ISSN: 0974-5823 Vol. 7 No. 1 January, 2022

International Journal of Mechanical Engineering

Relevance of Queuing theory to relegate waiting epoch at the Covid-19 vaccination center using simulated approach

[1]Parvathy. G and [2]Sreelatha. K. S

[1]Department of Mathematics, Amrita School of Arts and Science, Amrita Vishwa Vidyapeetham Amritapuri Campus, Kollam, Kerala, India.

[2]Assistant Professor, Department of Mathematics, Amrita School of Arts and Science, Amrita Vishwa Vidyapeetham, Amritapuri Campus, Kollam, Kerala, India

Abstract - Long queues are seen in our everyday life. A queue forms when the arrival of customers exceeds the number of customers taken to service. Queueing theory is modelling mathematical models of different queueing systems to identify the character of the queue. The Covid-19 pandemic has ravaged the world, posing an unprecedented threat to humanity. And Covid-19 vaccination programmes are underway globally. In this study, we try to understand the queue form in-frond of a Covid-19 vaccination centre and some of the particular models that are helpful in the evaluation of the queue of the Covid-19 vaccination centre. Data analysis can be done by collecting data from the primary health centre for a period of time.

Index Terms - Arrival rate, Covid-19, M/D/1 queueing model, M/M/1 queueing model, Service rate.

INTRODUCTION

Waiting in a queue for services is a common phenomenon in our daily life. To quantify the phenomenon of waiting in lines, researchers calculate relevant performance measures such as mean line length, mean waiting time in line, and mean facility usage. Agner Krarup Erlang, a Danish mathematician and engineer, first proposed queueing theory in the early twentieth century. Erlang was employed by the Copenhagen Telephone Exchange, and he intended to examine and improve the company's processes. Nowadays we know, Covid-19 pandemic hits all over the world. More than 50 lacks people lost their life due to the Covid-19 disease. Today as a partial cure the Covid-19 vaccines are discovered, and it is distributed all over the world. The India Government take initiative to distribute free vaccine through primary health centres, Hospitals Vaccination camps etc. In this paper, we used the queue from the Covid-19 vaccination centre. In this paper, we aim to find a suitable model which helps to reduce the waiting time in-frond of the Primary heath-centre.

METHODOLOGY

A queue system has a fairly straightforward mechanism. Customers are greeted by one or more waiters when they arrive at the services counter. A customer leaves the system as soon as he/she has been serviced. Customers arriving for service, waiting for service if it is not instantaneous, and if they have desired service, exiting the system after being serviced can be defined as a queuing system.

Queuing Theory Distribution

The arrival and departure of the customer are random and independent. The Poisson distribution is a good choice if the arrivals are all random and independent. The probability for Poisson process is:

$$F(x) = \frac{\lambda x e^{-\lambda}}{x!} \quad \forall\ x\geq 0,\ \lambda\geq 0 \qquad (1)$$

Little's Law

The average number of customers in a stationary system (L) is equal to the average arrival rate (λ) multiplied by the average time that a customer spends in the system (W), according to John Little's law.

$$L=\lambda\times W \qquad (2)$$

Model Used

In operations research, there are numerous queuing models that can be used.
The general form of a queuing model is Kendall's Notation.
Kendall's notation

(A/B/c): (N/D) (3)

A = Arrival time distribution
B = Service time distribution.
c = No: of servers on the system.
N = No: of clients on line.
D = Queue discipline.

M/M/1 Queueing Model

The model represented as
(M/M/1) : (∞ /FCFS)
This is a single server queue with infinite capacity, the service disciple is first come first serve. These M/M/1 queues are the simplest queuing model among queuing models and the arrival and departure follows Poisson distribution and the service rate is distributed exponentially.. The queue is considered as stable if the Traffic intensity $\rho<1$.
Mean arrival rate : λ Mean service rate : μ

Table 1. The steady-state parameters for M/M/1 Model

Attributes	Equation
Traffic intensity	$\rho = \frac{\lambda}{\mu}$
The mean no: of people in the system	$Ls = \frac{\lambda}{\mu - \lambda}$
The mean no: of people in the queue	$Lq = \frac{\lambda^2}{\mu(\mu - \lambda)}$
The mean waiting time in the system	$Ws = \frac{1}{\mu-\lambda}$
The mean waiting time in the queue	$Wq = \frac{\lambda}{\mu(\mu-\lambda)}$

M/D/1 Queuing model

The M/D/1 model represented as
(M/D/1): (FCFS:∞),
Which states that this is an endless queue, that the queue's arrivals follow a Poisson process, and that the service time is constant (deterministic). It has a single server. Also the queue disciple is first come first serve. Customers arrive at random and independently. Service is unaffected by the length of the queue.

Arrival Rate :λ Service Rate :μ

Table 2. The steady-state parameters for M/D/1 Model

Attributes	Equation
Traffic intensity	$\rho = \frac{\lambda}{\mu}$
The mean number of customers on system	$Ls = \rho + \frac{1}{2}(\frac{\rho^2}{1-\rho})$
The mean number of customers on line	$Lq = \frac{1}{2}(\frac{\rho^2}{1-\rho})$
The mean time delaying on system	$Ws = \frac{1}{\mu} + \frac{\rho}{2\mu(1-\rho)}$
The mean time delaying on line	$Wq = \frac{\rho}{2\mu(1-\rho)}$

DATA ANALYSIS

The data was collected from the Primary Health centre where the Covid-19 vaccination provided, by the observed data provided by the authorities. The cost of the vaccine is free. The number of slot or people arrive at the centre is depends on the availability of vaccine. On the day of observation there are 72 customers arrived for vaccination, and vaccination service starts at 9:54am and ends at 2:14pm. The customers arrived from 9:00am.

Table 3: Arrival and departure of customers in 30 min interval

Intervals	Arrival/30 min	Departure /30 min
9:00-9:30	21	0
9:30-10:00	15	0
10:00-10:30	15	5
10:30-11:00	3	25
11:00-11:30	6	23
11:30-12:00	2	4
12:00-12:30	5	5
12:30-1:00	0	4
1:00-1:30	3	1
1:30-2:00	2	3
2:00-2:30	0	2

Table 3. Analysed data and its values using M/M/1

Attributes	Symbol	Values
Number of customers	n	72
Arrival rate	λ	1.886
Service rate	μ	1.945
Traffic intensity	ρ	0.96967
Mean waiting time in the queue	Wq	16.435
Mean waiting time in the system	Ws	16.949
The mean no: of people on line	Lq	30.996
The mean no: of people on system	Ls	31.9661

Table 4. Analysed data and its values using M/D/1

Attributes	Symbol	Values
Number of customers	n	72
Average rate of customers arrived per unit time	λ	1.886
Average rate of customers served per unit time	μ	1.945
Utilization factor	ρ	0.96967
Mean delaying time on the line	Wq	8.4667
Mean delaying time on the system	Ws	8.73164
The mean no: of people on line	Lq	15.9683
The mean no: of people on system	Ls	16.4678

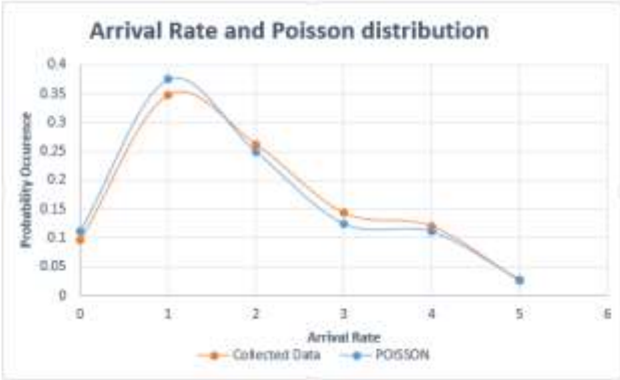

Figure 1: Calibration of Arrival rate and Poisson distribution

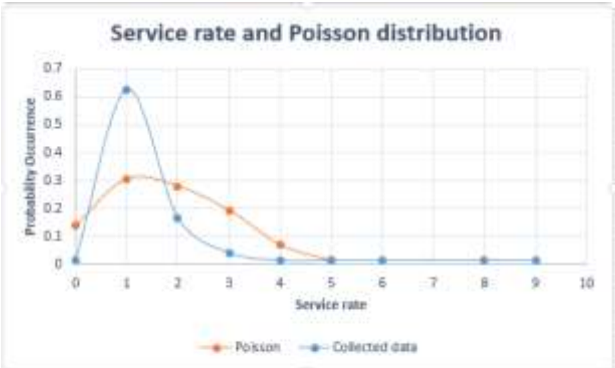

Figure 2: Calibration of service rate and Poisson distribution

LIKENESS OF M/M/1 AND M/D/1 MODEL TO THE OBSERVED DATA

Table 5. Likeness between the M/M/1 and M/D/1 model to Observed Data.

PM	Observed Data	Computed data		Relative error		Percentage error	
		M/M/1	M/M/1	M/M/1	M/D/1	M/M/1	M/D/1
Ls	25.23	31.96	16.467	0.2667	0.3461	26.67	34.61
Lq	24.15	30.99	15.986	0.2832	0.3383	28.32	33.83
Ws	15.04	16.949	8.731	0.1269	0.4194	12.69	41.94
Wq	14.43	16.435	8.446	0.1389	0.4146	13.89	41.46

(PM-Performance measures)

M/M/1 MODELING AND SIMULATION

Simulating a physical phenomena with a set of mathematical formulas is known as simulation. The mathematical model for M/M/1 queueing model is derived by using simulation on Microsoft excel with assumptions such as, the customers arrived via Poisson distribution. And also at an independent rate. The services is follows an exponential process.

entity	Arrival time	Random no:	Inter arrival Time	Starting Service	Random No:	Service time	Finishing Time	Wq	W
1	0	0.636633	0.536767	0	0.205434	0.118231	0.118231	0	0.514139
2	1.173401	0.233684	0.141125	1.173401	0.510849	0.367652	1.541053	0	0.514139
3	0.374809	0.315845	0.201257	1.541053	0.467025	0.323538	1.864591	1.166244	1.680383
4	0.517102	0.440241	0.307661	1.864591	0.992884	2.542632	4.407223	1.347488	1.861627
5	0.747903	0.707589	0.651959	4.407223	0.470713	0.327108	4.73433	3.65932	4.173459
6	1.359548	0.912807	1.293548	4.73433	0.355857	0.226136	4.960467	3.374782	3.888921
7	2.206356	0.101159	0.056548	4.960467	0.77562	0.768336	5.728802	2.754111	3.26825
8	0.157707	0.180829	0.105759	5.728802	0.913989	1.261329	6.990132	5.571095	6.085234
9	0.286588	0.4235	0.292036	6.990132	0.601518	0.473056	7.463188	6.703544	7.217683
10	0.715536	0.799235	0.851337	7.463188	0.576	0.441142	7.904329	6.747651	7.26179
11	1.650573	0.134749	0.076742	7.904329	0.707677	0.632337	8.536667	6.253757	6.767896
12	0.211491	0.806851	0.871843	8.536667	0.698113	0.615785	9.152452	8.325176	8.839315
13	1.678694	0.330017	0.212356	9.152452	0.671742	0.572728	9.72518	7.473758	7.987896
14	0.542372	0.928953	1.402131	9.72518	0.810852	0.856158	10.58134	9.182808	9.696946
15	2.331085	0.364111	0.240048	10.58134	0.138133	0.076429	10.65777	8.250254	8.764392
16	0.604159	0.811027	0.883432	10.65777	0.574369	0.439168	11.09694	10.05361	10.56775
17	1.69446	0.890058	1.170628	11.09694	0.187593	0.106815	11.20375	9.402476	9.916614
18	2.060686	0.977592	2.01397	11.20375	0.82418	0.893724	12.09747	9.143064	9.657202
19	2.991563	0.602672	0.489392	12.09747	0.592706	0.46181	12.55928	9.105911	9.62005
20	1.092064	0.654422	0.563381	12.55928	0.012754	0.0066	12.56588	11.46722	11.98136

Figure 3: M/M/1 Simulation of the observed data

Figure 1 shows apart of simulation on excel that use to model the M/M/1 model.

By using the simulation we calculated the M/M/1 model's simulated values such as Ws, Wq, Ls, Lq.

Table 6. Simulation results on M/M/1.

Wq	Ws	Lq	Ls
18.63962	18.12548	35.15432	35.25406

LIKENESS OF M/M/! MODEL'S SIMULATED AND THEORETIC VALUES.

Table 7. Simulated and theoretical values of M/M/1

	Wq	Ws	Lq	Ls
Simulated Values	18.6396	18.1254	35.15432	35.25406
Theoretical Values	16.435	16.949	30.996	31.966

RESULTS

A. Result

From table 5 the comparison between M/M/1 and M/D/1 to collected data we can observe that the M/M/1 model is much more accurate than the M/D/1 model. The percentage error on the M/M/1 model is in a range of 12.69% to 28.69% and the percentage error on the M/D/1 model is in a range of 33.83% to 41.94%. And the arrival rate and service rate of the queue follows the Poisson distribution.

B. Result

From table 6 and table 7 we can say that the simulated value of the M/M/1 model be near to the Theoretical value calculated ion table 3. This gives the idea that the simulated model for M/M/1 model is accurate.

CONCLUSION

This research report is based on a queue form in a single-server vaccination centre. Based on theoretical and simulated evidence, we may say that the M/M/1 model is accurate. Yet, when the data collected is evaluated to other analytical methods, it becomes evident that neither the M/M1 nor the M/D/1 models are correct. On the other side, the M/M/1 model is marginally more precise than the M/D/1 model. Despite the fact that M/M/1 model has many of the same characteristics as the observed model, it can be used to improve the system and service by minimising client wait times in large queues..

REFERENCES

[1]. Hamdy A.Taha, "Operations research an introduction", Person Education, Edition- 9, ISBN 978-81-317-8594-2, 2011.

[2]. Lakhan Patidar "Queuing Theory: A Case Study to Improve the Quality Services of a Restaurant", International conference (ICIREMPS-2014) under TEQIP-II, SIRT, Bhopal. February 22-24, 2014, pp. 1808-1812 (ISBN: 978-93-82518-64-8).

[3]. Akhil M Nair, Sreelatha K S, P V Ushakumari "Application of Queuing Theory to a Railway ticket window", 2021 International Conference on Innovative Practices in Technology and Management (ICIPTM)

[4]. S Devi Soorya and K S Sreelatha, "Application of queuing theory to reduce waiting period at ATM using a simulated approach", IOP Conference Series: Materials Science and Engineering, Volume 1145, International Conference on Chemical, Mechanical and Environmental Sciences (ICCMES 2021), 25th-26th March 2021, Coimbatore, India

[5]. Thoshiba Sheikh, "APPLICATION OF QUEUING THEORY FOR THE IMPROVEMENOF BANK SERVICE", International Journal of Advanced Computational Engineering and Networking, ISSN: 2320-2106 Volume- 1, Issue- 4, June-2013

[6]. Sathidevi. C, "The Reasons for dropouts of adult learners from adult education centres in Kerala –

A Case Study", Intrnational Journal of Advance Research in Science and Engineering, vol. 07, issue 03,pp. 791-802,2018.

[7]. "Queuing theory: Definition, history & real-life applications"; https://queue-it.com/blog/queuing-theory/

[8]. "Queueing Theory", Wikipedia: https://en.wikipedia.org/wiki/Queueing_theory

[9]. "Lesson 16. INTRODUCTION AND CLASSIFICATION OF QUEUES" ;http://ecoursesonline.iasri.res.in/mod/resource/view.php?id=90033

[10]. "M/D/1queue", https://en.wikipedia.org/wiki/M/D/1_queue

[11]. P.V. Ushakumari, S. Devi Krishna, "OPTIMAL SERVER TIME MANAGEMENT IN A SINGLE SERVER QUEUE", InternationalJournal of Advanced Research in Engineering and Technology, Vol.11, Issue 6, pp. 50-58,2020

.