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Different Solutions Of A Vibro-Impact System with Non-Ideal Excitation Based on the Misalignment of Frequency Response Curves Obtained by A Numerical and Analytical Analysis

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Abstract - The paper concerns with the analysis of different solutions of a vibro-impact system with non-ideal excitation. The system composed of a driving source with limited power supply (non-ideal excitation) in the form of a DC electro motor which is linked using a piston mechanism and a linear elastic element to a sliding body which can impact in a fixed wall. The paper presents the mechanical mode then the mathematical model of the whole system including the impact model, short overview of the analytical analysis procedure as the numerical analysis procedure. The mathematical model of the mechanical system is represented as a system of two coupled nonlinear differential equations. The impact model which was used in the analysis is the Newtonian inelastic impact model. Based on the two mentioned method of analysis frequency response diagrams are obtained which indicated discrepancies of the two frequency response curves (solutions) in some regions. Further analysis was conducted which explains the source of the discrepancy and how solutions can be obtained so that the results obtained numerically and analytically are overlapping. Steady state and transient behaviour of these solutions is indicated as well.

Keywords: *Vibro-impact systems; Non-ideal excitation; Numerical analysis; Analytical analysis*

Introduction

The driving source of a mechanical system can be ideal or non-ideal. The driving source or the excitation of the system is called ideal when the excitation frequency is constant or non-ideal when it is variable due to dissipating forces or torques in the system. Systems with non-ideal excitation are called also systems with limited power supply. The ideally excited systems are linear where systems with non-ideal excitation have one additional differential equation which describes the motion of the driving source and these systems are nonlinear. By adding a stop in which the oscillating body can impact and by that change suddenly direction with a lower value of velocity alternates a vibrating system into a vibro-impact system where such one is analysed in this paper and it is denoted as a vibro-impact system with non-ideal excitation. The nonlinearities of the system are two-layered, one is related to the variable driving source and the other nonlinearity is related to the impact or sudden changes of direction. Machines, such as hand-held percussion machines, cutting and grinding machines, pile driving machines, etc. are all working as a vibro-impact system. This paper will present such a system, define the system mathematically as a system of two second order coupled system of nonlinear differential equations.

Content in this paper is a continuation of the authors research from a numerical and analytical method based analysis [1,2] to an analysis of the discrepancy between the frequency response curves obtained by the mentioned methods. In [1] it is mentioned that this type of systems are not researched sufficiently and there are very small numbers of papers (or none by the authors knowledge) which are observing the behaviour of vibro-impact systems with non-ideal excitation. In this paper this system which will be analysed is similar to that one analysed in the papers [1,2] where in [1] the method used for the analysis is purely numerically and in [2] the system is analysed using direct analytical and approximate analytical methods. Influence of some parameters are analysed in [1], where in paper [2] influence of every important parameter on the system is shown. Besides the analytical analysis, stability analysis is also conducted. In paper [2] a comparison between the frequency response of an analytical and numerical analysis is shown where a discrepancy between results can be noticed. The discrepancy is not addressed and explained in paper [2] and because of this it will be thoroughly analysed in this paper.

In paper [3] a self-excited vibro-impact rotary drill penetration is modelled and analysed where an viscos-elastic impact model is used. Experimental validation of the numerically obtained results is shown. With the gathered results it is shown how the impact motion influences the drilling efficiency. The system is assumed to be with an ideal excitation where the excitation frequency is constant.

In [4] the use of a vibro-impact system as a dynamic absorber is analysed using numerical methods. Similar analysis can be found in wide range of published papers. The influence of two different types of excitation, random and deterministic, on a vibro-impact system is shown in [5]. Analysis of a vibro-impact capsule with various friction models is shown in [6], where the motion control by a forward and backward loop of the same system is shown in [7]. Analysis of a two-frame structure system is shown in [8] where on one frame's platform an unbalanced electromotor is rigidly connected which is causing a horizontal motion of the structure due to the pillars flexibility. Because the two frames are close to each other they will start to get in contact and by which an repeating impact motion will happen. The system which is analysed in [8] has the properties of an vibro-impact system with limited power supply but in the mathematical sense it is switching to partial differential equations where the analysis was focused on how different impact models are influencing the system's behaviour. Based on the analysis in paper [8] the inelastic impact model is used in this paper due to that the results when used this model give the best overlapping with the empirical results, or better than the other models. The energy density transferred during the impact motion of a ideally excited system is shown in [9] and this analysis is very helpful to see how energy could be dissipated using such a system as an dynamic absorber or an attachment to control motion. Similar analysis was mentioned and it is related to paper [4] and paper [10]. However, the analysis conducted in this paper is mainly focused on the behaviour of the impacting body and the excitation.

Repeated impact motion is analysed in [11,12] where an heavy mass particle is moving in a rough circular trajectory where two stops exists. Respectively in the mentioned papers the motion and the energy conservation and dissipation of such system is analysed.

In [13] the control law for a beneficial control of chaotic motion of a vibro-impact system with limited power supply is shown. Numerical analysis is conducted and results are shown. Analysis which is presented in the following text is related to steady-state motion of a vibro-impact system with limited power supply as a different form of motion and system. Stability analysis is found in [14] of a vibro-impact system with ideal excitation where the excitation function is a force function. Stability analysis of the system, which is the object of analysis in this paper, has an excitation in the form of displacement.

The use of a vibro-impact system as a control attachment to physical systems is shown in [15] where chatter is controlled during the turning operation. In [16-18], a wide range of nonlinear vibrating systems and methods for their analysis is presented. Paper [16] gives a general insight in nonlinear vibrations with an emphasis to oscillating systems with non-ideal excitation. The monography [17] analyses very through systems with non-ideal excitation in general, systems with non-ideal excitation that have some non-smoothness due clearance, systems where the observed body can impact in a stiff spring and systems where an opposing force exists during cutting operations. Pioneer literature in systems with limited power supply is [18], which terminologically describes systems with non-ideal excitation as systems with limited power supply. All the analysed systems in [16-18] do not have the sharp non-smoothens as a vibro-impact system has.

Mechanism with a non-ideal excitation is shown in [19] where the final/working body in the system is opposed by a cutting force which decreases the speed of the system. Sharp changes of the velocity value which are deterministic for vibro-impact system do not exist in this paper. A clear overview of systems with non-ideal excitation is given in [20].

In paper [21] the interaction between different excitation types is analysed of a system with ideal and non-ideal excitations. The physical model consists of a nonlinear Mathieu type spring with a periodically change stiffness and nonlinear Rayleigh damping. The excitation that is driving the system is given as a harmonic force that represent the ideal excitation and an electromotor that represents the non-ideal excitation. Both excitation are acting simultaneously. The electromotor model is given in two variants, one is the Kononenko variant and a full electromechanically model. In this paper, the Kononenko model of an electromotor is used with a linear characteristic. Quantitative and qualitative differences are shown in reference to the mentioned types of electro motors and it is shown if the transition in the resonance region is slow both electromotor models give similar results. It is also shown, if the dynamic motion of the system is more complex than the differences between the models are significant where the full electromechanical model tends more to chaotic motion. The difference between the researched system in [21] and in this paper is related to the impact motion and the spring characteristic is linear.

This study is focused on obtaining the same results in reference to frequency response diagrams and explaining how these results can be obtained. To fully show this analysis in the paper modelling and analysis of a system constructed from the limited power supply driving source, a slider mechanism and a impacting body will be shown as a physical model. Mathematical model of a vibro-impact system with non-ideal excitation is derived and given in the form of two coupled differential equations. First differential equation of the system represents the motion of the impacting oscillator, whereas the second one describes the motion of the excitation. Direct analytical method with some simplification is used for solving the differential equation related to the motion of the oscillator, whereas the differential equation that describes the motion of the excitation is solved using approximate analytical method of Harmonic Balance and it can be found in paper [2] because the behaviour of the power supply was not interesting in this paper. Numerical simulations are conducted also with the purpose to compare them with the analytical results. By comparing these two results it is pointed out that an discrepancy exists in a region of impact solutions of the frequency response diagram. Through an additional analysis it is pointed out how to get each of these two solutions and what is the cause of this discrepancy

In the following Section the description of the physical and mathematical model of the analysed mechanical system is shown. Section 3 presents the overlook of the analytical analysis of the vibro-impact system. In section 4 the results are presented and the analysis of the discrepancy is pointed out and explained. Summarised conclusions of the paper are given in Section 5.

PHYSICAL AND MATHEMATICAL MODEL

The physical model of the system under study (**FIG. 1.**) consists of an translating body C that is linked using a linear elastic element to a slider B on it's left side. On the other side of the body C is a fixed wall located. The slider B is part of a mechanism which consists, beside itself, of a lever AB, homogeneous disk with the radius a and moment of inertia J. The centre of the disk is located on the electromotor shaft which is driving the whole system. Slider B will be denoted as the driving slider whereas body C will be denoted as impacting oscillator.

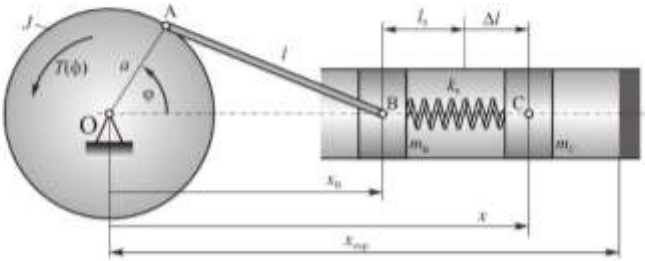


FIG. 1. MECHANICAL MODEL OF THE SYSTEM UNDER ANALYSIS

Geometrical and mechanical parameters are pointed out in **FIG. 1.** The lever AB with the length l is assumed to be of negligible mass. The mass of the driving slider is m_B and of the oscillator m_C; The stiffness of the spring is k_0 and the free length of the spring is l_0. The mass m_C can impact the wall on it's right side under certain regimes. The behaviour of the driving source will be described as a torque acting on the electromotor shaft $T(\dot{\varphi})$ and it is a function of the angular velocity of the shaft. The angular velocity of the electromotor shaft in this case is the angular velocity of the disk.

As noted in **FIG. 1.**, the oscillator C can impact into the fixed wall, where the coordinate marked by x_{stop} represents the position of a it. If the impact occurs periodically during the oscillating motion of the attachment, the system corresponds to a vibro-impact system.

The system has two degrees of freedom with the generalized coordinates being the displacement of the oscillator x and the angle of rotation of the eccentric drive φ, as shown in **FIG. 1.**.

The equations of motion are formed based on Lagrange's equations of motion:

$$\begin{aligned} &\ddot{x} + 2\delta\dot{x} + \omega^2 x = \omega^2 l_0 + \omega^2 u(\varphi) \\ &(m_B u'(\varphi)^2 + J)\ddot{\varphi} + m_B\dot{\varphi}^2 u'(\varphi)u''(\varphi) \\ &\quad = \kappa T_0\left(1 - \frac{\dot{\varphi}}{\kappa\Omega_0}\right) + cu'(\varphi)(x \\ &\quad - l_0 - u(\varphi)) \end{aligned} \tag{1}$$

where:

$$\omega^2 = \frac{k_0}{m_C}, 2\delta = \frac{c}{m_C} \tag{2}$$

The driving torque is assumed to be a linear function of the angular velocity of the eccentric drive:

$$T(\dot{\varphi}) = \kappa T_0\left(1 - \frac{\dot{\varphi}}{\kappa\Omega_0}\right) \tag{3}$$

Equation (3) describes how the driving torque changes with the angular velocity. The control parameter of the driving source behaviour is the parameter κ.

The first approximation of the function $u(\varphi)$ which is representing the motion of slider B is given by:

$$u(\varphi) = l + a\cos\varphi \tag{4}$$

In the case of the first approximation, a simplified mathematical model of the system can be formed which can be analysed analytically. The corresponding equations of motion in the first approximation are:

$$\begin{aligned} &\ddot{x} + 2\delta\dot{x} + \omega^2 x = \omega^2(l_0 + l) + \omega^2 a\cos\varphi \\ &(m_B a^2\sin^2\varphi + J)\ddot{\varphi} + m_B\dot{\varphi}^2 a^2\sin\varphi\cos\varphi \\ &\quad = \kappa T_0\left(1 - \frac{\dot{\varphi}}{\kappa\Omega_0}\right) - ca\sin\varphi\,(x \\ &\quad - l_0 - l - a\cos\varphi) \end{aligned} \tag{5}$$

Based on **FIG. 1.**, the following substitution can be introduced

$$x = l_0 + l + y \tag{6}$$

The equations of motion in the first approximation now become:

$$\ddot{y} + 2\delta\dot{y} + \omega^2 y = \omega^2 a\cos\varphi \tag{7}$$

$$(m_B a^2 \sin^2 \varphi + J)\ddot{\varphi} + m_B \dot{\varphi}^2 a^2 \sin\varphi \cos\varphi = \kappa T_0 \left(1 - \frac{\dot{\varphi}}{\kappa \Omega_0}\right) - ca \sin\varphi \,(y - a\cos\varphi)$$

Given the form of Eq. (7) and the existing coupling between the generalized coordinates φ and x, the angular velocity will not be constant, but it will change with time. These types of systems are known as systems with non-ideal excitation.

The system parameters used in this analysis are given in Table 1.

Parameter	Notation	Value	Unit
Mass of oscillator C	m_C	0.35	[kg]
Mass of slider B	m_B	0.2	[kg]
Stiffness of the spring	k_0	$9\pi^2$	[N/m]
Damping coefficient	c	0.5	[Ns/m]
Radius of the disc	a	0.04	[m]
Length of the lever	l	0.12	[m]
Moment of inertia of the lever	J	0.06	[kgm^2]
Restitution coefficient	k	0 - 1	[-]
Initial torque	T_0	2 - 100	[Nm]
Torque parameter	κ	0.5 - 1.9	[-]
Length of the underformed spring	l_0	0.36	[m]

TABLE 1. SYSTEM PARAMETERS

ANALYTICAL SOLUTION PROCEDURE

The analytical analysis is conducted more deeply in the paper [2] and here it will be shown an overlook of it because it is needed to point out why the discrepancy between the analytical solution and the numerical one is occurring.

In paper [2] the analytical analysis is based on the period of oscillation of 2π whereas in this paper the analysis will be based on the period of oscillation of 4π. The hypothesis is that in that specific regions besides the shown solution with the oscillation period of 2π solutions with the period of 4π are existing as well.

If it is introduced:

$$y = y(\varphi) \qquad \dot{\varphi} = \dot{\varphi}(\varphi) = \Omega(\varphi) \tag{8}$$

By introducing Eqs. (8) into Eq. (7), one obtains:

$$\Omega\Omega' y' + \Omega^2 y'' + 2\delta\Omega y' + \omega^2 y = \omega^2 a \cos\varphi$$
$$(m_B a^2 \sin^2\varphi + J)\Omega\Omega' + m_B \Omega^2 a^2 \sin\varphi\cos\varphi = \kappa T_0\left(1 - \frac{\Omega}{\kappa\Omega_0}\right) - ca\sin\varphi\,(y - a\cos\varphi) \tag{9}$$

where $y(\varphi)$ and $\Omega(\varphi)$ are unknown and should be determined and $y'(\varphi)$, $y''(\varphi)$ and $\Omega'(\varphi)$ are derivatives over φ. The first of Eq. (9) is a second order differential equation, while the second one is first order differential equation.

We introduce the assumption that implies the change of the angular velocity about its average value Ω_a:

$$\Omega = \Omega_a + \varepsilon f(\varphi)\,, \Omega' = \varepsilon f'(\varphi) \tag{10}$$

where: ε is a small parameter and $f(\varphi)$ is an unknown continuous function of φ.

In this case, the system of differential equations (9), can be expressed as

$$y'' + 2\frac{\delta}{\Omega_a}y' + \left(\frac{\omega}{\Omega_a}\right)^2 y = \left(\frac{\omega}{\Omega_a}\right)^2 a\cos\varphi + O(\varepsilon)$$

$$(m_B a^2 \sin^2\varphi + J)\Omega_a\, \varepsilon f'(\varphi) + m_B\Omega_a^2 a^2 \sin\varphi\cos\varphi = \kappa T_0\left(1 - \frac{\Omega_a}{\kappa\Omega_0}\right) - ca\sin\varphi\,(y - a\cos\varphi) \quad (11)$$

or with the additional assumption $\delta = O(\varepsilon)$

$$y'' + \frac{1}{\zeta^2}y = \frac{1}{\zeta^2}a\cos\varphi \quad (12)$$

where the parameter ζ is a substitution which represents the average angular frequency versus the natural frequency ratio:

$$\zeta = \frac{\Omega_a}{\omega} \quad (13)$$

The solution of Eq. (12) is

$$y(\varphi) = C_1\cos\left(\frac{\varphi}{\zeta}\right) + C_2\sin\left(\frac{\varphi}{\zeta}\right) + A\cos\varphi$$
$$y'(\varphi) = -\frac{1}{\zeta}C_1\sin\left(\frac{\varphi}{\zeta}\right) + \frac{1}{\zeta}C_2\cos\left(\frac{\varphi}{\zeta}\right) - A\sin\varphi \quad (14)$$

where

$$A = \frac{a}{1-\zeta^2} \quad (15)$$

and it represents the amplitude of non-damped forced oscillations.

Boundary conditions for when in reference to the period of the oscillation the impact happens are:

$$y(\varphi_0) = \Delta\,;\ y(4\pi + \varphi_0) = \Delta \quad (16)$$

while the boundary conditions based on the inelastic impact model and the coefficient of restitution are:

$$y'(4\pi + \varphi_0) = v_0\,;\ y'(\varphi_0) = -kv_0 \quad (17)$$

where v_0 is the velocity before the impact, and $-kv_0$ is the velocity after the impact, with k standing for the coefficient of restitution (see **Table 1**). Combining the two last conditions from Eq. (17), yields:

$$y'(\varphi_0) = -ky'(4\pi + \varphi_0) \quad (18)$$

where φ_0 is an unknown impact phase.

There are four unknowns: C_1, C_2, v_0 and φ_0 in Eqs. (14) and (18). From the boundary conditions (16), the following can be obtained:

$$y(\varphi_0) = C_1\cos\left(\frac{\varphi_0}{\zeta}\right) + C_2\sin\left(\frac{\varphi_0}{\zeta}\right) + A\cos\varphi_0 = \Delta$$
$$y(4\pi + \varphi_0) = C_1\cos\left(\frac{4\pi + \varphi_0}{\zeta}\right) + C_2\sin\left(\frac{4\pi + \varphi_0}{\zeta}\right) + A\cos(4\pi + \varphi_0) = \Delta \quad (19)$$

where the constants C_1 and C_2 are:

$$C_1 = (\Delta - A\cos\varphi_0)\cos\left(\frac{2\pi + \varphi_0}{\zeta}\right)\sec\left(\frac{2\pi}{\zeta}\right)$$
$$C_2 = (\Delta - A\cos\varphi_0)\sin\left(\frac{2\pi + \varphi_0}{\zeta}\right)\sec\left(\frac{2\pi}{\zeta}\right) \quad (20)$$

The boundary conditions for the velocity are:

$$y'(\varphi_0) = -\frac{1}{\zeta}C_1\sin\left(\frac{\varphi_0}{\zeta}\right) + \frac{1}{\zeta}C_2\cos\left(\frac{\varphi_0}{\zeta}\right) - A\sin\varphi_0 = -kv_0$$
$$y'(4\pi + \varphi_0) = -\frac{1}{\zeta}C_1\sin\left(\frac{4\pi + \varphi_0}{\zeta}\right) + \frac{1}{\zeta}C_2\cos\left(\frac{4\pi + \varphi_0}{\zeta}\right) - A\sin(4\pi + \varphi_0) = v_0$$

and yield the solution for the phase φ_0:

$$\sin\varphi_0 = -\frac{1-k}{2A}v_0\,;\ \cos\varphi_0 = \frac{\Delta}{A} + \frac{1+k}{2A}\cot\left(\frac{2\pi}{\zeta}\right)v_0 \quad (21)$$

In paper [2] the solution is obtained for the period of oscillation 2π.

RESULTS

The analytical results are obtained by the analysis conducted in the previous section, the numerical results are obtained based on the paper [1] using the continuation method.

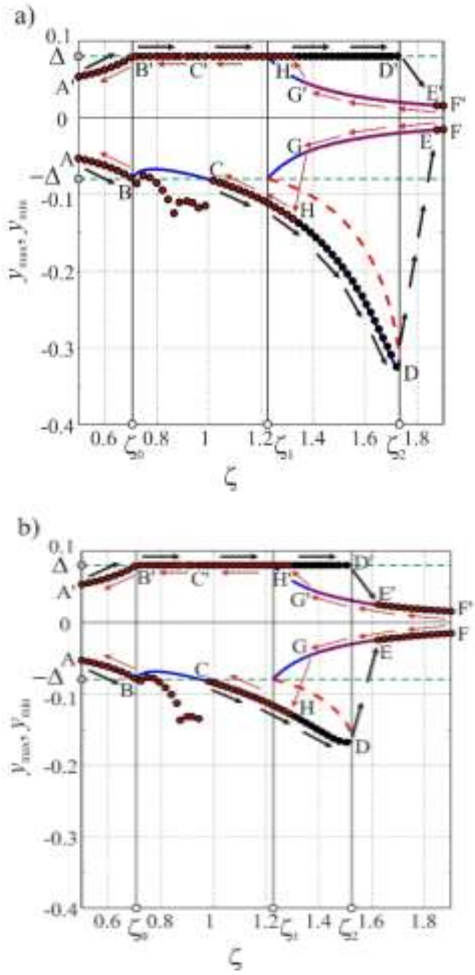


FIG. 2. AMPLITUDE-FREQUENCY DIAGRAM FOR VIBRO-IMPACT SYSTEM WITH IDEAL EXCITATION FOR Δ=2A: A) *K*=0.8; B) *K*=0.5; ANALYTICAL SOLUTIONS – BLUE AND RED SOLID LINES; NUMERICAL SOLUTIONS OBTAINED DURING RUN UP – BLACK CIRCLES; NUMERICAL SOLUTIONS OBTAINED DURING CLOSE DOWN – RED DOTS; ARROWS INDICATE THE SYSTEM BEHAVIOUR DURING RUN UP OR CLOSE DOWN. [2]

Summarized results of the analytical and numerical analysis are shown in **Fig. 2.** in the form of frequency response diagrams. These results can be found in [2]. Black and red circles represent the numerical solution, whereas the solid blue curve and the dashed red curve represent the analytical solution. As it can be seen on the diagrams shown in **Fig. 2.**

A discrepancy between the results in the region between the vertical lines BB' and CC' occurs for the minimum solution. For the maximum solution because the solutions are impact ones there is no discrepancy between the analytical and numerical solutions. For the minimum solution the discrepancy has an importance if an additional impact wall is located on this side. The lower dashed black horizontal line indicated a potential position of this wall an as it can be seen the analytical solution gives an value of the displacement which is away from the wall whereas the value of the numerical solution is "higher" than the position of the wall and by that it the numerical solution would be an impact solution and the analytical would be a non-impact solution.

On **Fig. 3.** results are shown based on an calculation for the period of oscillation of period 2π besides the dashed pink curve which is representing the analytical analysis for the period of oscillation 4π. Results based on the calculation for the period of oscillation 4π are shown on **Fig. 4.** on **Fig. 3.a)** and **Fig. 4.a)** the initial motion of the impacting slider for the period of oscillation of 4π is shown. As it can be seen on **Fig. 3.a)** the numerical and analytical solution is shown to oscillate with a period of 2π(dashed red curve and solid blue curve). On this figure the analytical solution for the period of oscillation 4π is as well added.

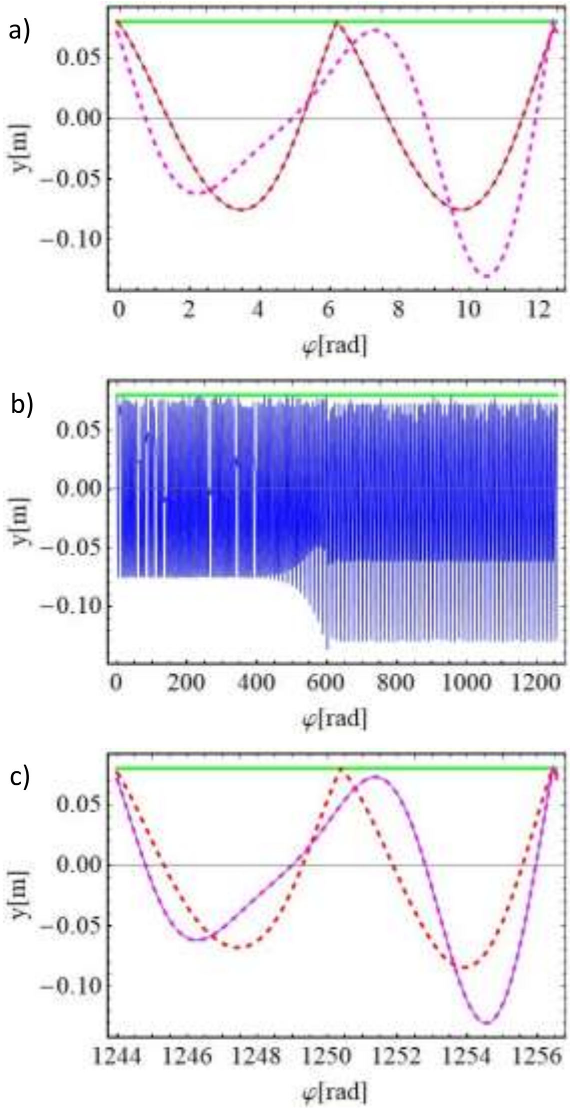
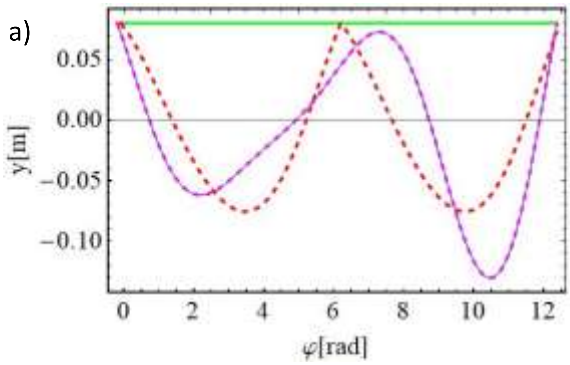


FIGURE 3. SOLUTION ANALYSIS FOR Z = 0.90, k = 0.50 AND THE OSCILLATION PERIOD OF 2Π; SOLID GREEN LINE – IMPACT WALL, SOLID BLUE LINE – NUMERICAL SOLUTION, DASHED RED LINE – ANALYTICAL SOLUTION FOR THE PERIOD OF OSCILLATION OF 2Π, DASHED PINK LINE–ANALYTICAL SOLUTION FOR THE PERIOD OF OSCILLATION OF 4Π

By this the minimum value of the oscillation has a lower value (bigger negative value) which directly explains the discrepancy on the frequency response curve. To get a bigger picture what's happening during the whole motion of the slider oscillation graphs are formed for a much longer period of oscillation.



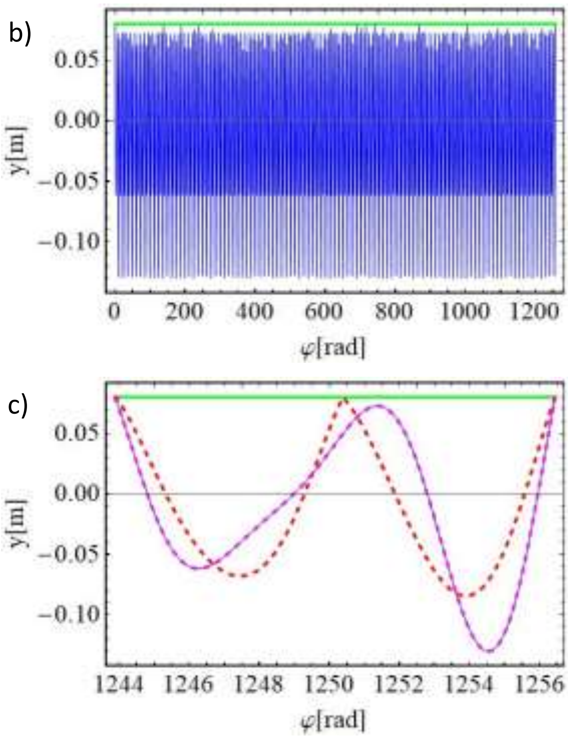


FIGURE 4. SOLUTION ANALYSIS FOR Z $= 0.90$, $k = 0.50$ AND THE OSCILLATION PERIOD OF 4Π; SOLID GREEN LINE – IMPACT WALL, SOLID BLUE LINE – NUMERICAL SOLUTION, DASHED RED LINE – ANALYTICAL SOLUTION FOR THE PERIOD OF OSCILLATION OF 2Π, DASHED PINK LINE–ANALYTICAL SOLUTION FOR THE PERIOD OF OSCILLATION OF 4Π

On **Fig. 3.b)** the numerical solution as shown in a blue solid curve gives a change in the minimum amplitude of oscillation which indicates a transient solution where the slider goes from an unstable to an stable solution because for the used numerical analysis it was only possible to find a stable solution. The reason to obtain a solution of 4π oscillation period is that during the calculation of the numerical solution there is always an calculation error caused by the rounding up of the decimals by a predefined number of decimals and due to this at some point the errors will add up to cause the small perturbation which will push the moving body from an unstable motion to an stable motion. This part of unstable motion can be skipped by applying accurate initial conditions which will cause a direct numerical solution with a period of oscillation of 4π, this was done and it is shown on **Fig. 4.b).** This can be seen as well on **Fig. 4.a)** where the solid blue line represents the numerical solution. On the same figure analytical solutions are included for the period of oscillation of 2π and 4π as well.

On **Fig. 3.c)** and **Fig. 4.c)** the last 8π periods of motions are shown. Comparing the **Fig. 3.a)** and **Fig. 3.c)** it can be seen that in the end of the motion the numerical solution changed from the period of a full oscillation 2π to 4π. Comparing on the other side the **Fig. 4.a)** and **Fig. 4.c)** it can be seen that for the numerical solution no change happens at the initial 4π of oscillation to the last 8π periods of oscillation or the last 4π periods of oscillations which more clearly shown the motion of the impacting slider.

Conclusion

In this paper the occurrence of different solutions in a specific frequency region of a vibro-impact system with non-ideal excitation is presented. Using Lagrange's equation of motion the mathematical model was derived in the form of a coupled system of second order nonlinear differential equations. The impact model was used as an inelastic impact model with a coefficient restitution. The driving power source was described in equation (3) with a varying parameter κ.

The mathematical model was solved analytically and numerically and the results were summarised in frequency response diagrams. A discrepancy in these results was noticed. A deeper analysis was conducted to find the cause and clarify the discrepancy. It has been shown that the reason for the discrepancy in the region between the lines BB' and CC' is that the analytical solution is based for the analysis of a period of oscillation which is 2π and that the numerical solution in the steady state motion is for the period of oscillation of 4π. By varying the equation used for the analytical analysis related to the boundary conditions solutions were obtained for the period of oscillation of 4π and compared to the final periods of oscillation with the numerical solutions it was shown that these solutions align perfectly. By an additional analysis and varying the initial conditions direct numerical solutions were obtained with the period of oscillation 2π. Additionally transient motions obtained numerically are obtained and shown. Through these results it was indicated that the numerical solution for the zero values of the initial position and velocity values is in the beginning unstable with the period of oscillation 2π where it changes to an stable motion with the period of oscillation 4π.

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REFERENCES

[1] Zukovic, M., Hajradinovic, D., Kovacic, I.: On the dynamics of vibro-impact systems with ideal and non-ideal excitation. Meccanica 56, 439-460 (2021)
[2] Zukovic, M., Hajradinovic, D. & Kovacic, I. Vibro-impact system with non-ideal excitation: analytical investigations. Nonlinear Dyn 106, 105–123 (2021).
[3] Pavlovskaia E, Hendry D C, Wiercigroch M (2015) Modelling of high frequency vibro-impact drilling. International Journal of Mechanical Sciences 91: 110–119
[4] Batako AD, Babitsky VI, Halliwell NA (2003) Modelling of vibro-impact penetration of self-exciting percussive-rotary drill bit. Journal of Sound and Vibration 271: 209–225
[5] Karayannis I, Vakakis AF, Georgiades F (2008) Vibro-impact attachments as shock absorbers. Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science 222: 1899-1908
[6] Rong H, Wang X, Xu W, Fang T (2010) Resonant response of a non-linear vibro-impact system to combined deterministic harmonic and random excitations. International Journal of Non-Linear Mechanics 45(5): 474–481
[7] Liu Y, Pavlovskaia E, Hendry D, Wiercigroch M (2013) Vibro-impact responses of capsule system with various friction models. International Journal of Mechanical Sciences 72: 39–54
[8] Liu Y, Pavlovskaia E, Wiercigroch M, Peng Z (2015) Forward and backward motion control of a vibro-impact capsule system. International Journal of Non-Linear Mechanics 70: 30–46
[9] Marcus VM, Arthur GM, Anderson LS, Jose MB, Angelo MT, Clivaldo O (2019) Modeling and Experimental Validation of Two Adjacent Portal Frame Structures Subjected to Vibro- Impact. Latin American Journal of Solids and Structures
[10] Marzbanrad J, Shahsavar M, Beyranvand B (2017) Analysis of force and energy density transferred to barrier in a single degree of freedom vibro-impact system. Journal of Central South University 24(6): 1351–1359
[11] Gendelman OV, Alloni A (2015) Dynamics of forced system with vibro-impact energy sink. Journal of Sound and Vibration Volume 358: 301-314
[12] Hedrih KR, Raičević V, Jović S (2010) Vibro-impact of a Heavy Mass Particle Moving along a Rough Circle with Two Impact Limiters. International Journal of Nonlinear Sciences and Numerical Simulation 11
[13] Jović S, Raičević V (2010) Energy Analysis of Vibro-Impact System Dynamics Based on a Heavy Mass Particle Free Oscillations Along Curvilinear Rough Trajectories. Scientific Technical Review 60: 9-21
[14] De Souza SLT, Caldas IL, Viana RL (2007) Damping control law for a chaotic impact oscillator. Chaos, Solutons & Fractals 32(2): 745–750
[15] Babitsky VI (1998) Theory of vibro- impact systems and applications. Springer
[16] Li T, Qiu D, Seguy S, Berlioz A (2017) Activation characteristic of a vibro-impact energy sink and its application to chatter control in turning. Journal of Sound and Vibration 405: 1–18
[17] Nayfeh AH, Mook DT (1995) Nonlinear Oscillations. Wiley Classics Library Edition Published
[18] Cveticanin L, Zukovic M, Balthazar JM (2018) Dynamics of Mechanical Systems with Non-Ideal Excitation. Mathematical Engineering, Springer
[19] Kononenko VO (1969) Vibrating systems with a limited power supply. Iliffe
[20] Zukovic M, Cveticanin L, Maretic R (2012) Dynamics of the cutting mechanism with flexible support and non-ideal forcing. Mechanism and Machine Theory 58: 1–12
[21] Kovacic I, Zukovic M (2016) Coupled purely nonlinear oscillators: normal modes and exact solutions for free and forced responses. Nonlinear Dynamics 87(1): 713–726