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RANDOM VIBRATIONAL ANALYSIS OF A DRILL STRING FOR A PORTABLE WATER WELL RIG

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ABSTRACT

One of the methods used nowadays in simulating dynamic behaviour in drillingoperations is by random vibrational analysis. In many engineering and drilling operations, thedynamic loading is random vibration. A random vibrational analysis of a drill string was donein a vertical water well by Ansys workbench 2020. Random vibrational analysis was performed to identify frequency response of the drill string. The probability of deformation for each sigmaunder random vibration was carried out. There is higher probability about 99.7% of failure in the axis of supplied PSD acceleration. The response power spectral density (RPSD) was carriedout in x, y and z taking any point of the (BHA) bottom hole assembly, mainly the bit. From theresults of response PSD, it is shown that there is higher value of resonance (spikes) as the drillpipe (DP) increases in both x-axis, y-axis and z-axis which are lateral and longitudinal movements which causes lateral and longitudinal vibrations to the drill string. The results canbe used to understand the most cause, which is lateral vibrations, for parameter optimization. With this for a critical component, on a range of frequencies we can determine lateral displacements.

KEY WORDS: Random Vibrations, Ansys workbench, Random Frequency, (PSD) Power Spectral Density, Root Mean Square (RMS).

1.0INTRODUCTION

One of the safest means of clean water is by boreholes. Drilling is being much expensivedue to drill rig cost caused by failure of various parts due to vibrations. Excessive drill stringvibrations causing secondary damage such as drill pipe fracture, bearing damage, decrease inrate of penetration and early failure of bits resulting in high maintenance and drilling cost ofrigs. About 40% of all exploration and production costs are found in drilling operations [28].

Drilling efficiency can be improved by preventing drill string failures, optimization and good interpretation of the drill string dynamics. On the (BHA) Bottom Hole Assembly, the drillstring is used to supply adequate weight to the bit (WOB), transmit torque and transport fluids

down hole. Excluding the drill pipe on the drill string the bottom part is called the Bottom HoleAssembly (BHA) [29].

Random vibrations deal with the probabilistic analysis of the response of structures. Inrandom vibrations, random parameters and initial condition ae used. In this it can be used withrandom excitation and random boundary conditions. Experiments of real structures are initially simulated in random environments. Random vibration analysis ae used in optimization and other purposes [25].

Drill string vibrations are difficult so there is need for predictive ability more accurately. Random vibrations has no clear pattern, it's not easy to define a vibration environment. This environment vary in a totally unpredictable manner. Random vibrations cannot be predicted. In random vibrations, many frequencies are excited on the same time. There is no periodicities. Resonances of different components on drill string can be excited simultaneously.

2.0 DRILL STRING VIBRATIONS

Random vibration motion is non deterministic. We cannot accurately predict the future behaviour. Random vibration analysis can be done to investigate the statistical properties of drill string or a response of a mechanical system, mainly standard deviations, stresses and forces. Random excitation are represented by power spectral density (PSD). The amplitude of excitation can constantly change at any given frequencies. If we look in many processes, average value of random excitation is uniform, so we characterize random excitation as a statistical process [27].

Dynamic loading can be explained as random vibration. The figure 2.0 is a graph of random vibrations on amplitude versus time histories. The three parameters of interest are frequency, time and amplitude so we can analyze a random process in a statistical manner. Random vibration characterization results in a frequency spectrum of Power Spectral Density(PSD). The Power Spectral Density defines the distribution of power over the frequency range excitation [24].

Amplitude

Time

Figure 2.0: The graph of Random Vibration for amplitude vs time [30].

Random vibrations follow a normal o the Gaussian distribution as figure 2.1.



Figure 2.1: Normal or the Gaussian distribution [26]

2.1 PROBLEMS CAUSED BY DRILL STRING VIBRATIONS

Drill string vibrations results drill bit failures, bearing damage and drill pipe fracture. Drill string vibration results in the unstableness of wellbore and a lot of energy is wasted. Some studies proves that the most dangerous effect of vibration is subjected to the drill collars and adjacent drill pipe. Each vibration model has its own effect on the drilling operation [5].

3.0. MATERIALS AND METHODS

The material used in this analysis is mainly structural steel as shown in table 1. Also the specifications of parts of drill strings (DS) used in the analysis are shown in table 2.

Table 1: Material used

| | Property | value | Unit |
|---|-----------------|--------------|-------------------|
| 1 | Density | 7850 | kgm ⁻³ |
| 2 | Young Modulus | 2E + 11 | Ра |
| 3 | Poisson's Ratio | 0.3 | |
| 4 | Bulk Modulus | 1.6667E + 11 | Pa |
| 5 | Shear Modulus | 7.6923E + 10 | Pa |

Table 2: Specifications of DS parts used

| Part | OD | ID | Length | Units |
|------|-------|-------|--------|-------|
| DC | 0.076 | 0.060 | 1 | m |
| DP | 0.076 | 0.068 | 20 | m |
| DP | 0.076 | 0.068 | 40 | m |
| DP | 0.076 | 0.068 | 60 | m |

3.1 RANDOM VIBRATIONS

Scalar or vector representation of random vibration is given by

$$\dot{x} = g(x, y, a)$$
 $x(0) = x_0, t \ge 0$ (1)

Then **x** represents system response; **y** is system excitation; **a**, is for system parameters; the dotrepresents the differentiation with respect to time; and $\mathbf{g}(.)$ is the deterministic functional form. Several problems with this aspect were solved in the case were $\mathbf{g}(.)$ is a function of linear excitation while the excitation cannot move, Gaussian random process.

3.2 STATISTICAL APPROACH

The most commonly used probability of distribution is the Normal or the Gaussian distribution. The probability density function (PDF) for a normal distribution is given by:

| 1 | x | | |
|------------|-------|--|--|
| 1 | [(2)(| | |
| - | | | |
| | | | |
|)2] | - | | |
| <i>p</i> = | _e | | |
| √2п | | | |
| x_{rr} | ns | | |
| (2) | | | |

Equation 1 is plotted as figure 3.0. In equation 2, X and Xrms can have displacement, velocity and acceleration units, or derivatives of these terms.



Figure 3.0: PDF for a Normal Distribution

3.3 INPUT ACCELERATION AND DISPLACEMENTCHARACTERIZATION

The simplest random excitation to analyze is a band limited white spectrum (BLWS) shown inFigure 3.1.



$$G_{rms} = \sqrt{w(f_2 - f_1)} \tag{3}$$

This value could be used in equation 2 to predict the probability of occurrence of instantaneousvalues of acceleration for a random signal. The RMS displacement is the square root of the area under the curve of m^2/Hz . For a BLWS, the RMS displacement can be shown to be given by

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Where,

Grms = input acceleration g = acceleration constant (9.81ms⁻²) f_1 = lower frequency, Hz f_2 = upper frequency, Hz For most cases, f_2 is significantly higher than f_1 and equation 4 can be approximated by:

$$X_{rms} = \frac{565Grms}{\sqrt{f^3 f^2}}$$

(5)

4.0 RESULTS AND DISCUSSIONS

The DS random vibrational analysis was modelled in Ansys shown in figure 4.0. The drillingfluid impact was considered negligible and insignificant.



Figure 4.0: Drill string modelling in Ansys

4.1 MODAL FREQUENCY

The figure 4.1, 4.2 and figure 4.3 shows the modal frequency of first 26 modes (20m DP),49modes (40m DP) and 73modes (60m DP). The range of frequency was set from 0 to 62Hz.



Figure 4.2: Modal frequency of first 49 modes (40m DP).



Figure 4.3: Modal frequency of first 73 modes (60m DP).

4.2 RANDOM VIBRATION

The table 3 and figure 4.4 is PSD acceleration against frequency. The accelerations supplied were given at random. The acceleration was applied in the longitudinal axis that is the y axis.

Table 3: PSD acceleration against frequency

| Frequency [Hz] | Acceleration[(m/s ²) ² /Hz] |
|----------------|--|
| 5. | 100. |
| 15. | 200. |
| 20. | 350. |
| 30. | |
| 45. | 200. |
| 60. | |



frequency Figure 4.4: PSD acceleration against frequency.

4.3 DIRECTIONAL DEFORMATION

The table 4, table 5 and table 6 shows directional deformation in x, y and z directions for 1 sigma, 2 sigmas and 3 sigmas respectively. The probability of deformation for each sigma under random vibration is given. There is higher probability about 99.73% of deformation in ysince the PSD acceleration was supplied along the axial/longitudinal movement(y direction) asshown by table 4, table 5 and table 6 results below. If the PSD acceleration was along the lateral directions, it means more deformation was to be experienced along the lateral directions. This shows that under random vibrational analysis a drill string can experience both lateral and longitudinal deviations with a measurable deformation depending on the supplied PSD acceleration at random. The results of deformation are same in x and z directions which are

lateral movements. The larger the drill string, the lower the value of maximum deformation. 1x RMS (1 sigma) accounts for 68.27% of total response, 95.45% and 99.73% for 2 x RMS (2 sigma) and 3 x RMS (3 sigma) respectively.

| | Direction | Direction | Direction | Direction | Direction | Direction | Direction | Direction | Direction |
|---------------|-------------|-----------|-----------|-----------|------------|-----------|-----------|-----------|-----------|
| Object | al D r | al D (| al D r | al D (| a/ | al D (| a/ | a/ | a/ |
| Name | tion v 1 | tion x 2 | tion v 3 | tion v 1 | tion v 2 | tion v 3 | tion z 1 | tion 7.2 | tion z 3 |
| | sioma | sioma | siama | sioma | siama | sioma | sioma | siama | sioma |
| State | <u> - 9</u> | argina | | | Solved | | | | arginia |
| | | | | Se | one | | | | |
| Scoping | | | | | | | | | |
| Method | | | | Geor | metry Sele | ction | | | |
| Geometr | | | | | All Dedice | | | | |
| y | | | | | All Dooles | | | | |
| | | | | Defi | nition | | | | |
| Туре | | | | Directi | onal Defor | mation | | | |
| Orientati | X Axis | | | | Y Axis | | Z Axis | | |
| on | | | | | | | | | |
| Referen | | | | Relativ | e to base | motion | | | |
| Scale | | | | | | | | | |
| Factor | 1 Sigma | 2 Sigma | 3 Sigma | 1 Sigma | 2 Sigma | 3 Sigma | 1 Sigma | 2 Sigma | 3 Sigma |
| Probabili | 68.269 | 05 45 % | 00 72 % | 68.269 | 05 45 04 | 00 72 84 | 68.269 | 05.45.04 | 00 72 84 |
| ty | % | 80.40 % | 88.73.70 | % | 80.40 70 | 88.1370 | % | 80.40 % | 88.1370 |
| Coordin | | | | | _ | | | | |
| ate Surtem | | | | Solution | Coordinate | e System | | | |
| System | | | | | | | | | |
| Suppres | | | | | | | | | |
| sed | | | | | No | | | | |
| | Results | | | | | | | | |
| Minimu | | | | | 0 | | | | |
| m | | | | | u. m | | | | |
| Maximu | 7.4821e- | 1.4984e- | 2.2446e- | 5.8307e- | 0.11661 | 0.17492 | 2.6843e- | 5.3686e- | 8.0529e- |
| m | 006 m | 005 m | 005 m | 002 m | m | m | 003 m | 003 m | 003 m |
| Average | 2.224e- | 4.4479e- | 8.6719e- | 3.7657e- | 7.5314e- | 0.11297 | 4.6462e- | 9.2924e- | 1.3939e- |
| | 006 m | 006 m | 006 m | 002 m | 002 m | m | 004 m | 004 m | 003 m |

| · | | | | | | | | | |
|-----------------|-------------|-----------|-----------|-------------|-------------|-----------|-------------|-----------|-----------|
| | Direction | Direction | Direction | Direction | Direction | Direction | Direction | Direction | Direction |
| Object | al | al | al | al | al | al | al | al | al |
| Nama | Deforma | Deforma | Deforma | Deforma | Deforma | Deforma | Deforma | Deforma | Deforma |
| THE HEL | tion x 1 | tion x 2 | tion x 3 | tion y 1 | tion y 2 | tion y 3 | tion z 1 | tion z 2 | tion z 3 |
| | sigma | sigma | sigma | sigma | sigma | sigma | sigma | sigma | sigma |
| State | | | | | Solved | | | | |
| | | | | Sc | ope | | | | |
| Scoping | | | | Geor | metry Sele | ction | | | |
| Method | | | | | neary ease | ener: | | | |
| Geometr | | | | | All Bodies | | | | |
| γ | | | | | | | | | |
| | Definition | | | | | | | | |
| Туре | | | | Directi | onal Defor | mation | | | |
| Orientati | | X Axis | | | Y Axis | | Z Axis | | |
| on | | | | | | | | | |
| Referen | | | | Relativ | e to base | motion | | | |
| ce | | | | | | | | | |
| Factor | 1 Sigma | 2 Sigma | 3 Sigma | 1 Sigma | 2 Sigma | 3 Sigma | 1 Sigma | 2 Sigma | 3 Sigma |
| Probabili | 68.269 ~ | 95.45 % | 99.73 % | 68.269 ~ | 95.45 % | 99.73 % | 68.269 ~ | 95.45 % | 99.73 % |
| TV Conselier | 70 | | | 96 | | | 76 | | |
| Coordin | | | | Calution | Consultant. | . C | | | |
| Suctors | | | | Solution | Coordinate | e bystem | | | |
| Identifier | | | | | | | | | |
| Suppres | | | | | | | | | |
| sed | | | | | No | | | | |
| | | | | Res | ults | | | | |
| Minimu | | | | | | | | | |
| m | | | | | 0. m | | | | |
| Maximu | 4.0231e- | 8.0462e- | 1.2069e- | 2.9221e- | 5.8442e- | 8.7664e- | 1.436e- | 2.8721e- | 4.3081e- |
| m | 006 m | 006 m | 005 m | 002 m | 002 m | 002 m | 003 m | 003 m | 003 m |
| A | 1.404e- | 2.8081e- | 4.2121e- | 2.2531e- | 4.5083e- | 6.7594e- | 2.5908e- | 5.1816e- | 7.7724e- |
| Average | 006 m | 006 m | 006 m | 002 m | 002 m | 002 m | 004 m | 004 m | 004 m |
| | | | | | | | | | |

| Object Name | Direction al Deforma tion x 1 sigma | Direction al Deforma tion x 2 sigma | Direction al Deforma tion x 3 sigma | Direction al Deforma tion y 1 sigma | Direction al Deforma tion y 2 sigma | Direction al Deforma tion y 3 sigma | Direction al Deforma tion z 1 sigma | Direction al Deforma tion z 2 sigma | Direction al Deforma tion z 3 sigma |
|--------------------------|---|---|---|---|---|---|---|---|---|
| State | | | | - | Solved | | | | |
| | | | | Sc | ope | | | | |
| Method | | | | Geor | metry Sele | ction | | | |
| Geometr V | | | | | All Bodies | | | | |
| | | | | Defir | nition | | | | |
| Type | | | | Directi | onal Defor | mation | | | |
| Orientati on | | X Axis | | | | Y Axis | | Z Axis | |
| Referen ce | | Relative to base motion | | | | | | | |
| Scale Factor | 1 Sigma | 2 Sigma | 3 Sigma | 1 Sigma | 2 Sigma | 3 Sigma | 1 Sigma | 2 Sigma | 3 Sigma |
| Probabili ty | 68.269 % | 95.45 % | 99.73 % | 68.269 % | 95.45 % | 99.73 % | 68.269 % | 95.45 % | 99.73 % |
| Coordin ate System | | Solution Coordinate System | | | | | | | |
| Identifier | | | | | | | | | |
| Suppres sed | | No | | | | | | | |
| | | | | Res | ults | | | | |
| Minimu m | | | | | 0. m | | | | |
| Maximu m | 1.9488e- 005 m | 3.8976e- 005 m | 5.8464e- 005 m | 2.6555e- 002 m | 5.311e- 002 m | 7.9665e- 002 m | 1.2297e- 003 m | 2.4594e- 003 m | 3.689e- 003 m |
| Average | 4.0772e- 006 m | 8.1545e- 006 m | 1.2232e- 005 m | 2.1288e- 002 m | 4.2576e- 002 m | 6.3865e- 002 m | 2.2336e- 004 m | 4.4671e- 004 m | 6.7007e- 004 m |

4.4 RESPONSE PSD

The response power spectral density (PSD) was carried out in x, y and z taking any point of the bottom hole assembly, mainly the bit and the results are as below. From the results on figure4.5, figure 4.6 and figure 4.7 below of response PSD, it is shown that there is more resonance in all axis when drill pipe size increases which are lateral and longitudinal movements which causes lateral and longitudinal vibrations to the drill string. The resonance is more and not deterministic for a 60m DP than a 20m DP at shown in both lateral and longitudinal movements. There is lack of periodicity. Many frequencies may be excited at the same time. The resonance is shown by spikes in the figures below. The area under PSD response represents the Root Mean square (RMS) value. Table 7, 8 and 9 shows the root mean square values and the expected frequencies.

Table 7: RMS value and expected frequency in x direction (lateral) for 20m, 40m and 60mrespectively

| Res | ults | | |
|--------------------|------------------------------|--|--|
| RMS Value | 1.635e-003 m/s ² | | |
| RMS Percentage | 100. % | | |
| Expected Frequency | 39.654 Hz | | |
| Res | ults | | |
| RMS Value | 1.185e-003 m/s ² | | |
| RMS Percentage | 100.% | | |
| Expected Frequency | 48.443 Hz | | |
| Res | ults | | |
| RMS Value | 9.9422e-004 m/s ³ | | |
| RMS Percentage | 100. % | | |
| | | | |

Table 8: RMS value and expected frequency in y direction (longitudinal) for 20m, 40m and60m respectively

| Resul | ts |
|--------------------|-------------------------|
| RMS Value | 85.116 m/s ² |
| RMS Percentage | 100. % |
| Expected Frequency | 22.305 Hz |
| Resul | ts |
| RMS Value | 58.254 m/s ² |
| RMS Percentage | 100. % |
| Expected Frequency | 23.027 Hz |
| Result | ts |
| RMS Value | 47.205 m/s ² |
| RMS Percentage | 100. % |
| Expected Frequency | 22.491 Hz |

Table 9: RMS value and expected frequency in z direction (lateral) for 20m, 40m and 60mrespectivelyThe figure 4.5, figure 4.6 and figure 4.7 shows the RPSD in x-axis (lateral) for 20m, 40m and60m DP respectively

| Res | ults | | |
|--------------------|------------------------------|--|--|
| RMS Value | 6.7387e-003 m/s ² | | |
| RMS Percentage | 100. % | | |
| Expected Frequency | 54.741 Hz | | |
| Rest | ults | | |
| RMS Value | 2.9807e-003 m/s ² | | |
| RMS Percentage | 100.% | | |
| Expected Frequency | 47.234 Hz | | |
| Resu | ults | | |
| RMS Value | 8.6972e-003 m/s ² | | |
| RMS Percentage | 100. % | | |
| Expected Frequency | 46.571 Hz | | |
| | | | |

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Figure 4.7: Response PSD in x-axis (lateral) for 60m DP (amplitude(y) vs frequency (x))

The figure 4.8, figure 4.9 and figure 4.10 shows the RPSD in z-axis (lateral) for 20m, 40m and60m DP respectively:



Figure 4.8: Response PSD in z-axis (lateral) for 20m DP (amplitude(y) vs frequency (x))



Figure 4.9: Response PSD in z-axis (lateral) for 40m DP (amplitude(y) vs frequency (x))



Figure 4.10: Response PSD in z-axis (lateral) for 60m DP (amplitude(y) vs frequency (x))

The figure 4.11, figure 4.12 and figure 4.13 shows the RPSD in y-axis (longitudinal/axial)) for 20m DP, 40m and 60m DP respectively:



Figure 4.11: *RPSD in y-axis (longitudinal) for 20m DP (amplitude (y) vs frequency (x))*



Figure 4.12: *RPSD in y-axis (longitudinal) for 40m DP (amplitude(y) vs frequency (x))*



Figure 4.13: *RPSD in y-axis (longitudinal) for 60m DP (amplitude(y) vs frequency (x))*

5.0. CONCLUSIONS

Drilling string vibration is a phenomenon that has attracted great interest of researchers. Lateralvibrations are the most cause of failure in drill string vibrations. The shocks produced by lateralvibrations can be higher than those which result from longitudinal/axial vibrations. This is because under lateral vibrations, the drill string collides with the wellbore wall, creating huge shocks. Random vibrational analysis was performed to identify the frequency response of the drill string at any point mainly the bit. The probability of deformation for each sigma under random vibration was carried out. There is higher probability about 99.7% of failure in the axis of supplied PSD acceleration. The response power spectral density (PSD) was carried out in x, y and z taking any point of the bottom hole assembly, mainly the bit. From the results of response PSD, it is shown that there is higher value of resonance (spikes) as the DP length increases in both x-axis, y-axis and z-axis which are lateral and longitudinal movements which causes lateral and longitudinal vibrations to the drill string. The drill pipe size of 40m or between 20 and 60 is considered optimum in water well drilling to avoid resonance.

There is need for a further study with bit/rock interaction, mud friction boundaryconditions and including drilling fluids in the model. A good computer with big ram and processor is needed for more fine meshing in FEA.

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