ISSN: 0974-5823 Vol. 7 No. 7 July, 2022)

*International Journal of Mechanical Engineering*

# Construction and Selection of Multiple Deferred State Repetitive Group Sampling Inspection Plan for Variables

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***Abstract –***

The acceptance sampling inspection plan, one of the realistic inspection methodology in statistical quality control, includes explicit inspection procedures for quality assurance and thereby taking decision on sentencing the submitted lots of manufactured products, based on the quality of sampled units. The multiple deferred state sampling inspection plan, one of the conditional sampling plans, take advantage of the quality of current lot together with the forthcoming lots to reach a decision on the present lot, and with a smaller average sample number. The repetitive group sampling inspection plan, a special purpose sampling plan also helps to bring down the average sample number. This article proposes a Multiple Deferred State Repetitive Group Sampling inspection plan for variables when the quality characteristics follows a normal distribution. The proposed plan embodies the characteristics of multiple deferred state sampling plan for variables along with repetitive group sampling plan for variables. The optimum parameters of the proposed plan are attained by considering two points on the operating characteristic curve and guaranteeing the producer and consumer preservation under specific protection, when the standard deviation is known and unknown. The proposed plan has been compared with the existing sampling inspection plans.

*Index Terms* **-** Average Sample Number, Multiple Deferred State Repetitive Group Sampling Plan, Normal Distribution, Operating Characteristic Curve, Variable Sampling Plan.

## Introduction

At present scenario, the producer focuses on manufacturing the finest quality products so as to meet the consumers demand. When the manufactured products are destructive in nature or the method of inspection is costly or time consuming, a sampling inspection procedure generally called as acceptance sampling is preferred than a 100% inspection for ensuring the requisite quality. In acceptance sampling either attribute or variable quality characteristics can be assessed for judging a lot submitted for inspection. The variable sampling inspection plans are more economic than attribute sampling plans as it assures the same protection with relatively less sample size, and is more efficient in the sense that they provide more information regarding the sampled units. (Montgomery [4], Bowker and Goode [1]).

The conventional single sampling plans will accept the lots of best quality products and reject the lots with bad quality products; but not giving further chance to the lots of moderate quality products. Hence for the sake of products possessing moderate quality, the conditional sampling plans (or special purpose sampling plans) are utilized, which make use of the quality of nearby lots to reach a decision on the current lot, with comparably smaller sample size. One of the special purpose sampling plans, the multiple deferred state sampling inspection plan for attributes introduced by Wortham and Baker [3] (1976), depends on the upcoming lot quality to reach a decision on sentencing a lot of moderate quality. Vaerst [9] (1982) modified the operating procedure and characteristic function of multiple deferred state sampling plan to make it on a par with chain sampling (ChSP-1) plan. Further, following the operating procedure and characteristic function developed by Wortham and Baker, Soundararajan and Vijayaraghavan [14], Govindaraju and Subramani [6] proposed multiple deferred (dependent) state sampling plan for attributes by minimizing the sum of risks. Recently, Geetha and Raichel [13] designed the multiple deferred state sampling (MDSS) inspection plan for variables by minimizing average sample number (ASN). Furthermore, another special purpose sampling plan, the repetitive group sampling plan for attributes proposed by Sherman [10] (1965), also considers additional samples to reach a decision on disposition of a lot with products of moderate quality. Balamurali .et. al [11] proposed repetitive group sampling plan for variables by minimizing the ASN. Moreover, the multiple deferred state sampling plan and repetitive group sampling (RGS) plan can be implemented when the lots are submitted for inspection serially in the order of their production. The sampling inspection plan assimilating the characteristics of multiple deferred state sampling plan and repetitive group sampling plan are the multiple deferred (dependent) state repetitive group sampling inspection plans. Balamurali, Jeyadurga and Usha [12] designed multiple deferred (dependent) state repetitive group sampling (MDSRGS) plan for attributes assuring mean life under Weibull and Gamma distributions and compared it with Birnbaum Saunders distribution. Navjeeth Singh, Anju Sood and Buttar [7] developed a MDSRGS plan for attributes for Inverse Weibull distribution based on life test. Kannan, Jeyadurga and Balamurali [5] proposed MDSRGS plan for attributes assuring mean and median life time of products under Birnbaum- Saunders distribution. Jeyadurga and Balamurali [8] recommended MDSRGS plan for variables by minimizing average of average sample number. The MDSRGS plan for attributes as well as for variables in the literature pursue the operating procedure and characteristic function of multiple deferred (dependent) state sampling plan modified by Vaerst (1982).

In this article we propose the construction of a Multiple Deferred State Repetitive Group Sampling Inspection plan for variables when the quality characteristics following a normal distribution, with the operating procedure and characteristic function in accordance with Wortham and Baker (MDSSRGS). A comparison of the suggested plan with respect to the ASN is carried out.

**Multiple Deferred State Repetitive Group Sampling Plan for Variables.**

### 2.1. Basic Assumptions for Implementation of the Plan

The basic assumptions for implementing the proposed plan are as follows:

1. The production is steady so that the results of current and future lots are largely indicative of a continuing process.
2. Lots are submitted serially in the order of their production.
3. The operating characteristic (OC) curves are obtained for inspection by variables with stable quality between lots.
4. The characteristics under consideration follows normal distribution.

### 2.2. Operating Procedures

Suppose the quality characteristics of interest be normally distributed with unknown mean $\mu\mu$ and unknown standard deviation. Let the upper specification limit be $UU$, specified. The proposed variable multiple deferred state repetitive group sampling plan performs as follows.

1. From each lot set forth for inspection, draw a random sample of $nn$ units. Record the measurement on each inspected unit in the sample, ($x_{1,}\ x_2, \dots\ x_n) x_{1,}\ x_2, \dots\ x_n)$, say.
2. Calculate the statistic $v = \frac{U-\bar{x}}{S} v = \frac{U-\bar{x}}{S}$ ,where $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ and $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$, is an unbiased estimate of $\sigma^2\sigma^2$.
3. Accept the present lot for $v \geq k_a v \geq k_a$. Reject or discard the present lot for $v < k_r v < k_r$

Defer the decision on the present lot up to successive $mm$ lots, for $k_r \leq v < k_a\, k_r \leq v < k_a$; and if the future $mm$ lots are all accepted, the current lot is accepted.

If any of the future $mm$ lots are rejected, repeat sampling until a decision is attained on the present lot.

Thus the suggested variable multiple deferred state repetitive group sampling plan is characterized by four parameters, namely, $nn$ – the sample size, $k_a k_a$ – the unconditional acceptability constant, $k_r k_r$ – the conditional acceptability constant and $mm$ – the number of upcoming lots for conditional acceptance.

If the lower specification limit, $LL$ is specified, the proposed plan executes as same as above with a corresponding change in the required statistic in step 2 as:

$$v = \frac{\bar{x} - L}{S}$$

When standard deviation is known, the proposed multiple deferred state repetitive group sampling plan for variables performs as the same as that of unknown standard deviation plan, with a corresponding change in the statistic as $v = \frac{U-\bar{x}}{\sigma} v = \frac{U-\bar{x}}{\sigma}$ and the plan parameters may be replaced by $(n', k_a{}', k_r{}', m')(n', k_a{}', k_r{}', m')$.

### 2.3. Performance Measures

The sampling inspection plans look over the fulfillment of the requisite quality characteristics of the products submitted for inspection, which is mainly achieved by examining the performance measures. A distinguished tool used for discriminating the performance of any sampling inspection plan is the operating characteristic (OC) function, which gives the probability of acceptance of a lot $P_a(p) P_a(p)$, as a function of incoming lot quality, $pp$. The OC curve is drawn by plotting $P_a(p) P_a(p)$ against $pp$. The expected number of samples inspected for different incoming quality is the average sample number (ASN), a function of lot quality, $pp$. The curve obtained by plotting ASN against $pp$ is the ASN curve. The corresponding OC function and ASN function of the proposed multiple deferred state repetitive group sampling plan for standard deviation unknown as well as standard deviation known cases are mentioned respectively as follows.

#### 2.3.1. Multiple Deferred State Repetitive Group Sampling Plan when Standard Deviation is Unknown

For a pre fixed upper specification limit, $U, U,$ the lot quality is defined as

$$p = P\{X > U|\mu\} = 1 - \Phi\left(\frac{U-\mu}{S}\right)\; p = P\{X > U|\mu\} = 1 - \Phi\left(\frac{U-\mu}{S}\right) \qquad (1)$$

Where $\Phi(.)\Phi(.)$ is the cumulative distribution function of standard normal random variable.

The operating characteristic function of the proposed unknown sigma plan is obtained based on the sampling distribution of $\bar{x} \pm k_a S\, \bar{x} \pm k_a S$. According to Duncan [2] (1986), $\bar{x} \pm k_a S \bar{x} \pm k_a S$ is asymptotically normal with $mean = \mu \pm k_a \sigma mean = \mu \pm k_a \sigma$ and $variance = \frac{\sigma^2}{n}\left(1 + \frac{k_a{}^2}{2}\right) variance = \frac{\sigma^2}{n}\left(1 + \frac{k_a{}^2}{2}\right)$.

i.e., $\bar{x} \pm k_a S \sim N\left(\mu \pm k_a \sigma, \frac{\sigma^2}{n}\left(1 + \frac{k_a{}^2}{2}\right)\right) \bar{x} \pm k_a S \sim N\left(\mu \pm k_a \sigma, \frac{\sigma^2}{n}\left(1 + \frac{k_a{}^2}{2}\right)\right)$

The operating characteristic function of the proposed plan is as follows

$$P_a(p) = \frac{P\{v \geq k_a|p\} + P\{k_r \leq v < k_a|p\}[P_a(p)]^m}{1 - P\{k_r \leq v < k_a|p\}[1-[P_a(p)]^m]} P_a(p) = \frac{P\{v \geq k_a|p\} + P\{k_r \leq v < k_a|p\}[P_a(p)]^m}{1 - P\{k_r \leq v < k_a|p\}[1-[P_a(p)]^m]} \qquad (2)$$

where, $P\{v \geq k_a|p\} P\{v \geq k_a|p\}$ is the unconditional probability of acceptance based on single sample

$P\{k_r \leq v < k_a|p\} P\{k_r \leq v < k_a|p\}$ is the conditional probability that the decision on current lot will be

deferred till the decision on forthcoming $mm$ lots.

Based on the sampling distribution of $\bar{x} \pm k_a S \bar{x} \pm k_a S$, the RHS of equation (2) can be expressed as

$$P_a(p) = \frac{\Phi(w_1)+[\Phi(w_2)-\Phi(w_1)][P_a(p)]^m}{1-[\Phi(w_2)-\Phi(w_1)][1-[P_a(p)]^m]} P_a(p) = \frac{\Phi(w_1)+[\Phi(w_2)-\Phi(w_1)][P_a(p)]^m}{1-[\Phi(w_2)-\Phi(w_1)][1-[P_a(p)]^m]} \quad (3)$$

By a similar argument used to attain the OC function, the ASN can be expressed as

$$ASN(p) = \frac{n}{1-[\Phi(w_2)-\Phi(w_1)][1-[P_a(p)]^m]} \; ASN(p) = \frac{n}{1-[\Phi(w_2)-\Phi(w_1)][1-[P_a(p)]^m]} \quad (4)$$

$$w_1 = \left(\frac{U-\mu}{\sigma} - k_a\right)\sqrt{\frac{n}{1+\frac{k_a{}^2}{2}}} = \left(Z_p - k_a\right)\sqrt{\frac{n}{1+\frac{k_a{}^2}{2}}}$$

where,

$$w_1 = \left(\frac{U-\mu}{\sigma} - k_a\right)\sqrt{\frac{n}{1+\frac{k_a{}^2}{2}}} = \left(Z_p - k_a\right)\sqrt{\frac{n}{1+\frac{k_a{}^2}{2}}} \quad (5)$$

$$w_2 = \left(\frac{U-\mu}{\sigma} - k_r\right)\sqrt{\frac{n}{1+\frac{k_r{}^2}{2}}} = \left(Z_p - k_r\right)\sqrt{\frac{n}{1+\frac{k_r{}^2}{2}}}$$

$$w_2 = \left(\frac{U-\mu}{\sigma} - k_r\right)\sqrt{\frac{n}{1+\frac{k_r{}^2}{2}}} = \left(Z_p - k_r\right)\sqrt{\frac{n}{1+\frac{k_r{}^2}{2}}} \quad (6)$$

For specific $AQLAQL$ and LQL, the lot acceptance probabilities and ASN can be expressed respectively as:

$$P_a(p_1) = \frac{\Phi(w_{11}) + [\Phi(w_{21}) - \Phi(w_{11})][P_a(p_1)]^m}{1 - [\Phi(w_{21}) - \Phi(w_{11})][1 - [P_a(p_1)]^m]}$$

$$P_a(p_1) = \frac{\Phi(w_{11})+[\Phi(w_{21})-\Phi(w_{11})][P_a(p_1)]^m}{1-[\Phi(w_{21})-\Phi(w_{11})][1-[P_a(p_1)]^m]} \quad (7)$$

$$P_a(p_2) = \frac{\Phi(w_{12})+[\Phi(w_{22})-\Phi(w_{12})][P_a(p_2)]^m}{1-[\Phi(w_{22})-\Phi(w_{12})][1-[P_a(p_2)]^m]} P_a(p_2) = \frac{\Phi(w_{12})+[\Phi(w_{22})-\Phi(w_{12})][P_a(p_2)]^m}{1-[\Phi(w_{22})-\Phi(w_{12})][1-[P_a(p_2)]^m]} \quad (8)$$

$$ASN(p_1) = \frac{n}{1-[\Phi(w_{21})-\Phi(w_{11})][1-[P_a(p_1)]^m]} ASN(p_1) = \frac{n}{1-[\Phi(w_{21})-\Phi(w_{11})][1-[P_a(p_1)]^m]} \quad (9)$$

$$ASN(p_2) = \frac{n}{1-[\Phi(w_{22})-\Phi(w_{12})][1-[P_a(p_2)]^m]} ASN(p_2) = \frac{n}{1-[\Phi(w_{22})-\Phi(w_{12})][1-[P_a(p_2)]^m]} \quad (10)$$

where, $w_{11}w_{11}$ is the value of $w_1w_1$ at $p = AQL\ (or\ p_1) p = AQL\ (or\ p_1)$ , $w_{21}w_{21}$ is the value of $w_2w_2$ at $p = AQLp = AQL$, $w_{12}w_{12}$ is the value of $w_1w_1$ at $p = LQL\ (or\ p_2) p = LQL\ (or\ p_2)$ , $w_{22}w_{22}$ is the value of $w_2w_2$ at $p = LQLp = LQL$

### 2.3.2. Multiple Deferred State Repetitive Group Sampling Plan when Standard Deviation is known

The fraction non - conforming is defined as,

$$p = P\{X > U|\mu\} = 1 - \Phi\left(\frac{U-\mu}{\sigma}\right) \; p = P\{X > U|\mu\} = 1 - \Phi\left(\frac{U-\mu}{\sigma}\right) \quad (11)$$

The operating characteristic function can be obtained as

$$P_a(p) = \frac{\Phi(y_1)+[\Phi(y_2)-\Phi(y_1)][P_a(p)]^{m'}}{1-[\Phi(y_2)-\Phi(y_1)]\left[1-[P_a(p)]^{m'}\right]} P_a(p) = \frac{\Phi(y_1)+[\Phi(y_2)-\Phi(y_1)][P_a(p)]^{m'}}{1-[\Phi(y_2)-\Phi(y_1)]\left[1-[P_a(p)]^{m'}\right]} \quad (12)$$

The average sample number can be expressed as

$$ASN(p) = \frac{n'}{1-[\Phi(y_2)-\Phi(y_1)]\left[1-[P_a(p)]^{m'}\right]} ASN(p) = \frac{n'}{1-[\Phi(y_2)-\Phi(y_1)]\left[1-[P_a(p)]^{m'}\right]} \quad (13)$$

where, $y_1 = (Z_p - k_a')\sqrt{n'}$ $y_1 = (Z_p - k_a')\sqrt{n'}$ (14)

$$y_2 = (Z_p - k_r')\sqrt{n'}\ y_2 = (Z_p - k_r')\sqrt{n'} \quad (15)$$

Now, the lot acceptance probabilities and ASN at AQL and LQL can be expressed respectively as:

$$P_a(p_1) = \frac{\Phi(y_{11})+[\Phi(y_{21})-\Phi(y_{11})][P_a(p_1)]^{m'}}{1-[\Phi(y_{21})-\Phi(y_{11})]\left[1-[P_a(p_1)]^{m'}\right]} P_a(p_1) = \frac{\Phi(y_{11})+[\Phi(y_{21})-\Phi(y_{11})][P_a(p_1)]^{m'}}{1-[\Phi(y_{21})-\Phi(y_{11})]\left[1-[P_a(p_1)]^{m'}\right]}$$

$$P_a(p_1) = \frac{\Phi(y_{11})+[\Phi(y_{21})-\Phi(y_{11})][P_a(p_1)]^{m'}}{1-[\Phi(y_{21})-\Phi(y_{11})]\left[1-[P_a(p_1)]^{m'}\right]}$$

$$P_a(p_1) = \frac{\Phi(y_{11})+[\Phi(y_{21})-\Phi(y_{11})][P_a(p_1)]^{m'}}{1-[\Phi(y_{21})-\Phi(y_{11})]\left[1-[P_a(p_1)]^{m'}\right]}$$

$$P_a(p_2) = \frac{\Phi(y_{12})+[\Phi(y_{22})-\Phi(y_{12})][P_a(p_2)]^{m'}}{1-[\Phi(y_{22})-\Phi(y_{12})]\left[1-[P_a(p_2)]^{m'}\right]}$$

(16)

$$P_a(p_2) = \frac{\Phi(y_{12})+[\Phi(y_{22})-\Phi(y_{12})][P_a(p_2)]^{m'}}{1-[\Phi(y_{22})-\Phi(y_{12})]\left[1-[P_a(p_2)]^{m'}\right]} \quad (17)$$

$$ASN(p_1) = \frac{n'}{1-[\Phi(y_{21})-\Phi(y_{11})]\left[1-[P_a(p_1)]^{m'}\right]} ASN(p_1) = \frac{n'}{1-[\Phi(y_{21})-\Phi(y_{11})]\left[1-[P_a(p_1)]^{m'}\right]} \quad (18)$$

$$ASN(p_2) = \frac{n'}{1-[\Phi(y_{22})-\Phi(y_{12})]\left[1-[P_a(p_2)]^{m'}\right]} ASN(p_2) = \frac{n'}{1-[\Phi(y_{22})-\Phi(y_{12})]\left[1-[P_a(p_2)]^{m'}\right]} \quad (19)$$

Where, $y_{11}y_{11}$ is the value of $y_1y_1$ at $= AQL\ (or\ p_1) = AQL\ (or\ p_1)$ , $y_{21}y_{21}$ is the value of $y_2y_2$ at $p = AQLp = AQL$, $y_{12}y_{12}$ is the value of $y_1y_1$ at $p = LQL\ (or\ p_2)p = LQL\ (or\ p_2)$ , $y_{22}y_{22}$ is the value of $y_2y_2$ at $p = LQLp = LQL$

**2.4. Designing of Variable Multiple Deferred State Repetitive Group Sampling Plan**

The optimal plan parameters are achieved by adopting the two point approach on the OC curve through a search procedure by minimizing the average of ASN at AQL and ASN at LQL. A sampling plan which ensures the producer and consumer conservation with minimum average sample number is preferable, as it is more economical. Accordingly the problem for evaluating the plan parameters are formulated as follows:

For prefixed values of $mm$ (or $m'm'$), $(AQL, 1-\alpha)AQL, 1-\alpha)$ and $(LQL,\beta)(LQL,\beta)$

Minimize $ASN(p) = \frac{1}{2}[ASN(P_1) + ASN(P_2)]ASN(p) = \frac{1}{2}[ASN(P_1) + ASN(P_2)]$

Subject to

$$P_a(p_1) \geq 1-\alpha P_a(p_1) \geq 1-\alpha$$

$$P_a(p_2) \leq \beta P_a(p_2) \leq \beta \quad (20)$$

$n(or\ n') \geq 1n(or\ n') \geq 1$ , $k_a(or\ k_a') > k_r(or\ k_r') > 0k_a(or\ k_a') > k_r(or\ k_r') > 0n(or\ n') \geq 1$,

$k_a(or\ k_a') >$
$k_r(or\ k_r') > 0$

where, $P_a(p_1)P_a(p_1)$ , $P_a(p_2)P_a(p_2)$ , $ASN(P_1)ASN(P_1)$ and $ASN(P_2)ASN(P_2)$ are defined respectively in equations (7), (8), (9) and (10) ;when standard deviation is unknown.

When standard deviation is known, the acceptance probabilities as well as ASN at AQL and LQL can be obtained from equations (16), (17), (18) and (19) respectively.

**3. Application of the proposed plan**

The following example is used to illustrate the implementation of the proposed variable multiple deferred state repetitive group sampling inspection plan. Suppose the desired quality standards and associated risks are respectively specified as $AQL = 0.001\, AQL = 0.001$, $LQL = 0.006\, LQL = 0.006$, $\alpha = 0.05\ \alpha = 0.05$, and $\beta = 0.10\beta = 0.10$. Let standard deviation be unknown and $m = 2\ m = 2$. The optimal plan parameters can be obtained as: $n = 36n = 36$, $k_a = 3.18k_a = 3.18$ and $k_r = 2.52k_r = 2.52$

The proposed plan executes as follows:

Step:1 From lot A, a sample of 36 units is drawn and inspected for the requisite quality characteristics. Measure the quality characteristics and record it. Obtain the sample

mean $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ and $S = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}\ S = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$

Step 2: For specific upper specification limit, U (or lower specification limit, L). Evaluate the statistic $v = \frac{U-\bar{x}}{S}$ $v = \frac{U-\bar{x}}{S}$ or $\frac{\bar{x}-L\bar{x}-L}{S\quad S}$

Step 3: If $v \geq 3.18 v \geq 3.18$, accept lot A. If $v < 2.52 v < 2.52$, reject lot A. If $2.52 \leq v < 3.18$ $2.52 \leq v < 3.18$, defer the decision on lot A until the next 2 lots , say lot B and lot C are both accepted. If any one of the lot B or lot C is rejected, then take another sample of 36 units from lot A and continue the procedure till we reach a decision on lot A.

**4. Comparison of Sampling Plans**

The efficiency of the proposed plan is studied through its performance measures. The performance measures of the proposed variable multiple deferred state repetitive group sampling (MDSSRGS) inspection plan is compared with that of variable MDSRGS plan (proposed by Balamurali and Jeyadurga, 2020) and variable RGS plan (proposed by Balamurali .et.al., 2005). The discriminating power of the proposed plan regarding producer and consumer conservation at desired protection is measured by drawing the OC curves of the corresponding plans.

In particular, for specified protection at $AQL = 0.03 AQL = 0.03$, $\alpha = 5\%\alpha = 5\%$, $LQL = 0.06\ LQL = 0.06$ and $\beta = 10\%\beta = 10\%$, the proposed plan parameters for unknown standard deviation can be obtained as:

- $n = 56n = 56$ , $k_a = 2.22k_a = 2.22$ and $k_r = 1.55k_r = 1.55$; when $m = 1m = 1$
- $n = 57n = 57$, $k_a = 1.93k_a = 1.93$ and $k_r = 1.55k_r = 1.55$; when $m = 2m = 2$
- $n = 57n = 57$ , $k_a = 1.92k_a = 1.92$ and $k_r = 1.55k_r = 1.55$; when $m = 3m = 3$

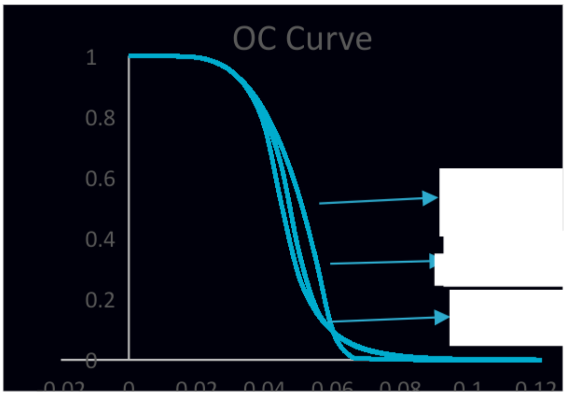


FIGURE 1: OC CURVE FOR PROPOSED VARIABLE MULTIPLE DEFERRED STATE REPETITIVE GROUP SAMPLING PLAN AT $m = 1,\ \ m = 2,\ \ and\ \ m = 3 m = 1,\ \ m = 2,\ \ and\ \ m = 3$
$\boldsymbol{m = 1,\ \ m = 2,\ \ and\ \ m = 3}$

Now for the same protection of $(AQL, 1-\alpha)(AQL, 1-\alpha)$ and $(LQL, \beta)(LQL, \beta)$ and when $m = 1m = 1$, the optimal plan parameters of variable RGS plan and variable MDSRGS plans for unknown standard deviation are respectively as follows:

- $n = 79n = 79$, $k_a = 1.84k_a = 1.84$ and $k_r = 1.57k_r = 1.57$
- $n = 68n = 68$, $k_a = 1.92k_a = 1.92$ and $k_r = 1.55k_r = 1.55$

In this case, it is observed that the proposed variable multiple deferred state sampling repetitive group sampling (MDSSRGS) unknown sigma plan ($n = 56n = 56$, $k_a = 2.22k_a = 2.22$, $k_r = 1.55k_r = 1.55$; when $m = 1m = 1$) have comparatively lesser sample size than existing RGS and MDSRGS plans for variables. The OC curve is drawn in Figure2 for these three sampling inspection plans in order to examine the performance of the proposed plan in discriminating the lot submitted for inspection in respect of the incoming lot quality.

**PAPER LENGTH**

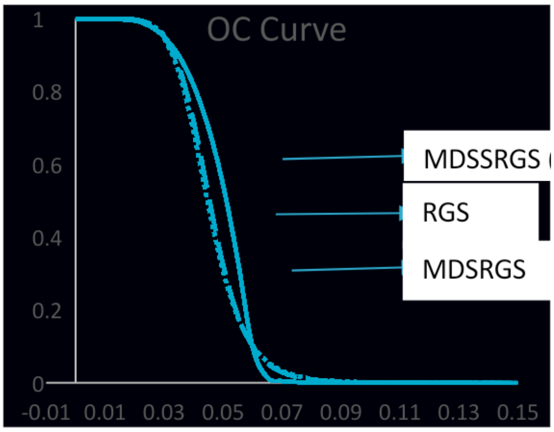


FIGURE 2: OC CURVE FOR VARIABLE MDSRGS (PROPOSED), VARIABLE RGS AND VARIABLE MDSRGS (JEYADURGA AND BALAMURALI)

From the OC curve it is clear that the proposed variable MDSSRGS plan ensures better producer and consumer conservation together with comparably smaller sample size as well as ASN.

Further we compare the ASN of the proposed MDSSRGS plan with variable RGS plan (Balamurali. et.al, 2005) and variable MDSRGS plan (Jeyadurga and Balamurali, 2020). Table1 is constructed for the ASN values of the aforementioned three plans at different combination of AQL and LQL, when standard deviation is known and unknown.

TABLE: 1 ASN VALUES OF VARIABLE RGS PLAN, VARIABLE MDSRGS PLAN AND VARIABLE MDSSRGS PLAN (PROPOSED)

| | | **Average Sample Number** | | | | | |
|---|---|---|---|---|---|---|---|
| | | Known Standard Deviation | | | Unknown Standard Deviation | | |
| **AQL** | **LQL** | RGS | MDS RGS | MDSS RGS (proposed) | RGS | MDS RGS | MDSS RGS (proposed) |
| **0.001** | 0.004 | 30.256 | 25.300 | 20.394 | 695.16 | 579.890 | 92.898 |
| **0.005** | 0.012 | 57.189 | 47.886 | 39.183 | 219.76 | 183.995 | 134.911 |
| **0.030** | 0.080 | 25.667 | 21.432 | 17.205 | 58.78 | 49.286 | 34.493 |
| **0.040** | 0.100 | 26.393 | 22.086 | 18.524 | 55.32 | 46.461 | 32.827 |
| **0.050** | 0.120 | 26.310 | 22.001 | 17.548 | 51.20 | 43.041 | 30.171 |

From the table it is clear that the proposed variable MDSSRGS plan is efficient regarding ASN when compared to other two plans. For example, when standard deviation is known and $AQL = AQL =$0.005 and $LQL = 0.012 LQL = 0.012$, the ASN values for the proposed variable MDSSRGS plan, variable MDSRGS plan (Jeyadurga and Balamurali) variable RGS plan (Balamurali.et.al.) and can be observed from Table1 as 39.183, 47.886 and 57.189 respectively. For the same protection when standard deviation is unknown, a comparably larger ASN values can be obtained from Table1. In this instance when standard deviation is known as well as unknown the ASN value is minimum for the proposed plan. Also ASN curves are drawn in Figure 3 for the proposed variable MDSSRGS plan, variable MDSRGS plan (Jeyadurga and Balamurali) and variable RGS plan when standard deviation is unknown, with the specific protection at $AQL = 0.001 AQL = 0.001$, $\alpha = 0.05 \alpha = 0.05$, $LQL = 0.004 LQL = 0.004$ and $\beta = 0.10 \beta = 0.10$ and for $m = 1 m = 1$ .

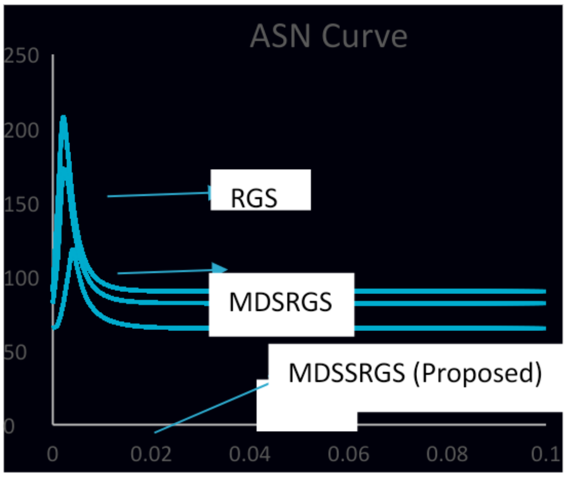


Figure 3: ASN curve for proposed MDSSRGS plan, variable MDSRGS plan (Jeyadurga and Balamurali, 2020) and variable RGS plan.

It can be observed that at any quality level the ASN is minimum for the proposed plan, and hence reduces the inspection time and cost of inspection. So the proposed plan can be implemented for industrial purposes.

## 5. Conclusion

A Multiple Deferred State Sampling Repetitive Group Sampling Plan for variables is recommended in this article for the acceptance or non-acceptance of lots of manufactured products, whose quality characteristics following a normal distribution. The proposed plan provides a better protection for producer as well as consumer together with smaller sample sizes. Moreover, the ASN of the suggested sampling inspection plan is comparably smaller than the existing variable RGS plan and variable MDSRGS plan (Jeyadurga and Balamurali). Hence, the proposed plan is an alternative to the existing conventional sampling plans, as it is more economical.

## References


[1] A. H Bowker and H. P. Goode, "Sampling Inspection by Variables", McGraw-Hill, New York, NY, USA,1952.

[2] A.J. Duncan, "Quality Control and Industrial Statistics", fifth ed., Richard D Irwin, Homewood, Illinois, 1986.

[3] A.W. Wortham, and R. C. Baker, "Multiple Deferred State Sampling Inspection", The International Journal of Production Research, Vol.14, No.6, 719-731, Int. J. Prod. Res., 1976. http://dx.doi.org/10.1080/00207547608956391

[4] D. C. Montgomery, Introduction to Statistical Quality Control, John Wiley & Sons, New York, NY, USA, 2004.

[5] G. Kannan, P. Jeyadurga, S. Balamurali, "Determination of multiple deferred state repetitive group Sampling Plan for Life Time Assurance under Birnbaum–Saunders Distribution", Journal of Statistical Computation and Simulation, 2021. https://doi.org/10.1080/00949655.2021.1934838

[6] K. Govindaraju, K. Subramani, "Selection of Multiple Deferred (Dependent) State Sampling Plans for given Acceptable Quality Level and Limiting Quality Level", Journal of Applied Statistics, Vol.20, No. 3, J. Appl.Stat., 1993. http://dx.doi.org/10.1080/02664769300000041

[7] Navjeet Singh , Anju Sood, G.S Buttar, "Design of Multiple Deferred State Repetitive Group Sampling Plan for Inverse Weibull Distribution Based on Life Test", International Journal of Scientific Research and Review, Volume 07, Issue 03, March 2019. ISSN No.: 2279-543X

[8] P. Jeyadurga, S. Balamurali, "Optimal Designing of Multiple Deferred (Dependent) State Repetitive Group Sampling Plan for Variables Inspection", Communications in Statistics - Theory and Methods, 2020 . https://doi.org/10.1080/03610926.2020.1814815

[9] R. Vaerst, "A Procedure to Construct Multiple Deferred State

Sampling Plan", Methods of Operations Research, Vol.37, pp.477-485, Math Methods Oper Res.,1982.

[10] Robert. E. Sherman, "Design and Evaluation of a Repetitive Group Sampling Plan", American Society for Quality, Technometrics, Vol. 7, No. 1, pp. 11-21, 1965. http://www.jstor.org/stable/1266124

[11] S. Balamurali, Heekon Park , Chi-Hyuck Jun , Kwang-Jae Kim & Jaewook Lee, "Designing of Variables Repetitive Group Sampling Plan Involving Minimum Average Sample Number", Communications in Statistics - Simulation and Computation, 34:3, 799-809, 2005. http://dx.doi.org/10.1081/SAC-200068424

[12] S. Balamurali, P. Jeyadurga, M. Usha, "Optimal Design of Repetitive Group Sampling Plans for Weibull and Gamma Distributions with Applications and Comparison to the Birnbaum–Saunders Distribution", Journal of Applied Statistics, https://doi.org/10.1080/02664763.2018.1426740

[13] S Geetha and Raichel Mathew, "Design of Multiple Deferred State Sampling Inspection Plan for Variables", Seventh International Conference on Statistics for Twenty-first Century-2021, University of Kerala, Trivandrum, 15 – 19, December, 2021

[14] V. Soundararajan and R. Vijayaraghavan, "On Designing Multiple Deferred State Sampling (MDS-1(0, 2)) Plans Involving Minimum Risks, Journal of Applied Statistics, 16:1, 87-94, J. Appl. Stat.,1989 http://dx.doi.org/10.1080/02664768900000010