

Free Integrated Heat Dissemination through Radiating MHD Casson Fluid via Stretched Surface Thick Parameter

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Abstract

Through means of this specific paper, an endeavor is made towards dealing with “Free Integrated Heat Dissemination in a 2D Magneto Hydrodynamic stream of Casson liquid over a Non-uniform thickness extending sheet within the sight of Heating radiation alongside Non-uniform source/sink of Heat”, particularly. In addition, various administering conditions pertaining to Dissemination of Heat & flow, will likely be transformed into resource of non-linear “ODEs and settled mathematically utilizing *bvp4c* package”, specifically. Further, impact of appropriate boundaries, to be specific, “attractive field boundary, Casson boundary, Heat radiation boundary, Non-uniform Heat source/sink boundaries”, pertaining towards heat as well as flow dissemination will thereby be worked upon through means of mathematical applications, including diagrams as well. Thus, through means of such application, various relevant outcomes are figured for subsequent grinding factor alongside diminished “Nusselt number”, from where it shall be concluded that radiative properties of Heat, results in improving temperature aspects of “Casson liquid”, respectively.

1. Introduction

As of late, works undertaken on “Non-Newtonian liquids” are altogether improved inferable from their colossal practical usage into aspects of “science as well as designing”. As a result, it is formulated that, width of these “non-Newtonian liquids, gradually turns on shear rate, wherein stuffs of daily use like- paints, shampoos, blood, and tooth glue are examples of such non-Newtonian liquids”. Although, the prescribed comprising conditions are exceptionally non-linear for these liquids, as there isn't any model in writing, which tends to portray each resource of such “Non-Newtonian liquids”. Thus, on considering prompted impact on attractive field of peristaltic transport relating to Carreau Liquid, was specifically examined by “Hayat et al.” [1]. Similarly, “Heat radiation impact on free convective nanofluid stream over an upward plate was hypothetically researched, through works of Sandeep et al.” [2]. Further, “Sulochana and Sandeep [3] undertook work on ascertaining Heat & flow vehicle in MHD dusty stream over an extending/contracting chamber by thinking about the different temperatures, whereas Cattaneo-Christov heat motion model for stream of variable warm conductivity summed up through Burgers liquid was specifically being studied by Waqas”[4], respectively. Moreover, study on altered attractive field impact pertaining to “MHD nanofluid stream” was thereby being undertaken by “Sandeep”[5].

In a relative manner, hypothetical examination of “Magneto hydrodynamic nanofluid stream inserted with the magnetite nanoparticles, was basically undertaken by Sandeep et al”[6]. Further, “Jayachandra Babu and Sandeep, [7] considered work on Upper form of Maxwell liquid stream over a dissolving surface with twofold definition, whereas examination of limit layer framed on an upper level surface of a paraboloid of upset inside a nano-fluid stream within the sight of thermophoresis and Brownian dissemination, was particularly being concentrated by Koriko et. al.”[8]. In addition, “Magneto hydrodynamic Oldroyd-B liquid stream with Soret and DuFour impacts was mathematically investigated by Sandeep and Ganeswara Reddy” [9], while “Mohan Krishna et al. [10] explored the allegorical progression of MHD Carreau liquid involving lightness impacts”. Likewise, the frictional warming impact onto “ferro-fluid stream over a criticizing sheet was accounted for by Ramana Reddy et al”.[11]. Thereafter, “MHD stagnation stream of Casson liquid over an extending surface with Integrated limiting conditions was mathematically concentrated by Ibrahim and Makinde” [12], whereas, “Hayat et al. [13], undertook work on examining 3D limit layer stream of Sisko nanofluid through means of cross over attractive field impact”, respectively.

Moreover, blended convectional stream of “Maxwell nanofluid within the sight of Heating age alongside assimilation was concentrated by Abbasi et al. [14], while 3-dimensional progression of radiative nanofluid with attractive field as well as internal source heating impacts was being worked upon by Abbasi et al.” [15]. Moving ahead, “Hayat et al. [16] considered the synthetic response impacts on radiative MHD stream, wherein specific technical outcomes pertaining to Heat & mass dissemination of blended hydrodynamic/warm slip stream over an extending sheet was thereby studied by Turkyilmazoglu” [17]. Double answers for unstable blended convection stream of miniature polar liquid over an extending and contracting sheet was concentrated by Sandeep and Sulochana [18]. Followed by this, impacts of radiation on “MHD convective stream over a porous extending surface with attractions & heating effect was explored by Mohan Krishna et al”. [19]. Comparatively, various other specialists, [20-23] considered study on “Heat dissemination via attractive non-Newtonian liquid streams over different stream calculations, wherein

resulting impact of Thermophoresis and Brownian second on nanofluid stream over a microchannel was specifically being focused upon by Fani et al". [24]. Thereafter, limiting layered flows pertaining to nano-fluids past an stretched upon sheet involving an integrated limiting conditions was particularly analyzed by "Makinde and Aziz" [25]. Lately, study on "MHD three-dimensional limit layering stream of Casson nanofluid past a straight sheet having convectational limiting condition, was given by Nadeem et al". [26], followed by works of various specialists [27-30], in relation to MHD flows.

2. Formulating of concerned problem

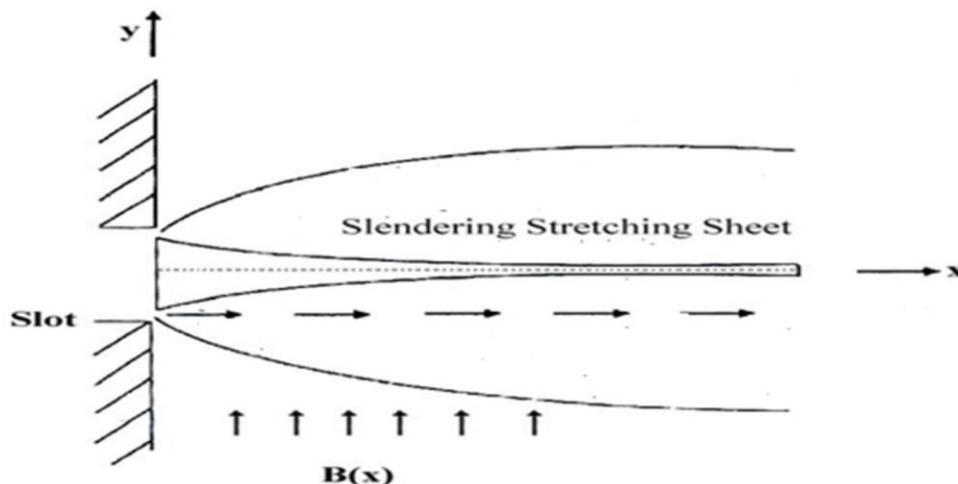


Image.1 Tangible Model

On taking into consideration, the stable "2D flow of Casson fluid through means of stretched sheet pertaining to variable width in such a way that the x-axis is along the sheet and y-axis is specifically appeared as to be perpendicular to it". As a result, it's being formulated in a manner wherein, $= A(x + b)^{\frac{1-m}{2}} u_w(x) = (x + b)^m U_0, v_w = 0, m \neq 1$. Hence, for our work, radiating as well as "non-uniform Heat source/sink impacts" are specifically being focused upon, in addition to "transverse attracting field of solidarity By is forced along the subsequent flow", which is thereby portrayed through means of Image.1, respectively. As a result, based on stated conditions, various set of expressions that are likely being used are given as under-

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = v \left(1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2(x)}{\rho} u, \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial x^2} + \frac{1}{\rho c_p} \frac{16\sigma^* T_\infty^3}{3\rho c_p k^*} \frac{\partial^2 T}{\partial y^2} + \frac{1}{\rho c_p} q''', \quad (3)$$

Representing conditions as-

$$u = U_w(x) + h_1^* \left(\frac{\partial u}{\partial y} \right), v = 0, T = T_w(x) + h_2^* \left(\frac{\partial T}{\partial y} \right), \quad (4,5)$$

and $u(\infty) = 0, T(\infty) = T_\infty$,

wherein,

$$h_1^* = \left[\frac{2 - f_1}{f_1} \right] \xi_1 (x + b)^{\frac{1-m}{2}}, \xi_2 = \left(\frac{2\gamma}{\gamma + 1} \right) \frac{\xi_1}{Pr}, h_2^* = \left[\frac{2 - a}{a} \right] \xi_2 (x + b)^{\frac{1-m}{2}},$$

$$B(x) = B_0 (x + b)^{\frac{m-1}{2}}, T_w - T_\infty = T_0 (x + b)^{\frac{1-m}{2}} \quad (6)$$

In equation $q''' = \frac{(T_w - T_\infty) k u_w}{xv} \left(A^* f' + B^* \frac{(T - T_\infty)}{(T_w - T_\infty)} \right)$ represents "non-uniform Heat source/sink parameter", considerably.

Thus, "similarity transformations" are thereto being highlighted as-

Wherein, we shall be suggesting about the presence of underlying conditions-

$$\Psi = f(\eta) \left(\frac{2}{m+1} v U_0 (x+b)^{m+1} \right)^{0.5}, \eta = y \left(\frac{m+1}{2} U_0 \frac{(c+b)^{m-1}}{v} \right)^{0.5}, \theta(T_w(x) - T_\infty) = T - T_\infty \quad (7)$$

Such that, Ψ is likely represented as $u = \frac{\partial \Psi}{\partial y}$ and $v = -\frac{\partial \Psi}{\partial x}$

$$u = U_0 (x+b)^m f'(\eta) \text{ and } v = -\sqrt{(m+1) \frac{v U_0}{2} (x+b)^{m+1} \left[f'(\eta) \eta \left(\frac{m-1}{m+1} \right) + f(\eta) \right]} \quad (8)$$

flow function is particularly presented as-

Through means of (7) & (8), expressions (2-3) are likely being transformed into-

$$\left(1 + \frac{1}{\beta} \right) f'''' + f'' f - \frac{2m}{m+1} f'^2 - M f' = 0 \quad (9)$$

$$\left(1 + \frac{4}{3} Ra \right) \theta'' + Pr f \theta' - Pr \frac{1-m}{m+1} f' \theta + A^* f' + B^* \theta = 0, \quad (10)$$

Wherein, resulting condition is stated as-

$$f(0) = \lambda \left(\frac{1-m}{m+1} \right) [1 + h_1 f''(0)], f'(0) = [1 + h_1 f''(0)], \left. \begin{array}{l} \theta(0) = [1 + h_2 \theta'(0)] \\ f'=0, \theta=0, \phi=0 \text{ as } \eta \rightarrow \infty \end{array} \right\}$$

Such that, “M, Pr, as well as Ra” are being highlighted as-

$$M = \frac{2\sigma B_0^2}{\rho U_w^2}, Pr = \frac{\mu C_p}{k}, Ra = \frac{4\sigma^* T_\infty^3}{k^* k}, \quad (12)$$

Moreover, the basic quantities in relation to “engineering interest, the friction factor and the local Nusselt number” are gradually presented through-

$$C_f = 2 \frac{\mu \frac{\partial u}{\partial y}}{\rho U_w^2}, Nu_x = \frac{(x+b) \frac{\partial T}{\partial y}}{T_w(x) - T_\infty}, \quad (13)$$

Such that, on taking into consideration, expression (5), the expression (19) gets converted to-

$$C_f (Re_x)^{0.5} = 2 \left(\frac{m+1}{2} \right)^{0.5} ((1 + \beta^{-1}) f''(0)), Nu_x = - \left(\frac{m+1}{2} \right)^{0.5} (Re_x)^{0.5} \theta(0), \quad (14)$$

Wherein, $Re_x = \frac{U_w X}{\nu}$ and $X = (x+b)$

3. Results alongside Discussions

Through means of above provided work, varied sets pertaining to ODEs (8 - 10) in respect to limiting conditions (11) thereby is particularly presented via mathematical formulation by making use of “bvp4c package”, respectively, wherein the “non-dimensional parameter” are variably selected as- Pr = 6, m = 0.5, M = 1, Ra = 0.5, h₁ = h₂ = A* = B* = 0.5, λ = 0.2. Moreover, considered upsides of these boundaries will in general be perpetual in present work, except if there exist any circumstance of being redirected in below given graphs.

Images 2 as well as 3, thereby portrays effect pertaining to outer “magnetic field on flow alongside temperature fields of Casson liquid”, based on which its determined that, rising worth of cross over attractive field boundary, specifically supports resulting “magnetic field and decay the velocity field”, comparatively. Truly, expanding upsides of such “magnetic field boundary fortify the Lorentz force, which acts inverse to the stream field, ultimately resulting in decaying the speed profiles”. Moreover, several comparative outcomes has been noticed for expanding upsides of “Casson boundary”, as portrayed in images 4 as well as 5,

wherein for most part, expanding upsides of “Casson boundary smothers the viscous attribute of these flows”.

Images 6-8 thereto represents impact pertaining to “Heat radiation and non-uniform Heat source/sink boundaries on temperature profiles of the Casson liquid, through which it was significantly formulated that, rising upsides of heat radiation and non-uniform source/sink boundaries, tends to improve the temperature profiles of respective flows”, considerably. In a likely manner, positive upsides of unstable “heat source/sink boundaries behave like generators of heat”, particularly, which results in building of “temperature fields”. Although, a contrary pattern to above aspects is thereto identified for rising worth of dividing width of such boundary, which is displayed in Image 9.

Moving ahead, impact pertaining to “dimension-less velocity slip boundary on heat and flow fields” is thereby being represented through image (10 as well as 11). As a result, unmistakably, the expanding worth of “velocity slip tends to decay resulting flow field, which subsequently lifts the Heat fields”. However, a contrary situation is identified in case of expanding upsides of “temperature bounce boundary and Prandtl number”, which is displayed through means of image (12 & 13), respectively.

Henceforth, the diverse properties in “skin grating coefficient and nearby Nusselt number at various appropriate boundaries is shown in Table 1, through which its determined that, rising upsides of magnetic field boundary, speed, Heat radiative elements, Heat slip boundaries and Casson boundary, gradually decreases the neighborhood Nusselt number”. As a result, expanding worth of dividing width for boundaries, particularly act on improving its heat dissemination rate.

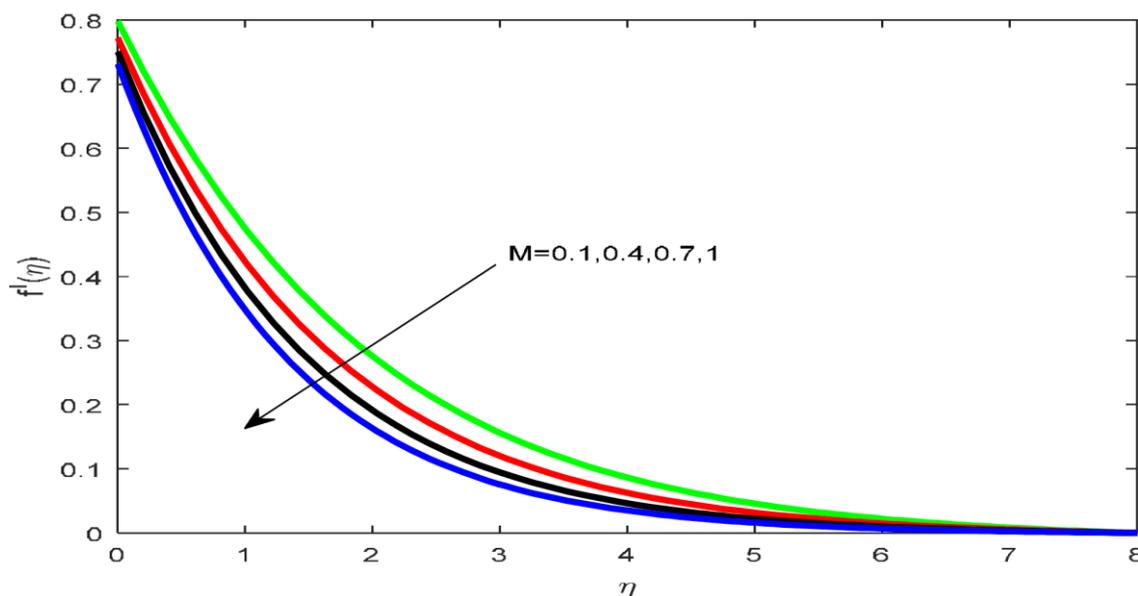


Image.2 Impact pertaining to M based upon velocity field.

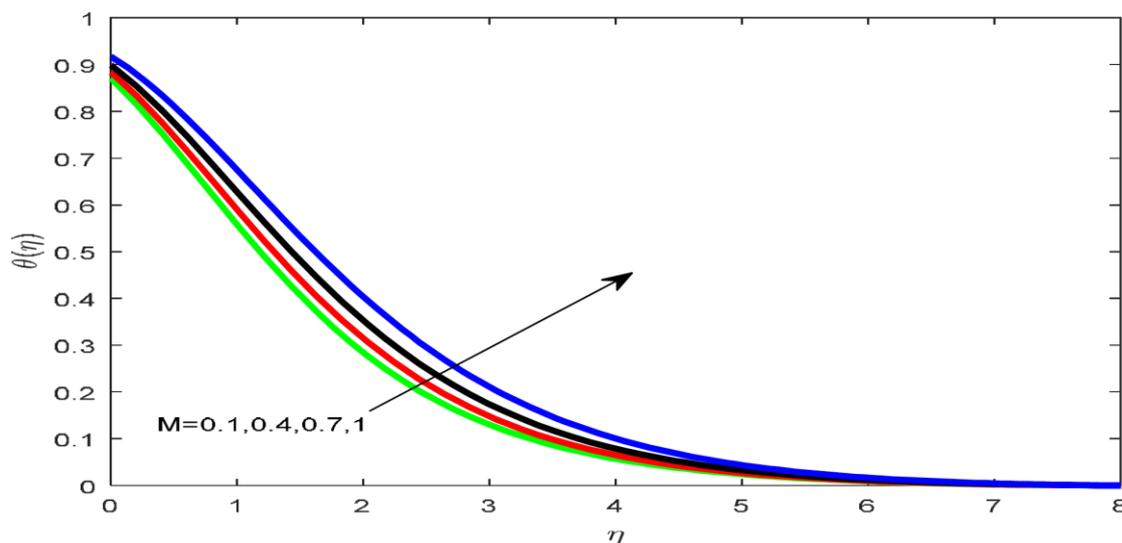


Image.3 Impact pertaining to M based upon thermal field.

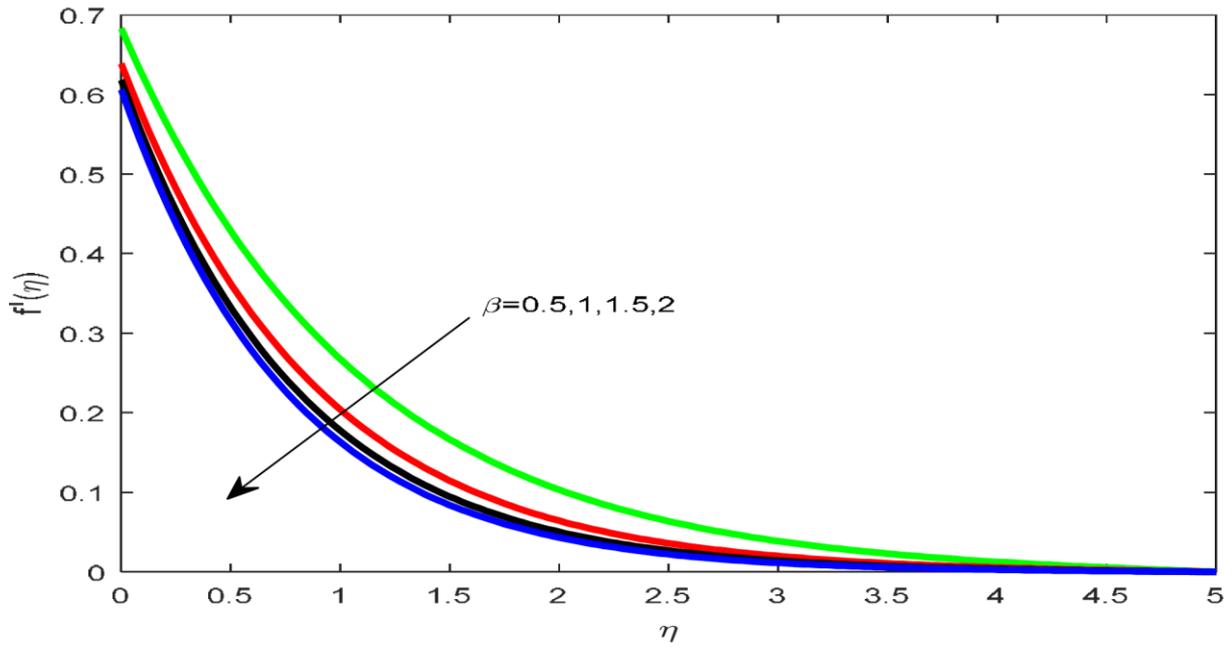


Image.4 Impact pertaining to β based upon velocity field.

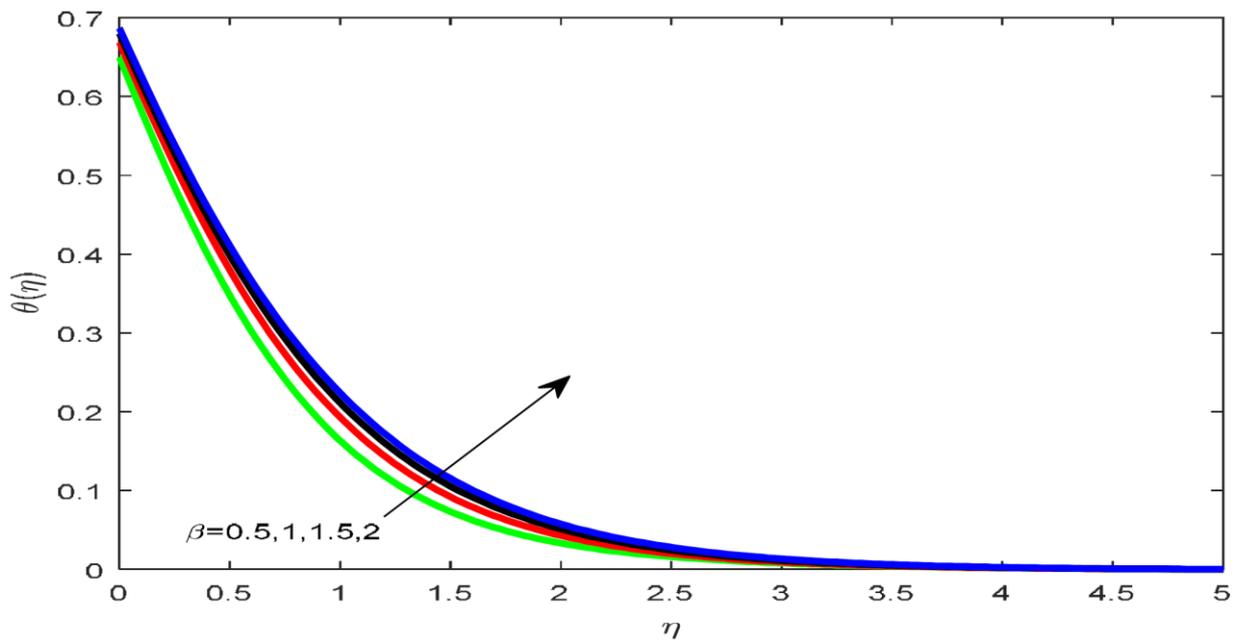


Image.5 Impact pertaining to β based upon thermal field.

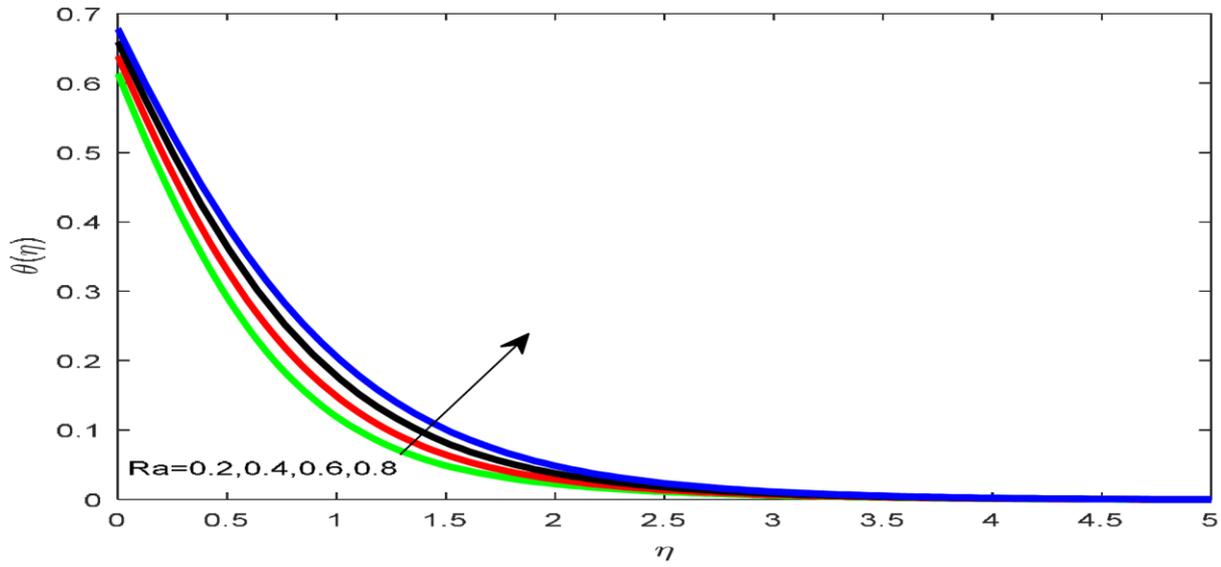


Image.6 Impact pertaining to Ra based upon thermal field.

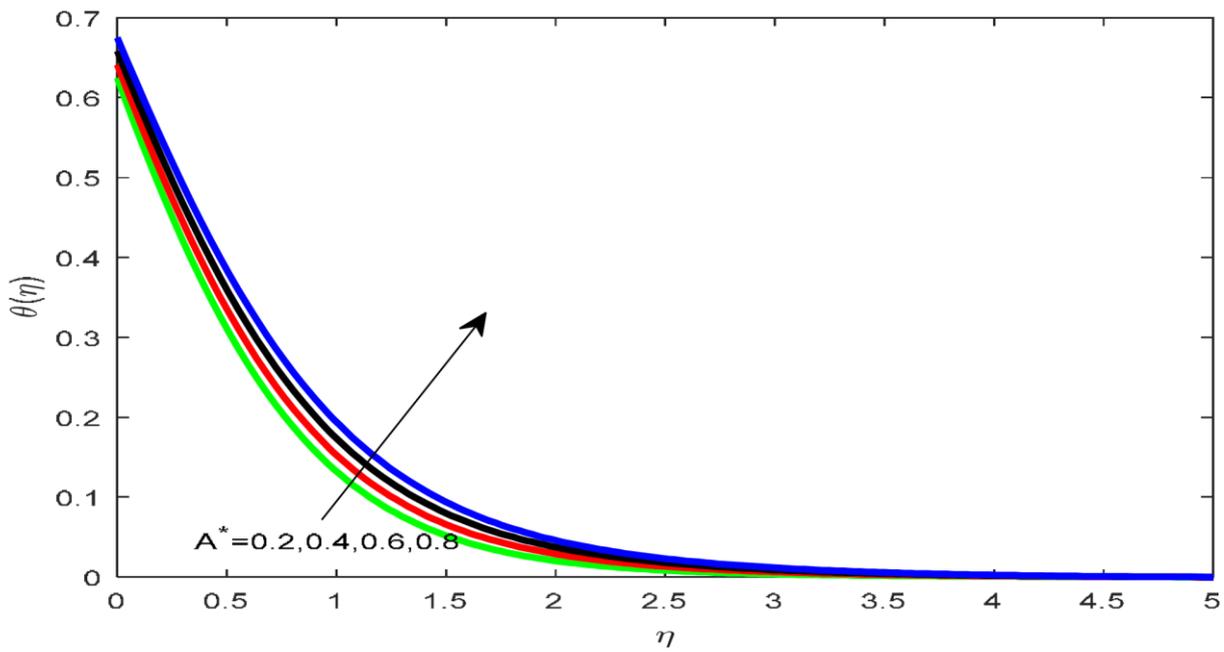


Image.7 Impact pertaining to A^* based upon thermal field.

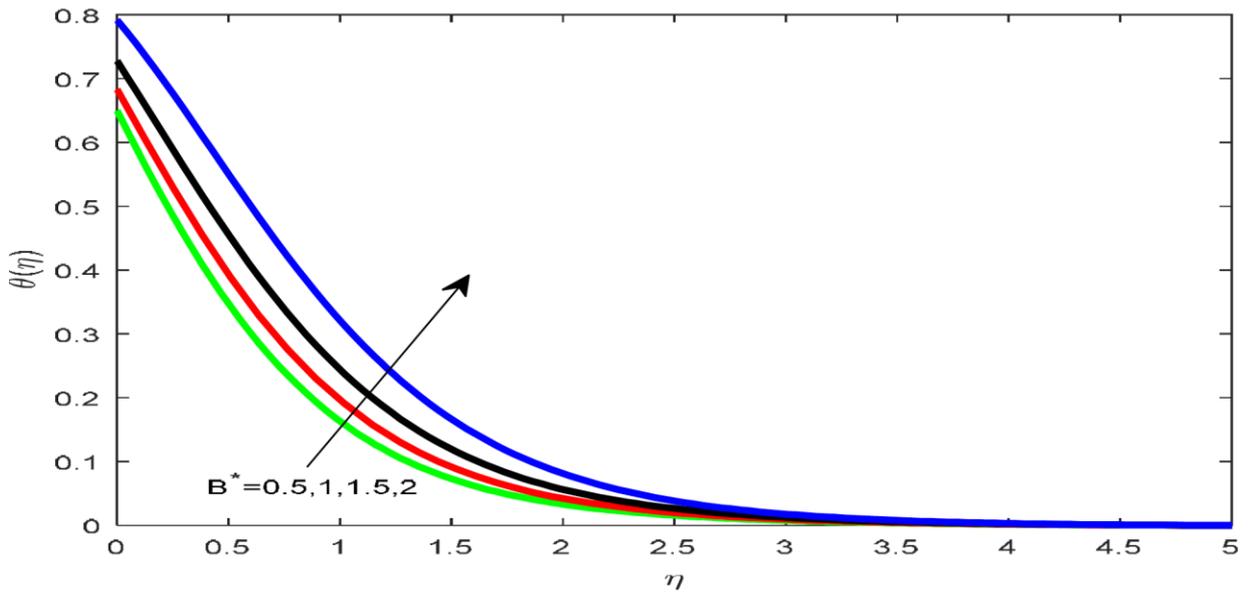


Image.8 Impact pertaining to B^* based upon thermal field.

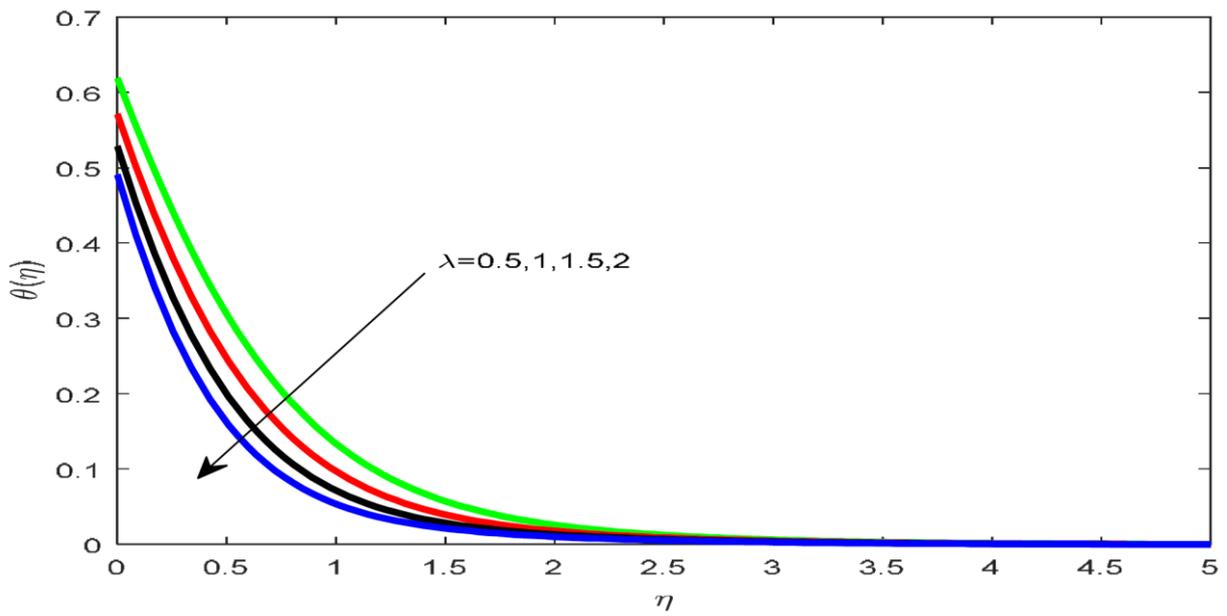


Image.9 Impact pertaining to λ based upon thermal field.

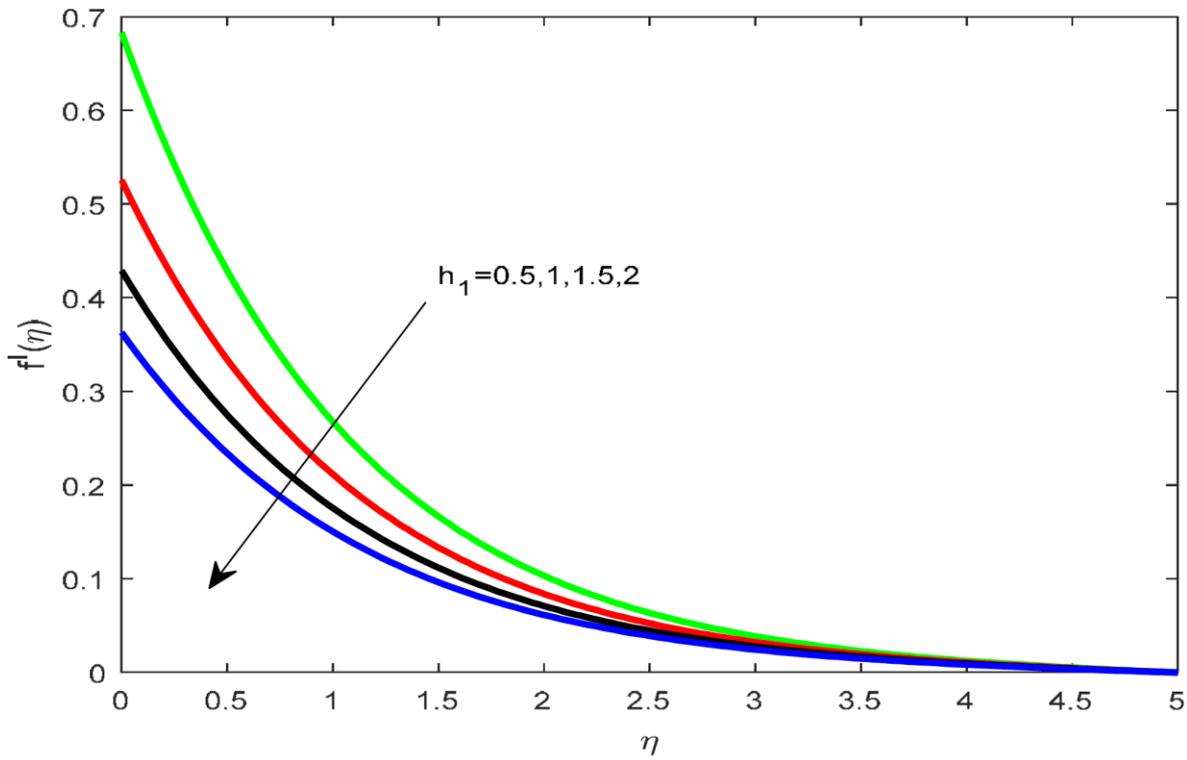


Image.10 Impact pertaining to h_I based upon velocity field.

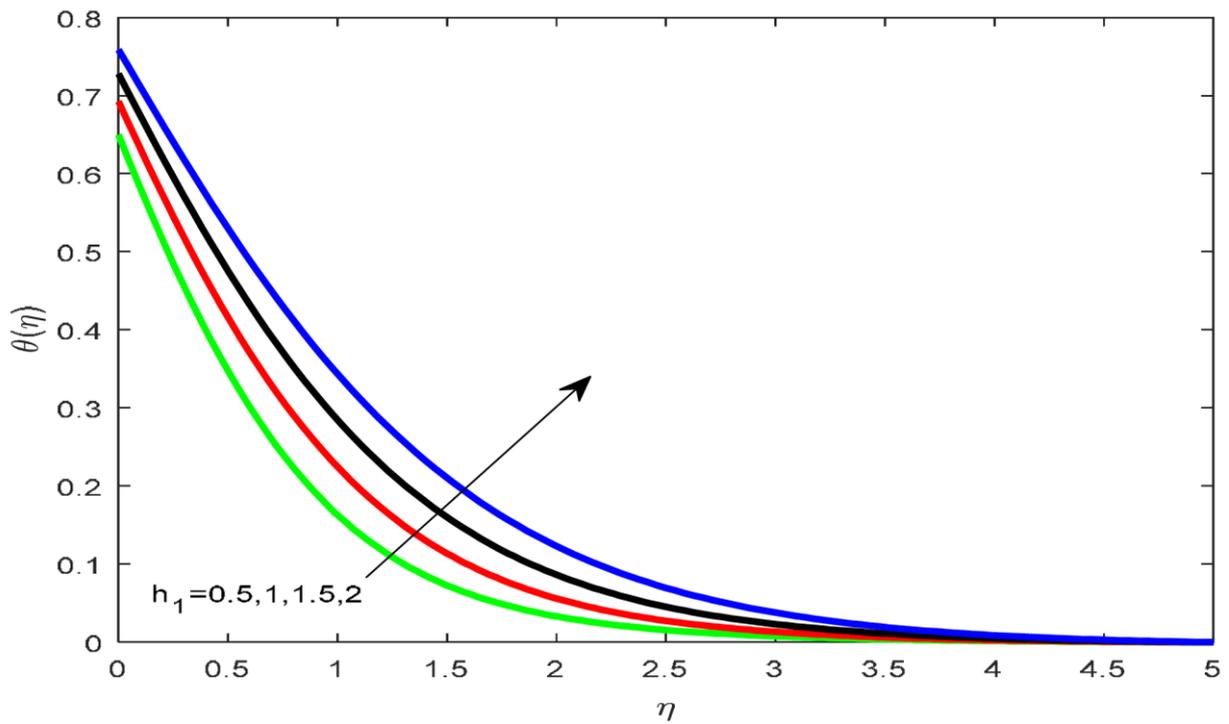


Image.11 Impact pertaining to h_I based upon thermal field.

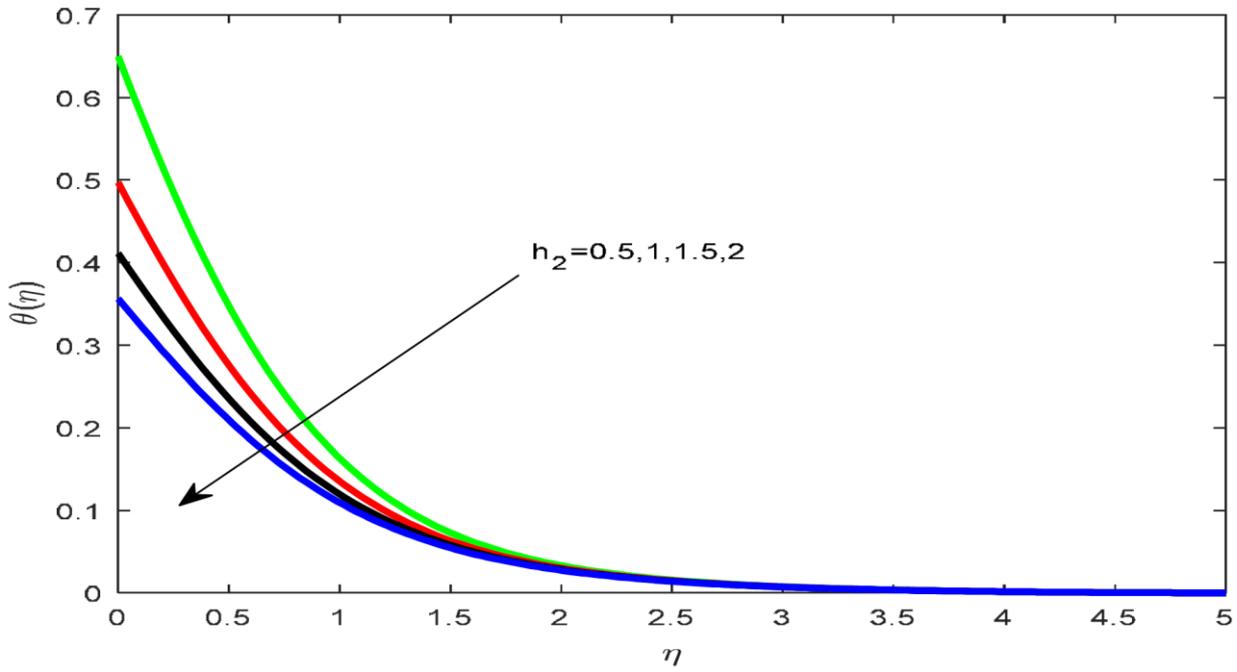


Image.12 Impact pertaining to h_2 based upon thermal field.

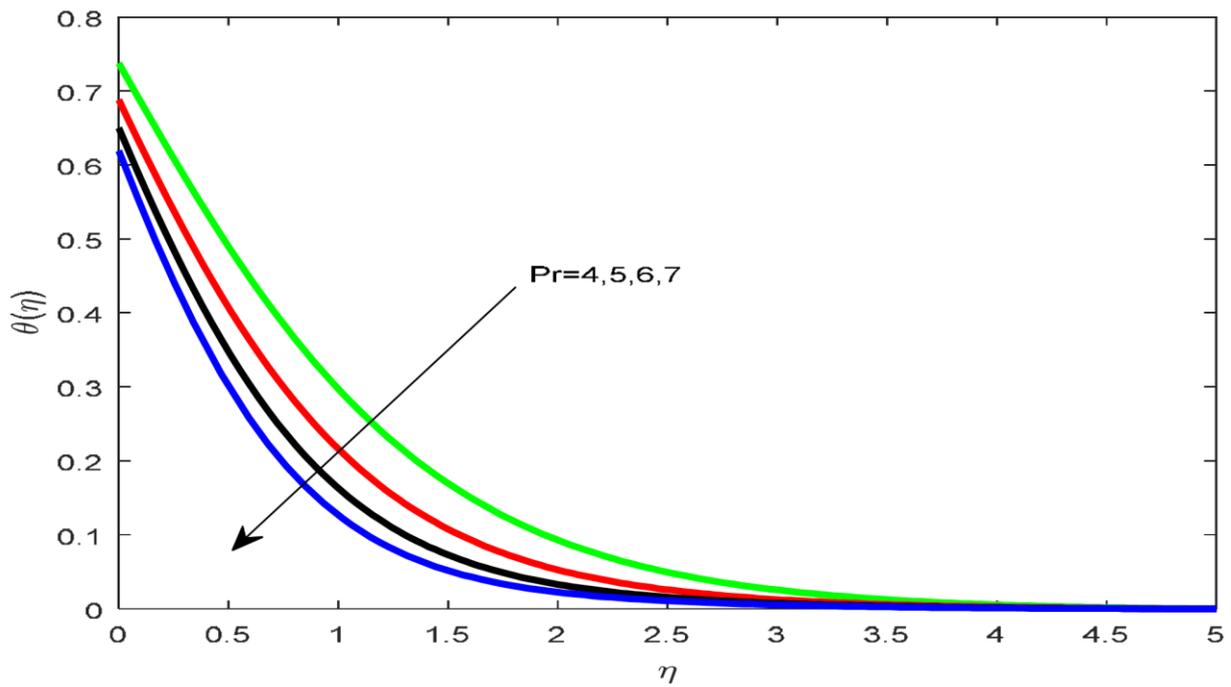


Image.13 Impact pertaining to Pr based upon thermal field.

Table 1 Changes pertaining to tangible quantities at diverse pertinent attributes.

M	Ra	λ	h_1	h_2	A^*	β	C_f	Nu_x
0.1							-0.402640	0.222246
0.4							-0.454791	0.200086
0.7							-0.498500	0.174230
	0.2						-0.633381	0.668461
	0.4						-0.633381	0.624995
	0.6						-0.633381	0.588461
		0.5					-0.638598	0.659612
		1.0					-0.647254	0.742026
		1.5					-0.655855	0.815724
			0.5				-0.633381	0.605993
			1.0				-0.474382	0.532111
			1.5				-0.380660	0.470960
				0.5			-0.633381	0.605993
				1.0			-0.633381	0.435435
				1.5			-0.633381	0.339799
					0.2		-0.633381	0.649458
					0.4		-0.633381	0.620481
					0.6		-0.633381	0.591505
						0.5	-0.633381	0.605993
						1.0	-0.721482	0.572876
						1.5	-0.762779	0.554049

4. Conclusion

Based on our above conducted work, we aimed at analyzing “Free Integrated Heat Dissemination in a 2D Magneto Hydrodynamic stream of Casson liquid over a Non-uniform thickness extending sheet within the sight of Heating radiation alongside Non-uniform source/sink of Heat”, particularly. In addition, various administering conditions pertaining to Dissemination of Heat & flow, are also being worked upon for transforming it into resource of non-linear “ODEs and settled mathematically utilizing bvp4c package”, specifically. Also, various graphs and Image are taken as for assistance in understanding the concept, while outcomes for “friction factor and Nusselt number”, were obtained through help of mathematical application.

As a result, below are given few of the outcomes of our work-

- The “thermal boundary layer”, is specifically being regulated through attributes of “Non-uniform Heat Source/sink”.
- Increasing attributes pertaining to Heat radiation decays the subsequent "Heat dissemination rate, while upgrading the temperature field".
- Expanding width of wall, gradually tends to improve the heat dissemination aspects.
- Resulting heat dissemination aspects subsequently gets diminish with "Casson boundary".
- The "thermal field" is thereby controlled through "slip boundaries".

Greek Symbols

σ : Electrical conductivity of the fluid

- θ : Dimensionless temperature
 ρ : Density of the fluid
 β : Casson fluid parameter
 ν : Kinematic viscosity
 λ : Wall thickness parameter

Nomenclature:

- u, v : Velocity parameters in x and y directions.
 x : Direction along the surface
 y : Direction normal to the surface
 C_p : Specific heat capacity at constant pressure
 A^*, B^* : non-Uniform heat source/sink parameters
 k : Thermal conductivity
 T_m : Mean fluid temperature
 T_∞ : Temperature of the fluid in the free stream
 h_1^* : Dimensional velocity slip parameter
 h_2 : Dimensionless temperature jump parameter
 C_f : Skin friction coefficient
 σ^* : Stefan-Boltzmann constant
 k^* : mean absorption coefficient
 Nu_x : Local Nusselt number
 Re_x : Local Reynolds number

References

1. T.Hayat, N.Saleem, N.Ali, Effect of induced magnetic field on peristaltic transport of a Carreau fluid. *Commun Nonlinear Sci Numeric Simul*, 15, 2407-2423, 2010.
2. N.Sandeep, V. Sugunamma, P Mohan Krishna, Effects of radiation on an unsteady natural convection flow of an EG-Nimonic 80a nanofluid past an infinite vertical plate. *Advances in Physics Theories and Applications*, 23, 36-43, 2013.
3. C.Su1ochana, N.Sandeep, Flow and heat transfer behavior of MHD dusty nanofluid past a porous stretching/shrinking cylinder at different temperatures, *Journal of Applied Fluid Mechanics*, 9(2) (2016)543-553.
4. M. Waqas, T. Hayat, M. Farooq, S.A. Shehzad, A. Alsaedi, Cattaneo-Christov heat flux model for flow of variable thermal conductivity generalized Burgers fluid, *Journal of Molecular Liquids*. 220 (2016) 642-648.
5. N.Sandeep, Effect of Aligned Magnetic field on liquid thin film flow of magnetic-nanofluid embedded with graphene nanoparticles, *Advanced Powder Technology*, 28, 865-875, 2017.
6. N.Sandeep, A.J.Chamkha, I.L.Animasaun, Numerical exploration of magnetohydrodynamic nanofluid flow suspended with magnetite nanoparticles, *J Braz. Soc. Mech. Sci. Eng.* DOI 10.1007/s40430-017- 0866-x, 2017.
7. M.Jayachandra Babu, N.Sandeep, UCM flow across a melting surface in the presence of double stratification and cross-diffusion effects, *Journal of Molecular Liquids*, 232, 27-35, 2017.
8. O.K.Koriko, A.J.Omowaye, N.Sandeep, I.L.Animasaun, Analysis of boundary layer formed on an upper horizontal surface of a paraboloid of revolution within nanofluid flow in the presence of thermophoresis and Brownian motion of 29 nm, *International Journal of Mechanical Sciences*, 124-125, 22-36, 2017.
9. N.Sandeep, M.Gnaneswara Reddy, MHD Oldroyd-B fluid flow across a melting surface with cross diffusion and double stratification, *European Physical Journal Plus*, 132: 147, 2017.
10. P. Mohan Krishna, N. Sandeep, Ram Prakash Sharma, Computational analysis of plane and parabolic flow of MHD Carreau fluid with buoyancy and exponential heat source effects, *European Physical Journal Plus*, 132: 202, 2017.
11. J.V. Ramana Reddy, V. Sugunamma, N. Sandeep, Effect of frictional heating on radiative ferrofluid flow over a slendering stretching sheet with aligned magnetic field, *European Physical Journal Plus*, 132:7, 2017.

12. W. Ibrahim, O. D. Makinde: Magnetohydrodynamic stagnation point flow and heat transfer of Casson nanofluid past a stretching sheet with slip and convective boundary condition. *Journal of Aerospace Engineering*, 29, Issue 2, Article number 04015037, 2016.
13. T. Hayat, Taseer Muhammad, S.A. Shehzad, A. Alsaedi, On three-dimensional boundary layer flow of Sisko nanofluid with magnetic field effects, *Advanced Powder Technology* 27 (2016) 504-512.
14. F.M. Abbasi, S.A. Shehzad, T. Hayat, B. Ahmad, Doubly stratified mixed convection flow of Maxwell nanofluid with heat generation/absorption, *Journal of Magnetism and Magnetic Materials* 404 (2016) 159-165.
15. S.A. Shehzad, Z. Abdullah , A. Alsaedi , F.M. Abbasi, T. Hayat, Thermally radiative three-dimensional flow of Jeffrey nanofluid with internal heat generation and magnetic field, *Journal of Magnetism and Magnetic Materials* 397 (2016) 108-114
16. T. Hayat, T. Muhammad, S.A. Shehzad, A. Alsaedi and F. Al-Solamy, Radiative Three-Dimensional Flow with Chemical Reaction, *International Journal of Chemical Reactor Engineering* 14 (2016) 79-91.
17. M. Turkyilmazoglu, Analytic heat and mass transfer of the mixed hydrodynamic/thermal slip MHD viscous flow over a stretching sheet. *Int J Mech Sci* 53(10) (2011) 886-96.
18. N. Sandeep, C. Sulochana, Dual solutions for unsteady mixed convection flow of MHD Micropolar fluid over a stretching/shrinking sheet with non-uniform heat source/sink. *Engineering Science and Technology, an International Journal* 18 (2015) 738-745.
19. P. Mohan Krishna, N. Sandeep, V. Sugunamma. Effects of radiation and chemical reaction on MHD convective flow over a permeable stretching surface with suction and heat generation. *Walailak Journal of Science and Technology Walailak J Sci & Tech* 2015; 12(9): 831-847.
20. E.M.A Elbashbeshy, M.A.A Bazid, Heat transfer in a porous medium over a stretching surface with internal heat generation and suction or injection, *Applied Mathematics and Computer Science* 158 (2004) 799-807.
21. A. Ishak, R. Nazar, I. Pop, Heat transfer over an unsteady stretching permeable surface with prescribed wall temperature, *Nonlinear Analysis: Real World Applications* 10 (2009) 2909-2913.
22. A. Subhas, P. Veena, Visco-Elastic fluid flow and heat transfer in a porous medium over a stretching sheet, *Int. J. Non-Linear Mechanics*, 33 (3) (1998) 531-540.
23. Rafael Cortell, A note on flow and heat transfer of a viscoelastic fluid over a stretching sheet, *International Journal of Non-Linear Mechanics*, 41 (2006) 78-85.
24. B. Fani, M. Kalteh, A. Abbassi, Investigating the effect of Brownian motion and viscous dissipation on the nanofluid heat transfer in a trapezoidal microchannel heat sink, *Advanced Powder Technology*. 26 (2015) 83-90. doi:10.1016/j.appt.2014.08.009.
25. O.D. Makinde, A. Aziz, Boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition, *International Journal of Thermal Sciences*. 50 (2011) 1326-1332. doi:10.1016/j.ijthermalsci.2011.02.019.
26. S. Nadeem, R.U. Haq, N.S. Akbar, MHD three-dimensional boundary layer flow of Casson nanofluid past a linearly stretching sheet with convective boundary condition, *IEEE Transactions on Nanotechnology*. 13 (2014) 109-115.
27. G. Sucharitha, P. Lakshmi Narayana, N. Sandeep, Joule heating and wall flexibility effects on the peristaltic flow of magnetohydrodynamic nanofluid, *Int. J. of Mechanical Sciences*, 131-132, 52-62, 2017.
28. N. Sandeep, Ram Prakash Sharma, M. Ferdows, Enhanced heat transfer in unsteady magnetohydrodynamic nanofluid flow embedded with aluminum alloy nanoparticles, *Journal of Molecular Liquids* 234, 437-443, 2017.
29. M.E. Ali, N. Sandeep, Cattaneo-Christov model for radiative heat transfer of magnetohydrodynamic Casson-ferrofluid: A numerical study, *Results in Physics*, 7, 21-30, 2017.
30. M. Jayachandra Babu, N. Sandeep, Three-dimensional MHD slip flow of a nanofluid over a slendering stretching sheet with thermophoresis and Brownian motion effects, *Advanced Powder Technology*, 27, 2039-2050, 2016.