labelling was first introduced by Rosa in mid sixties. In the year 1967, Rosa introduced a new type of graph labelling which he named as β-labelling. Let G be any graph and m be the number of edges in G. Rosa introduced a function f from the set of vertices of G to the set of the integers {0,1,2,…,m}, so that each edge is assigned the label ∣f(u) - f(v)∣, with all labels are distinct. Golomb independently studied the same type of labelling and named this labelling called graceful labelling. Since then, different properties of graceful labelling of graphs have been introduced and studied extensively by several graph theorists. The concept of square difference labelling was first introduced by J.Shiama in [2]. The square difference labelling for cycles, complete graphs, cycle cactus, ladder, lattice grids, wheels, quadrilateral snakes, the graph G = K_2+mK_1 has been shown by J.Shiama. In this study, we look into square difference labelling of crown graph c_nok_1, One point union of six copies of p_4.

The energy, E(G), of a graph G is defined to be the sum of the absolute values of its eigen values. Hence if A(G) is the adjacency matrix of G, and $\lambda_1,\ldots,\lambda_n$ are the eigen values of A(G), then E(G) = $\sum_{i=1}^{n} |\lambda i|$. The set { $\lambda_1,\ldots,\lambda_n$ } is the spectrum of G and denoted by Spec G. In this study, we also find Energy of the Crown graph c_nok_1.

2. Preliminary

2.1 Definition

A graph G consists of a pair (V(G), E(G)) where V(G) is a non - empty finite set whose elements are called vertices and E(G) is a set of unordered pair of distinct elements of V(G). The elements of E(G) are called edges of the graph G.

2.2 Definition

Let G = (V(G), E(G)) be a graph. G is said to be square difference labelling if there exist a bijection f: V(G)→{0,1,2,….p-1} such that the induced function f * :E(G) → N given by f * (u v) = |[f* (u)]^2 – [f* (v)]^2 | for every uv∈ E(G) are all distinct.

2.3 Definition

A vertex joined to itself by an edge is called a loop. Let G be a graph, if two or more edges of G have the same end vertices then these edges are called multiple edges.

2.4 Definition

A graph is called simple if it has no loops and no multiple edges.

2.5 Definition

A graph G is called connected if every pair of its vertices is connected by a path. A graph which is not connected is called disconnected.

2.6 Definition

Joining a pendant edge to each vertex of cycle yields the crown graph (CnoK1).

2.7 Definition

A comb graph is created by connecting each vertex of a route with a single pendant edge.

2.8 Definition

Energy of graph The energy E(G) of a graph G is defined to be the sum of the absolute values of Eigen values of G. Hence if A(G) is the adjacency matrix of G and λ_1, λ_2, ..., λ_n are Eigen values of A(G) then E(G) = $\sum_{i=1}^{n} |\lambda i|$. As the sum of the absolute values of Eigen values. The energy of any graph G, E(G) is always greater than or equal to zero. Since for the totaly disconnected graph K_c^n is the adjacency matrix is a zero matrix. There for it has no nonzero Eigen values.

3 - Square difference labelling of Some Graphs

3.1 Theorem

The crown graph $c_n ok_1$ admit square difference labelling.

Proof:

Draw a cycle graph which consists of two paths, a left path $u, u1, u2, . , u_{(\frac{n}{2})-1}$

and right path $v_1, v_2, \dots, v_{(\frac{n}{2})}$. In order to get a cycle, connect the vertex u with the vertex v_1 and connect the vertex $v_{(\frac{n}{2})}$ with the vertex $uu_{(\frac{n}{2})-1}$. Let the number of vertices $u, u_1, u_2, \dots, u_{(\frac{n}{2})-1}$ be x and the number of vertices $v_1, v_2, \dots, v_{(\frac{n}{2})}$ be y. Join one pendant edge for each vertex v_i on the right path ($v_i v_{i'}$) where $i = 1, 2, \dots, \frac{n}{2}$.

Join one *pendant* edge for each vertex u_i on the left path ($u_i u_i'$) where $i = 1, 2, \dots, \frac{n}{2} - 1$. Finally, join one pendant edge for the vertex (uu').

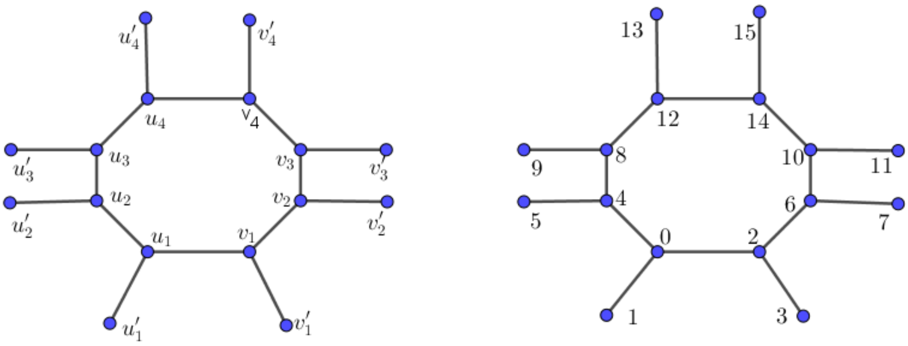
Case 1: n ≡ 0 (mod 2).

$$f(u_i) = \begin{cases} 8m(i-N)/m & (i \text{ is odd}) \\ 4m(i-1)/m & (i \text{ is even}) \end{cases} \quad \text{for } 1 \le i \le \frac{n}{2} - 1$$

$$f(v_i) = \begin{cases} 2m(i+2w)/m & (i \text{ is odd}) \\ 2m(2i-1)/m & (i \text{ is even}) \end{cases} \quad \text{for } 1 \le i \le \frac{n}{2}$$

$$f(u_i') = \begin{cases} 8m(i-N)+1/_m & (i \text{ is odd}) \\ 1+\frac{4m(i-1)}{m} & (i \text{ is even}) \end{cases} \text{ for } 1 \le i \le \frac{n}{2}-1$$

$$f(v_i') = \begin{cases} 1+\frac{2m(2i-1)}{m} & (i \text{ is odd}) \\ \frac{7m(i-N)}{m}-W & (i \text{ is even}) \end{cases} \text{ for } 1 \le i \le \frac{n}{2}$$



Vertex Labelling of the crown graph c_8ok_1

Let f* be the induced edge labelling:

$f^*(uv) = \frac{4n-i}{n}$; $i = 0$

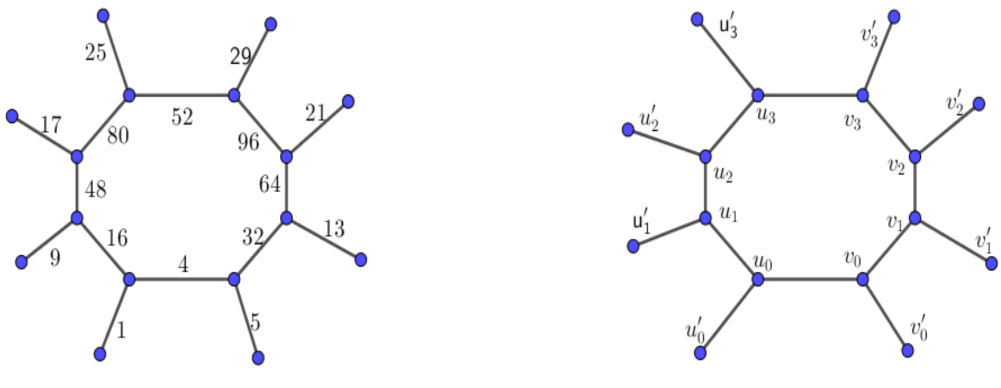
$f^*(u_iu_{i+1})$ = 16(2i+1) ; $i = 0,1,\ldots\frac{n}{2}-2$

$f^*(v_iv_{i+1})$ = 32(i+1) ; $i = 0, 1, \ldots\frac{n}{2}-2$

$f^*(u_iv_i)$ = 4[4(i-1)+5] ; $i = \frac{n}{2}-1$

$f^*(u_iu'_i)$ = 8i+1 ; $i = 0, 1, \ldots\frac{n}{2}-1$

$f^*(v_iv'_i)$ = [4(i+N) +1] ; $i = 0, 1, \ldots\frac{n}{2}-1$



Edge Labelling of the crown graph c_8ok_1

Case 2: n ≡ 1(mod 2).

$$f(u_i) = \begin{cases} 8m(i-N)/_m & (i \text{ is odd}) \\ 4m(i-1)/_m & (i \text{ is even}) \end{cases} \text{ for } 1 \le i \le \frac{n-1}{2}$$

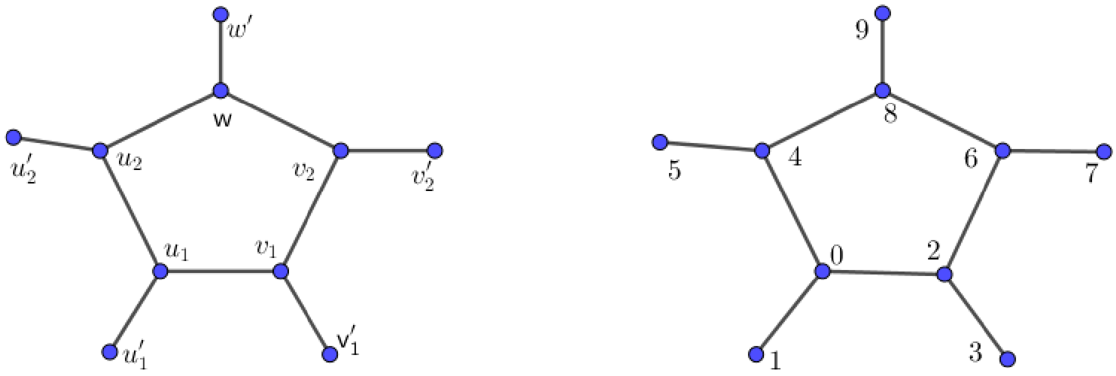
$$f(v_i) = \begin{cases} 2m(i+2w)/m & (i \text{ is odd}) \\ 2m(2i-1)/m & (i \text{ is even}) \end{cases} \text{ for } 1 \leq i \leq \frac{n-1}{2}$$

$$f(u_i') = \begin{cases} 8m(i-N)+1/m & (i \text{ is odd}) \\ 1+\frac{4m(i-1)}{m} & (i \text{ is even}) \end{cases} \text{ for } 1 \leq i \leq \frac{n-1}{2}$$

$$f(v_i') = \begin{cases} 1+\frac{2m(2i-1)}{m} & (i \text{ is odd}) \\ \frac{7m(i-N)}{m} - W & (i \text{ is even}) \end{cases} \text{ for } 1 \leq i \leq \frac{n-1}{2}$$

f(w)=2(n-1)

$f(w')$=2(n-1)+1



Vertex Labelling of the crown graph c_5ok_1

Let f* be the induced edge labelling:

f^*(uv) = $\frac{4n-i}{n}$; i = 0

$f^*(u_iu_{i+1})$ = 16(2i+1) ; i = 0,1,….. $\frac{n-1}{2}$

$f^*(v_iv_{i+1})$ = 32(i+1) ; i = 0, 1, ….. $\frac{n-1}{2} - 2$

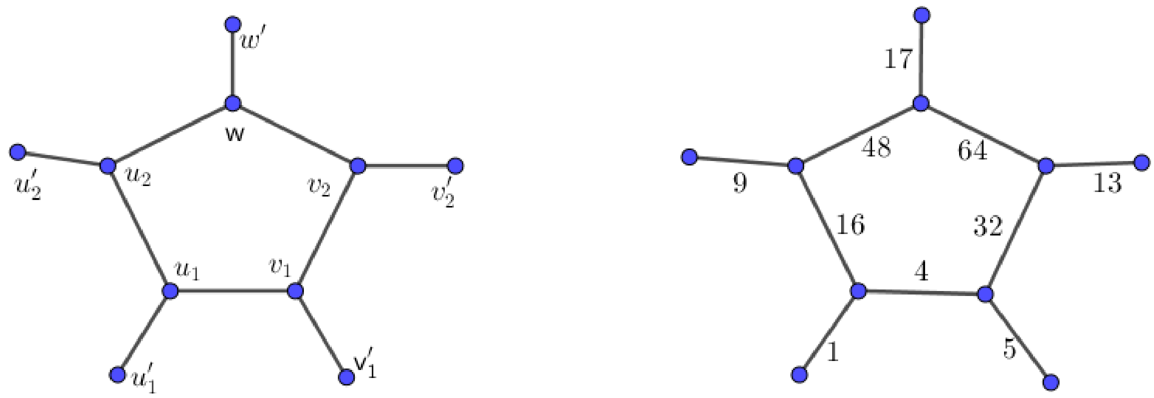
$f^*(u_iw)$ = 16(2i+1) ; $i = \frac{n-1}{2} - 1$

$f^*(v_iw)$ = 32(i+1) ; i = 0, 1, ….. $\frac{n-1}{2} - 1$

$f^*(u_iu'_i)$ = 8i+1 ; i = 0, 1, ….. $\frac{n-1}{2} - 1$

$f^*(v_iv'_i)$ = [4(i+N) +1] ; i = 0, 1, ….. $\frac{n-1}{2} - 1$

$f^*(ww')$ = 8($\frac{n-1}{2}$)+1



Edge Labelling of the crown graph c_5ok_1

Therefore, the whole of edges in G receive distinct labels. Thus the defined function provides square difference labelling for a graph.

(i.e) The crown graph $c_nok_1$is square difference labelling.

3.2 Theorem

One point union of six copies of p_4 admit square difference labelling.

Proof:

Let H be a graph with a route p_n of length n and k(1,t) . Let w be the central vertex for p_n^t and $v_{1,\,m}$($1 \leq l \leq t$, $1 \leq m \leq n$) be the consecutive vertices of each branch of p_n^t from v_o.

Let f be Vertex labelling, defined by

$f(v_i) = 2n + 1$ for $0 \leq i \leq \frac{n}{2}$

$f(u_i) = 2n$ for $1 \leq i \leq \frac{n}{2}$

Let f * be the induced edge labelling, defined by

$f^*(v_i v_{i+1}) = 8(i+1)$; $i = 0, 1, 2, 4, 5, 6, 8, 9, 10\ldots..\frac{n-3}{2}$

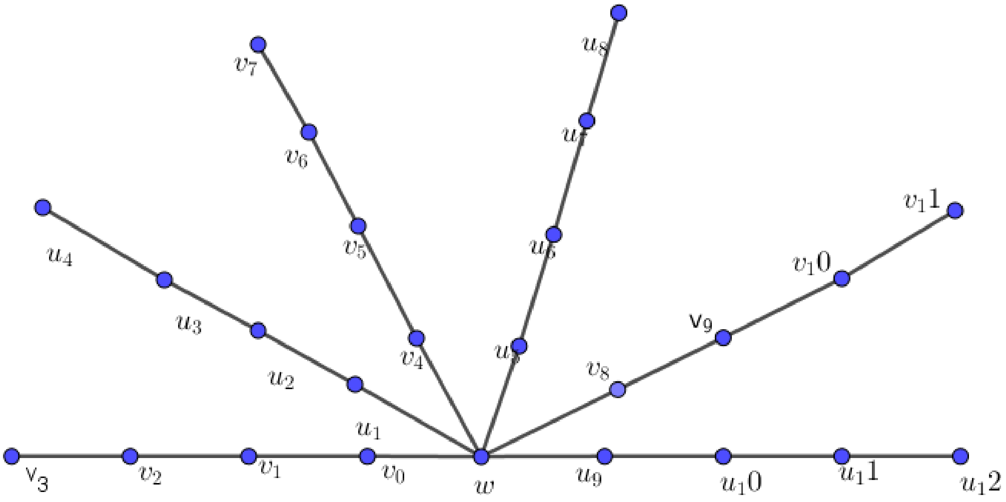
(Here we have to leave the I for every set of 1)

$f^*(u_i u_{i+1}) = 4(2i+1)$; $i = 0, 1,\ldots..\frac{n-1}{2}$

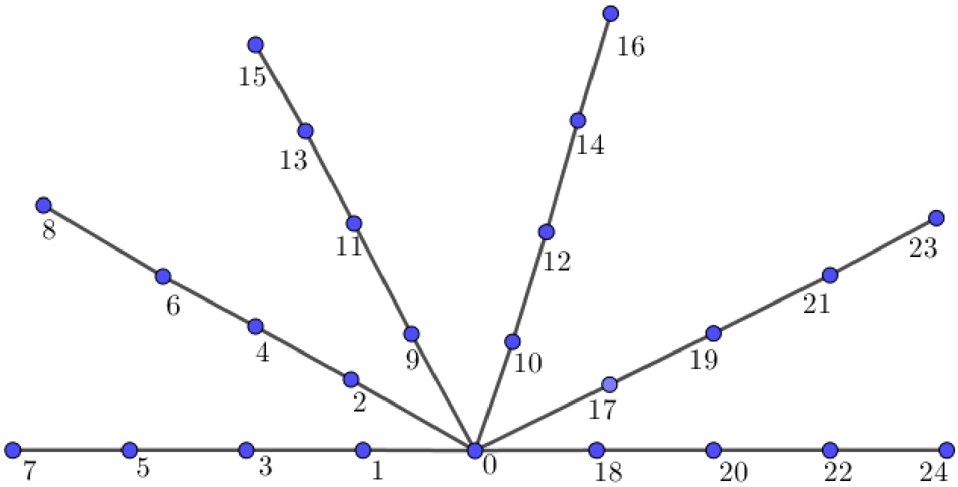
$f^*(wv_i) = i^2$; $i = 0$

$f^*(wv_i) = i^2$; $i = 4, 8\ldots$

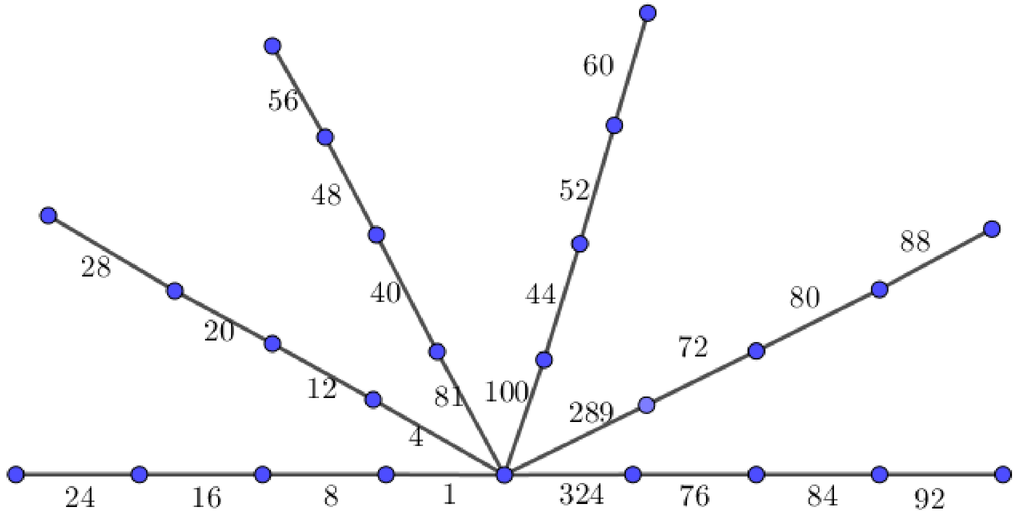
$f^*(wu_i) = 4(i)^2$; $i = 1, 5, 9\ldots$



One point union of six copies of p_4



Vertex labelling of the one point union of six copies of p_4



Edge labelling of the one point union of six copies of p_4

Therefore, the whole of edges in G receive distinct labels. Thus the defined function provides square difference labelling for a graph. (i.e) One point union of six copies of p_4 is square difference labelling.

4 Energy Of the Crown Graph:

The energy E(G) of a graph G is defined to be the sum of the absolute values of Eigen values of G. Hence if A(G) is the adjacency matrix of G and λ_1, λ_2, ..., λn are Eigen values of A(G) then E(G) = X_n i=1 $|\lambda_i|$ As the sum of the absolute values of Eigen values. The energy of any graph G, E(G) is always greater than or equal to zero. Since for the totally disconnected graph Kc n is the adjacency matrix is a zero matrix. There for it has no nonzero Eigen values. Thus the energy of totally disconnected graph is zero [2] that is $E(K^n{}_c) = 0$ and thus zero is connected as the lower bound for graph energy.

CROWN GRAPH $C_n oK_1$	ENERGY OF THE GRAPH
3	12.0
4	16.0
5	20.0
6	24.0
7	28.0
8	32.0
9	36.0
10	40.0

Property:

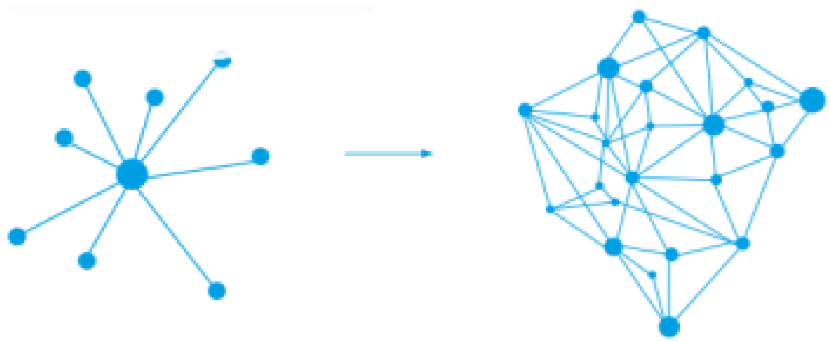
Energy of the Grown Graph is 4n Where n is Positive integer ≥ 3

5. Application

5.1 BLOCKCHAIN

Distributed ledger technology (DLT) is a relatively recent technological breakthrough that has far-reaching implications for many sectors. Although cryptographic technologies have been the basis for blockchains for some time, their inclusion as a useful package is truly innovative. In the electrical field, the combination of distributed energy resources and the multiplication of grid-interacting devices excites blockchain potential. But blockchains are exactly what blockchains are basically unchangeable digital ledgers that can be used to securely record all transactions that take place on a given network, which cannot be changed once the data is sealed within a block. This includes not only financial transaction data but also anything of value. Technology enables a new world of decentralized communication and integration by creating the infrastructure to allow one to connect with one another securely, cheaply and quickly without a centralized intermediary. Its range is linked like a star map.

Example



Graph in Block Chain Format

6. CONCLUSION

It's very Interesting to study graphs which admit of square difference labelling

of various classes of graphs such as the crown graph (cn × k1), single point union of six copies of $p_4$are established. Square difference labelling of other sorts of graphs is still a work in progress, and it will be completed later.

REFERENCES

[1] J. A. Gallian, Dynamic Survey of Graph Labeling, The Electron. J. Combin., DS6, (2016), 1- 408.

[2] Badra E.M , Moussa M.I, and Kathiresan K : Crown graphs and subdivision of ladders are odd graceful, International Journal of Computer Mathematics, Vol.88, No. 17, November 2011, 3570 – 3576.

[3] Bondy JA and Murthy USR : Graph theory with applications, Elesvier North Holland, Newyork, 1976.

[4] Lekha and Bijukumar : Some interesting results in the theory of Graphs, Ph.D thesis, Saurastra University, 2012.

[5] Uma R and Murugesan N : Graceful labelling of some graphs and their subgraphs, Asian Journal of Current Engineering and Maths, Vol.1, No.6, (2013), pp.307-370.

[6] M.Apostal, Introduction to Analytic Number Theory, Narosa Publishing House, Second edition,1991. 2 Frank Harary, Graph Theory, Narosa Publishing House, New Delhi,2001.

[7] S.W.Golomb, How to number a graph in Graph theory and Computing, R.C.Read, ed., Academic Press, Newyork, pp23-37,1972

[8] J.Shima "Square sum labelling for some middle and total graphs" International Journal of Computer Applications (0975-0887) Volume 37-No:4 January 2012.

[9] Shiama, J."Cube difference labelling of some graphs" International Journal of engineering Science and Innovative technology Volume 2(6), November 2013

[10] E. Esakiammal, B. Deepa and K. Thirusangu, Some labelling on square graph of comb, International Journal of Mathematics Trends and Technology, ISSN: 2231-5373.

[11] K. Manimekalai and K. Thirusangu, Pair sum labelling of some special graphs, International Journal of Computer Applications, 69(8) (2013).

[12] A. Rosa, on certain valuation of graph theory of graphs (Rome, July 1966), Golden and Breach, N.Y and Paris (1967), 349–355.

[13] Joan M Aldous and Robin J Wilson, Graphs and applications: an introductory approach, Springer Science & Business Media, 2003.

[14] R Balakrishnan, The energy of a graph, Linear Algebra and its Applications 387 (2004), 287–295. [3] Ravindra B Bapat, Graphs and matrices, vol. 27, Springer, 2010.

[15] Dragos M Cvetkovic, CVETKOVIC DM, et al., Spectra of graphs theory and application, (1980).

[16] Xing Duan and Bo Zhou, Sharp bounds on the spectral radius of a nonnegative matrix, Linear Algebra and Its Applications 439 (2013), no. 10, 2961–2970.