FUZZY QUEUEING MODEL WITH AN UNRELIABLE SERVER

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ABSTRACT

As a queuing model with an unstable server, the arrival rate and service rate of customers as well as the breakdown and repair rate of server are all fuzzy values, this work creates the system characteristics' membership functions. For a fuzzy queuing model with an unstable server, the service rate, entrance rate, breakdown (collapse) rate, and repair rate may be found in crisp numbers using this technique. Weird numbers are used for all these rates. Apply the traditional queueing performance metrics formulae after de-fuzzifying the fuzzy numbers. The method's implementation is shown using numerical examples.

Keywords: Fuzzy Queues, Nonlinear programming, Queue, Octagonal Fuzzy Number, Unreliable server.

INTRODUCTION

Most queueing models are not able to maintain the server functioning at all times, hence service might be disrupted. Because of the failure of the server. Scheduled service pauses, such as on weekends or public holidays, are also a possibility. As far as server breakdown models go, Gaver presented an M/G/I queueing system with interrupted service and priority. Sengupta expanded his method to the GI/G/I scenario. Unreliable servers have recently been studied by Li et al. and Wang for their effects on M/G/I queuing models and controlled M/HK/I queuing systems with an unreliable server. Gurukajan and Srinivasan presented a sophisticated two-unit system in which the repair facility is vulnerable to random breakdowns.

PRELIMINARIES

Definition 2.1: Let X be a universal set. Then the fuzzy subset A of X is defined by its membership function X, where the value of∈A(x) in the interval to each element xμ which assign a real number →A: X μ A(x) at x shows the grade of membership of x in A. The membership function of a fuzzy set is known as aμ possibility distribution.

Definition 2.2: Given a fuzzy set A in X and any real number α then the α-cut or α-level or cut worthyset of A, denoted by αA is the crisp set. The strong denoted by α+A is the crispset. For example, let A be a fuzzy set whose membership function is given as,

$$\mu_{A(x)} = \begin{cases} \frac{x-a}{b-a} & if a \le x \le b \\ \frac{c-x}{c-b} & if b \le x \le c \end{cases}$$

= (x – a) /α [0,1] to both left and right reference functions of A. That is, ∈To find the α-cut of A, we first set α, asα = (c –x) / (c – b). That is, x can be expressed in terms of α(b – a) and∈]. x α+a, c- (c-b)αA = [(b – a)α-cut of A as α which gives α+a and x = c-(c-b)αx = (b – a)[a,b] is called the left reference function, which is right continuous, monotone and non-decreasing, while the x [b, c] is called right reference function, which is left continuous, monotone and non- increasing. The above∈ definition of a fuzzy number is known as an L-R fuzzy number.

Definition 2.8: Triangular Intuitionistic Fuzzy Number TIFN is defined as follows:

Let A be a Intuitionistic fuzzy set defined by $A = \{(x, \mu_A(x), v_A(x)): x \epsilon X\}$ Let the membership function A is defined asμ

$$\mu_{A(x)} = \begin{cases} \frac{x-a_1}{b_1-a_1} & if a_1 \le x \le b_1 \\ \frac{c_1-x}{c_1-b_1} & if b \le x \le c_1 \end{cases}$$

A is given byvand non membership function

$$v_{A(x)} = \begin{cases} \frac{b_1-x}{b_1-a_1'} & if a_1' \le x \le b_1 \\ \frac{x-b_1}{c_1'-b_1} & if b_1 \le x \le c_1' \end{cases}$$

Where $a' < a < b < c < c'$ and $\mu_{A(x)}, v_{A(x)} \le 0.5$

$\mu_{A(x)}, v_{A(x)}$

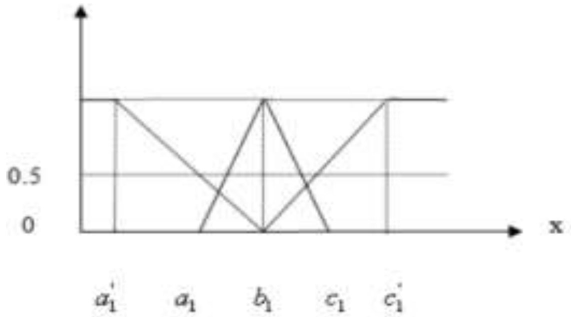

Figure 2.1 Membership and Non membership values of Triangular Intuitionistic Fuzzy Number

If we put $a' = a, and c' = c and v_{A(X)} = 1 - \mu_{A(X)} for all x \in X$ then TIFN becomes Triangular Fuzzy Number TFN.

Definition-2.9. Trapezoidal Intuitionistic Fuzzy Number (TrIFN): Ã is a subset of IFS in R with membership function and non-membership function as follows.

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1} & for a_1 \le x \le a_2 \\ 1 & for a_2 \le x \le a_3 \\ \frac{a_4 - x}{a_4 - a_3} & for a_3 \le x \le a_4 \end{cases}$$

0 Otherwise

$$v_{\tilde{A}}(x) = \begin{cases} \frac{a_2 - x}{a_2 - a_1'} & for a_1' \le x \le a_2 \\ 0 & for a_2 \le x \le a_3 \\ \frac{x - a_3}{a_4' - a_3} & for a_3 \le x \le a_4' \end{cases}$$

1 otherwise

Where $a_1' \le a_1 \le a_2 \le a_3 \le a_4 \le a_4' and is denoted by \tilde{A}_{TrIFN} = (a_1, a_2, a_3, a_4; a_1', a_1, a_2, a_3, a_4'$

$\mu_{\tilde{A}}(x), v_{\tilde{A}}(x)$

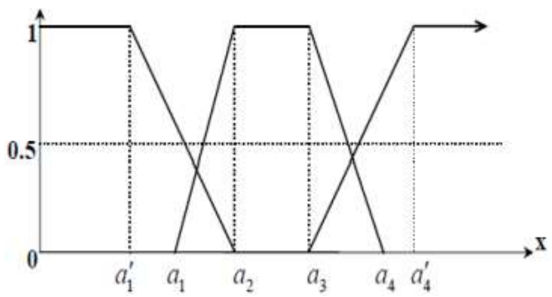

Figure 2.2 Membership and Non membership values of Trapezoidal Intuitionistic Fuzzy Number

UNRELIABLE SERVER

Let us take a fuzzy queueing system with an unreliable server and two types of breakdowns. In type I, there are no customers in the system even if the server is breakdown. In type II, there is at least one customer in the system even if the server is breakdown. Consider the customers arrive at a single server with fuzzy rate λ as a Poisson process, service time with fuzzy rate μ as an exponential distribution, a breakdown with fuzzy rate α as a Poisson process and the repair with fuzzy rate β as an exponential distribution respectively.

Let $\varphi_{\tilde{\lambda}}(x), \varphi_{\tilde{\mu}}(y), \varphi_{\alpha}(s), \varphi_{\beta}(t)$ be the membership functions of λ, μ, $\alpha and \beta$. Then the following fuzzy sets are:

$$\tilde{\lambda} = \{(x, \varphi_{\tilde{\lambda}}(x))/x \in X\}$$

$$\beta^{\sim} = \left\{ \left(y, \varphi_{\tilde{\beta}}(y) \right) / y \in Y \right\}$$

$$\tilde{\alpha} = \left\{ \left(s, \varphi_{\tilde{\alpha}}(s) \right) / s \in S \right\}$$

$$\tilde{\beta} = \left\{ \left(t, \varphi_{\tilde{\beta}}(x) \right) / t \in T \right\}$$

Where X, Y, S and T are the universal crisp sets of the entry, service, break down and repair rates, respectively.

Let f (x, y, s, t) be the system characteristic of interest. Since x, y, s, and t and f (x, y, s, t) are all fuzzy numbers.

Let A and B denotes the membership function of the expected time and the system is idle in type I and type II, respectively.

Type I

$$A = f(x, y, s, t) = \frac{ty - x(s+t)}{y(s+t)}$$

Type II

$$B = f(x, y, s, t) = \frac{ty - x(s+t)}{ty}$$

In steady state, it is required as $0 < \frac{ty - x(s+t)}{y(s+t)} < 1 and\ 0 < \frac{ty - x(s+t)}{ty} < 1$

TECHNIQUES FOR SOLVING PROBLEMS OF QUEUEING MODELS

Queueing models are classified as Markovian queueing models and non-Markovian queueing models. The techniques generally adopted to solve these types of queueing models are explained below.

Markovian Queueing Models

Queueing models with exponential interarrival time and exponential service time are called Markovian queueing models. Some of the techniques used to solve Markovian queueing models are:

1. Difference – differential equation method
2. Neuts matrix-geometric algorithm
3. Continued fraction method

Some queueing systems are studied analytically by deriving the corresponding difference - differential equations and solving them by applying Rouche’s theorem through suitable generating functions. The first method is discussed elaborately by Gross and Harris (1998) and Saaty (1961). Neuts (1981) developed the matrix-geometric algorithmic approach to study the steady state queueing models. This method involves real arithmetic and avoids the calculation of complex roots based on Rouche’s theorem.

CONCLUSION

A queuing model with an unstable server was studied using Fuzzy set theory in this research. In operations and service mechanisms, fuzzy queuing models with unreliable server models have been used to evaluate system performance. A system's performance may be measured using this method. The strong ranking indices distribute the fuzzy numbers. The comparison of the three solved fuzzy numbers. As a result, management may make the most effective and efficient judgments.

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