

A Comparative Study of Identifying the Most Prompting Nodes in a Network Using Efficiency of Navigating Agents

¹Malavika S, ²Sarga G, ³Manjusha R

¹Postgraduate student, Amrita Vishwa Vidyapeetham, Amritapuri, India

²Postgraduate student, Amrita Vishwa Vidyapeetham, Amritapuri, India

³Assistant Professor, Amrita Vishwa Vidyapeetham, Amritapuri, India

Abstract - Graph theory and network analysis are related in the sense that graphical measures are useful to understand and measure the influence of each node in the network. A properly designed robots called basis elements are placed in the network at distinctive locations to observe the activities of the elements in the network. Metric dimension of a graph is the least number of such robots and these robots navigating through the network can measure the shortest distance between any two nodes in the network. In this paper, we explicitly calculate the number of nodes identified by each of the robots and compares with those vertices located at a maximum distance from each of these robots. In that sense we identify the role of other nodes in a network by using two graphical measures namely capacity-I index and eccentricity of the robots.

Index Terms - capacity-I index, centrality, efficiency, eccentricity, metric dimension

INTRODUCTION

The traditional fundamental monitoring of a network system mainly depends on annual work, which require labor and it is a time overwhelming practice. Sensor monitoring system is deliberated as a hopeful resolution to discourse these problems. Monitoring the activities of the components involved in the network plays vital role in many fields such as industries that manufacturing robots and various other real life applications. It is not unusual to realize that most of the surveillance and security applications also depend on sensor grounded applications. The efficiency of the sensors placed on some isolated areas in the network should be analyzed by time to time as it is organized with batteries. Examining the functioning of the whole network system can be done effectively by applying mathematical aspects.

Any network can be characterized as a graph at which the vertices are the components and edges are the connection between them. Metric dimension of graph can be used as a measure for navigation in which the navigation agent (robot) moves from vertex to vertex of a graph. The robot can locate itself with respect to the labelled vertices in the graph. Also, the position of these robots are uniquely determined so as to monitor the activities of the vertices in an effective manner.

The theory of metric dimension of graph was found by P.J.Slater and developed by Harary and Melter. Metric dimension of graph is the least number of vertices or robots required for locating every vertices in the graph. It is assumed that these robots are properly designed so that it knows the shortest distance to every vertices and an locate all the vertices in the graph by means of distances from it.

The main encounter of a navigation problem is to find the minimum number of robots required to monitor the system components in the network. Sometimes more than one robot an locate one vertex. So, in order to reduce the cost a minimum number of robots can be assigned by avoiding such overlapping. In this paper, we analyze the navigation of robots by comparing the capacity-I index to determine the most important robot and its position. This robot or robots is considered as the navigation heads (NHs). We propose an enhanced approach to reorder the positioning of the NHs so that route of the navigation can be designed continuously from anywhere and anytime.

PROCEDURE FOR PAPER SUBMISSION

Definition

A graph G consisting of a vertex set $V(G)$, an edge set $E(G)$, and a relation that connects an edge having two end vertices. A loop is an edge with same end vertices and if two or more edges have same end vertices are termed as parallel edges. A simple graph is a graph that does not have any loops or parallel edges. We specify that an edge set as a set of pairs of vertices which is not ordered and writing $e = uv$ (or $e = vu$) for an edge e with endpoints u and v . When u and v are the end vertices of any edge, they are adjacent and are called neighbors. A graph G is linked if there is a path between each pair of vertices in G , otherwise, G is called disconnected. If G has a $u - v$ path, then the distance from u to v , denoted by $d(u, v)$ and is defined as the least length of $u - v$ path. If G has no such path, then we say that $d(u, v) = \infty$.

Definition

The least number of sensors needed to find every nodes of the network structure is called the metric dimension and the set of all least probable number of robots is called basis. Let $W = \{w_1, w_2, \dots, w_m\}$ be an ordered set of vertices of G and let v be a vertex of G and $d(v, w_i), i = 1, 2, \dots, m$ denote the least distance from v to w_i . The co-ordinate of a vertex v is an ordered pair $(d(v, w_1), d(v, w_2), \dots, d(v, w_m))$. If each vertex of G have different co-ordinates with respect to W , then W is said to be a resolving set of G . A resolving set or a location of least cardinality is said be a basis for G and this number is called the metric dimension or location number of G and is denoted by (G) . In figure.1, $(G) = 2$ and u and v are the basis elements.

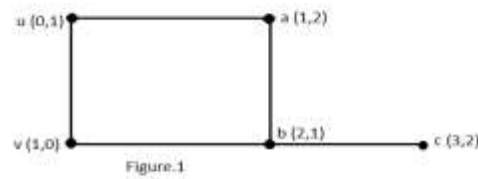


Figure.1
F
ig. 1. A connected simple graph with metric dimension 2

Definition

The *Capacity – l index* of an element in the basis set, v_j for $j = 1, 2, \dots, n$ is denoted by $Cal(v_j)$ and is the number of vertices of G recognized by v_j with respect to $d(v_i, v_j) = l$ for $i = 1, 2, \dots, m$. The graph given in the figure 2 has metric dimension two with respect to the basis $W = \{a, b\}$. Here $Cal(a) = 2$ and $Cal(b) = 2$ with regard to $d(v_i, v_j) = 1$.

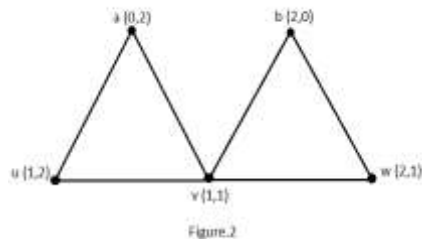


Figure.2
Fig. 2. The robots a and b and its capacity – l index with respect to $d(v_i, v_j) = 1$

LOCALIZATION OF NAIGATION HEADS IN A NETWORK CORRESPONDING TO CAPAITY-I INDEX

Localization is one of the most fundamental problem in any network. It is worth investigating to analyse the efficiency of the robots so as to monitor the activiteis of the constituents in the network. The *Capacity – l index* measure can be used to analyse the most important robot in the network.

Navigation heads in a data passing network

Identification of dominant nodes in complex networks is an area of moving development meanwhile it can help us to deal with various problems. The figure give below represents data communication and computer network. Various graphical measures have been proposed over the past years to identify the important nodes in a network. Here we propose a measure based on the efficiency of robots placed in the network for monitoring the activities of network components

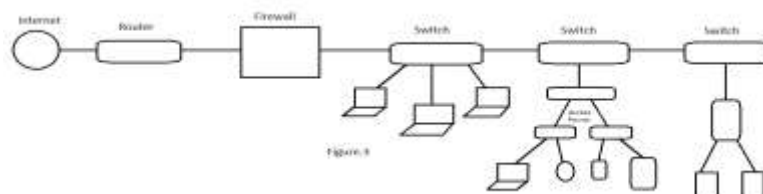


Figure.3
Fig. 3. A data communication and computer network

Graphically representing the above network, we obtain the metric dimension and locate the positions at which the robots are to be placed. Here $W = \{R_1, R_2, \dots, R_9\}$ is the basis and the coordinates of all the vertices are given in the graph given below.

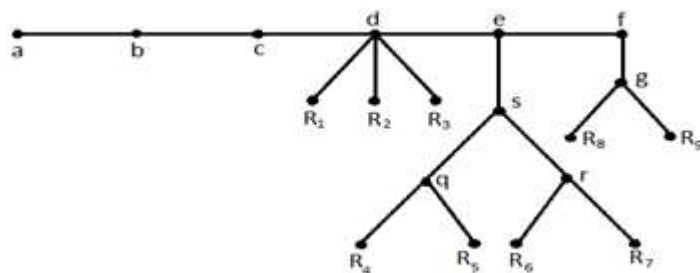


Fig. 4. Navigating agents and their positions in the network

The following table gives the coordinates of all the vertices in the network. Here shortest distance from each of the robots to vertices differ from 1 to 7. Therefore, the diameter and radius of the network is 7 and 1 respectively.

Table 1. Coordinates of the vertices relative to the basis W

Vertices	Coordinates
a	(4,4,4,7,7,7,7,7,7)
b	(3,3,3,6,6,6,6,6,6)
c	(2,2,2,5,5,5,5,5,5)
d	(1,1,1,4,4,4,4,4,4)
e	(2,2,2,3,3,3,3,3,3)
f	(3,3,3,4,4,4,4,2,2)
g	(4,4,4,5,5,5,5,1,1)
R1	(0,2,2,5,5,5,5,5,5)
R2	(2,0,2,5,5,5,5,5,5)
R3	(2,2,0,5,5,5,5,5,5)
R4	(5,5,5,0,2,4,4,6,6)
R5	(5,5,5,2,0,4,4,6,6)
R6	(5,5,5,4,4,0,2,6,6)
R7	(5,5,5,4,4,2,0,6,6)
R8	(5,5,5,6,6,6,6,0,2)
R9	(5,5,5,6,6,6,6,2,0)

Among these nine robots traversing within the network so as to detect the activities of the components, it is worth investigating to find the most important robots. Some of the graphical measures used for detecting the most prominent vertices in a graph are centrality measures. Here we try to relate centrality measures with the capacity index of the robots.

Table 2. Capacity – l index of the nine robots with respect to $d(v_i, v_j) = l$

Robots	Cal(R_i) for $l = 1$	Cal(R_i) for $l = 2$	Cal(R_i) for $l = 3$	Cal(R_i) for $l = 4$
R1	1	4	3	4
R2	1	4	3	4
R3	1	4	3	4
R4	1	2	2	4
R5	1	2	2	4
R6	1	2	2	4
R7	1	2	2	4
R8	1	2	1	2
R9	1	2	1	2

Robots	Cal(R _i) for $l = 5$	Cal(R _i) for $l = 6$	Cal(R _i) for $l = 7$
R1	6	0	0
R2	6	0	0
R3	6	0	0
R4	5	3	1
R5	5	3	1
R6	5	3	1
R7	5	3	1
R8	5	5	1
R9	5	5	1

By considering other centrality measures, we can make an analytical survey to evaluate how do network measures a correlation between network data components. Eccentricity and degree centrality are two important graphical measures to target the nodes that shows influences on the communication between other nodes in the network.

So, with respect to maximum geodesic distance from the robots to all other nodes, on the average we can indicate the targeted nodes among nine robots even if these measures vary very slightly in this network.

In Table 1. Each of the nine robots have same degree centrality means that they equally detecting the communication channels in the network. But, relative to geodesic from these robots to other nodes, we can identify all those robots that make sense in observing the activities of other nodes.

Table 3. Eccentricity and degree centrality of the nine robots

Robots	e(R _i)	D(R _i)
R1	5	0.055
R2	5	0.055
R3	5	0.055
R4	7	0.055
R5	7	0.055
R6	7	0.055
R7	7	0.055
R8	7	0.055
R9	7	0.055

Table 4. Degree centrality of vertices (non - robots)

Vertices	D(i)
a	0.055
b	0.111
c	0.111
d	0.277
e	0.166
f	0.111
g	0.166
s	0.166
q	0.166
r	0.166

On the average the maximum number of vertices identified by the robots is corresponds to $l = 2, 4, 5$ as described in Table 2. Now, consider the dominating data values of graphical measures mentioned above are with respect to $l = 2, 4, 5$.

In the following chart, series 1,2,3,4,5 represents eccentricity, capacity – l index for $l = 2, 4, 5$, degree centrality of robots and non – robots respectively. With respect to this dominating set of l values, it is recognized that robots R₄, R₅, R₆, R₇ are the most prominent. It is obvious that d is the vertex with high degree centrality and R₄, R₅, R₆, R₇ locate the vertex d through a geodesic

of length 4. Hence, the targeted nodes that influences the flow or activities of other nodes in the network we discussed in this paper is with respect to the capacity – l index 4.



Fig. 4. Multiple chart representing the relation between centrality measure and capacity – l index

CONCLUSION

In this paper, we analyze that centrality measures helps to locate the prominent robots and their position with respect to the capacity – l index. These robots can access the functioning of the other nodes in the network. We concentrated on some simple network structure however there are numerous revisions is worth investigating for finding the influential nodes in a complex network.

REFERENCES

- [1] P. J. Slater. Leaves of trees. In: Proc. 6th Southeastern Conf. on Combinatorics, Graph Theory and Computing, 14, pp 549-559, 1975.
- [2] Harary and R. A. Melter, "On the Metric Dimension of a Graph," *Ars Combinatorica*, Vol. 2, 1976, pp. 191-195.
- [3] G. Chartrand, L. Eroh, M. A. Johnson, and O. R. Oellermann. Resolvability in graphs and Metric Dimension of Graph. *Discrete Appl. Math.*, 105(1-3), pp. 99-113, 2000.
- [4] S. Khuller, B. Raghavachari and A. Rosenfeld. Landmarks in Graphs. *Discrete Appl. Math*, 70(3), pp. 217-229, 1996.
- [5] R. Manjusha, A. Sunny Kuriakose, Metric Dimension and Uncertainty of Traversing Robots in a Network, *International Journal on Applications of Graph Theory in Wireless Ad Hoc Networks and Sensor Networks (GRAPH-HOC)*, 7 (2015), no. 2-3, 1-9
- [6] NarsinghDeo, "Graph Theory with Application to Engineering and Computer Science", 1974 by Prentice Hall.
- [7] Malavika S, Sarga G, Manjusha R, "Integrated Monitoring of system components using metric dimension of graphs", *Material's Today Proceedings*. 2020.
- [8] Manjusha. R, Sunny Kuriakose A, "Efficiency of Basis Elements and Fuzzy Weighted graph", *Applied Mathematical Sciences*, Vol. 11, 2017, no. 59, 2941 – 2949.
- [9] Brandes, U.: Faster Evaluation of Shortest-Path Based Centrality Indices. *Kon-stanzer Schriften in Mathematik und Informatik*, 120 (2000).
- [10] Bader, D. A., Kintali, S., Madduri, K., Mihail, M.: Approximating Betweenness Centrality. *The 5th Workshop on Algorithms and Models for the WebGraph (2007)*124–137.
- [11] Athira Krishnan, Manjusha R, Sunny Kuriakose A, "A study on network analysis by navigation of robots using graphical measures to solve the complexity." *Journal of physics: Conference series*, 2021.
- [12] Devidas A R, Ramesh M V, Rangan V P, 'High performance communication architecture for smart distribution power grid in developing nations', *Wireless Networks*, 2018, 24(5).