ISSN: 0974-5823 Vol. 7 No. 1 January, 2022

International Journal of Mechanical Engineering

STUDY OF GENERAL BULK SERVICE QUEUEING SYSTEMWITH BATCH ARRIVAL

Dr. Naveen Kumar[1], Babita[2]

Department of Mathematics, Baba Mastnath University, Asthal Bohar, Rohtak.

ABSTRACT

Here, in this paper a bulk service queueing system with different threshold policies for secondary tasks is examined. To keep the queue from becoming too long, after service is completed, the server runs a secondary task of type one. As soon as it returns from a secondary task of type one, the server conducts another secondary job of type two and so on, until the queue length exceeds a threshold value 'N' (N ≥ b > a) at which point it serves another batch of consumers. A variety of performance metrics and a cost model are shown.

Keywords: Bulk Arrival, Single Server, Batch Service, Vacations, Queuing System.

INTRODUCTION

A complete literature review of bulk service queueing systems can be found. In majority of the works, customer service is supplied in batches of variable sizes with minimum batch size a and maximum batch size b-also termed general bulk service (GBS) rule. In, Ayappan and Renganathan discuss a single server preemptive priority queue in which high priority customers are given service in batches according to GBS Rule and with accessibility to batches, whereas low priority customers are offered service individually. Additionally, in, the service batch sizes are considered to be Markov dependent.Queueing theory was initiated by Erlang. Ke et al. have studied the queueing system with N-policy and at most j vacation. Lee and Kim analyzed the queueing system with vacation interruption and single working vacation. Krishna Reddy et al.analyzed bulk service queueing system with N-policy. Thangaraj and Rajendran studied the batch arrival queueing model with two types of service pattern and two types of vacation. Ke proposed the queueing system under two typesof vacation policies. Balasubramanian and Arumuganathan developed the bulkqueueing model with modified M-vacation policy and variant arrival rate. Recently,Singh et al. investigated the bulk service queueing system with unsatisfied customer, optional service, and multiphase repair. In multiphase repair process, the repaired server may not go under repair immediately due to unavailability of repair man, unavailability of spare parts, or other reasons. Similarly, we have discussed at most j vacation in this model. In at most j vacation, the server findsinadequate number of customers after completing first service; the server will take another vacation until required number of customers is in the system. Many oftheir application can be found in real life with bulk service such as CNC turning machines, soft flow dying machine, vegetable oil refinery, giant wheel, etc.

PRELIMINARIES

We first pack our known data specified in (1.1)–(1.3) into a few generating functions. For any $0 \leq i \leq N-1$, define

$$H_i(s) = \sum_{j=0}^{\infty} h_{ij} s^j \, (0 \leq i \leq N-1)$$

And

$$B(s) = \sum_{j=0}^{\infty} b_j s^j.$$

Here we view B(s) and Hi(s) $(0 \leq i \leq N-1)$ as complex functions. Note that B(s) and Hi(s) $(0 \leq i \leq N-1)$ may have their (usually different) convergence radii. But due to conditions (1.2) and (1.3), they are all well-defined at least on the closed unit disc {s; |s| ≤ 1} and analytic on the open unit disc {s; |s| < 1}. Considering that the q-matrix Q given in (1.1)–(1.3) is conservative and bounded, the corresponding Q-process is unique and is just the Feller minimal Q-process. It follows that the Feller minimal Q-resolvent R(λ) satisfies both the Kolmogorov backward and the forward equations. Using the Kolmogorov forward equations, we immediately obtain the following construction theorem which will be the starting point for our further analysis.

Theorem .1 For any i ≥ 0, the Feller minimal Q-resolvent R(λ) = {ri j(λ); i, j ∈ Z+} satisfies the equation

$$\sum_{j=0}^{\infty} r_{ij}(\lambda) s^j = \frac{\sum_{k=0}^{N-1} r_{ik} \lambda (B(s)s^k - H_k(s)s^N) - s^{i+N}}{B(s) - \lambda s^N},$$

Where B(s) and Hk (s) $(0 \leq k \leq N-1)$ are defined in (2.1) and (2.2), respectively.

Proof By the Kolmogorov forward equation λR(λ) − I = R(λ)Q, together with noting the form of Q given in (1.1), we immediately obtain that, for any i, j ∈ Z+,

$$\lambda r_{ij}(\lambda) - \delta_{ij} = \sum_{k=0}^{N-1} r_{ik}(\lambda) h_{kj} + \sum_{k=N}^{j+N} r_{ik}(\lambda) b_{j-k+N}.$$

Multiplying by sj, where |s| < 1, on both sides of (2.4) and summing over j from 0 to ∞ yields

$$\lambda \sum_{j=0}^{\infty} r_{ij}(\lambda) s^j - s^i = \sum_{j=0}^{\infty} \left(\sum_{k=0}^{N-1} r_{ik}(\lambda) h_{kj} \right) s^j + \sum_{j=0}^{\infty} \left(\sum_{k=N}^{j+N} r_{ik}(\lambda) b_{j-k+N} \right) s^j.$$

By noting the definitions given in (4.1) and (4.2), we immediately obtain

$$\lambda \textstyle\sum_{j=0}^{\infty} r_{ij}(\lambda) s^j - s^i = \sum_{k=0}^{N-1} r_{ik}(\lambda) h_k(s) + \frac{B(s)}{s^N} \sum_{k=N}^{\infty} r_{ik}(\lambda) s^k.$$

Now easily follows from, which ends the proof.

By **Theorem .2**, particularly by, it is clear we need to define, for each λ > 0,

$$B_\lambda(s) = B(s) - \lambda s^N$$

Which isC∞ at least on (−1, 1). Similarly to B(s), we view Bλ(s) as complex functions of s and note that Bλ(s) is well defined, at least on the closed unit disc {s; |s| ≤ 1}, and is analytic on the open unit disc {s; |s| < 1}.

We now provide a couple of fundamental lemmas which will be our stepping stones for further analysis. To make these lemmas, as well as the conclusions obtained thereafter, enjoy probabilistic meanings, let

$$m_b = \sum_{j=N+1}^{\infty} (j-N) b_j and\ m_d = \sum_{j=0}^{N-1} (N-j) b_j$$

denote the mean “arrival” and “service” rates, respectively. Note that$0 < m_d < +\infty \quad 0 < m_d \leq +\infty.$ Clearly,

$$B'(1) = m_b - m_d with - \infty < B'(1) \leq +\infty,$$

Which explains the probabilistic meaning of B (1).

The following conclusions are just corollaries of results obtained in Li and Chen.

Lemma 1 The equation B(s) = 0 has either N or N + 1 roots in the closed disc {s; |s| ≤ 1}. Moreover, it has N roots if and only if B (1) ≤ 0. More specifically,

(i) If B (1) < 0 (i.e. mb <md), then 1 is the only real and single root on the interval [0, 1], and, for any s ∈ [0, 1), B(s) > 0.

(ii) If B (1) = 0 (i.e. mb = md), then 1 is the only real root but is a “double” root (with multiplicity 2) on [0, 1] and, again, for any s ∈ [0, 1), B(s) > 0.

(iii) If B (1) > 0 (i.e. md < mb ≤ +∞), then, in addition to the root 1, we have another positive root, denoted by u, such that B(s) > 0 for all s ∈ [0, u) and B(s) < 0 for all s ∈ (u, 1).

(iv) All the other (N − 1) roots of B(s) = 0 in {s; |s| ≤ 1} are either negative or complex conjugate roots. Also, their moduli are strictly less than the smallest positive root of B(s) = 0. That is, if B (1) ≤ 0, then for any such root, the modulus is strictly less than 1, while if B (1) > 0, then all the moduli of these roots are less than u.

QUEUESWITH GENERAL BULK SERVICE

Service may be done in single or in bulk. Parallel processing in computers, sightseeing on guided tours, passengers boarding a train are some examples for bulk service. There are a number of variations ofbulk service rules or policies. The following bulk service rules are used frequently.

(i) The service is provided in batches consisting of ‘x! customers where 1 < x < b.

(ii) The service is provided in batches consisting of a fixed number of ‘k’ customers. On completion of service, the server will take up another service only if at least ‘k’ customers are in the queue or waits until ‘k’ customers are accumulated.

(iii) The size ofthe batch may be deterministic or probabilistic

(iv) The same is extended in batches consisting of ‘x’ customers, where a < x < b and a * 1. On completion of a service of a batch, if the server finds less than ‘a’ customers in the queue, he waits till atleast ‘a’ customers get accumulated in the queue and there upon he takes a batch of customers for service. If the server finds, on completion of a service, not less than ‘a’ customer but at the most ‘b’, then he takes all ofthem in a single batch for service and ifthe number of customers waiting is more than ‘b’ be takes a batch of ‘b’ customers only for service while others wait in the queue. This type of bulk service is called “general bulk service” and has been introduced by Neuts

It has been further Investigated by MedhiBorthakurand Medhi Chaudhry Chaudhry and Templeton Gross and Harris Sim and Templeton Nadarajan and SankaranarayananJayaraman and others

Mathematical Model

A single server batch arrival queueing system with server breakdown, multiphase repair, and different vacation policies is considered. In batch service, the server provides service to the batch of customer (minimum of ‘a’ customers and maximum of ‘b’ customer). In this model, the server begins the service, when at least ‘a’ customers are waiting in the queue. If the queue length reaches the value ‘a’, the server begins the bulk service. After completing bulk service, if the queue length, Q, is greater than or equal to a, then the server will continue the bulk service according to Neuts general bulk service rule. Whenever breakdown occurs in the main server, the failed server goes to the repair station. During the repair period, the server undergoes the k-different phases of repair. At the end of each phases of repair, the server either goes to next phase of repair or otherwise goes to the service station. After completion of repair process, the server either goes to the bulk service or the server goes to the different vacation policies (at most j vacation) according to the queue length. If the queue length is less than ‘a’, then the server goes to different vacation policies. After completing vacation if the queue length is less than ‘a’, then the server will be idle (dormant) until the queue length reaches ‘a’ and then provide bulk service. Otherwise, the server either goes setup time or then provides bulk service (Fig. 1).

Notation

λ, poisson arrival rate; Y, group size random variable of the arrival; gk, probability that k customer arrives in a batch; Nq (t), number of customers waiting for service at time t; Ns(t), number of customers under the service at time t.

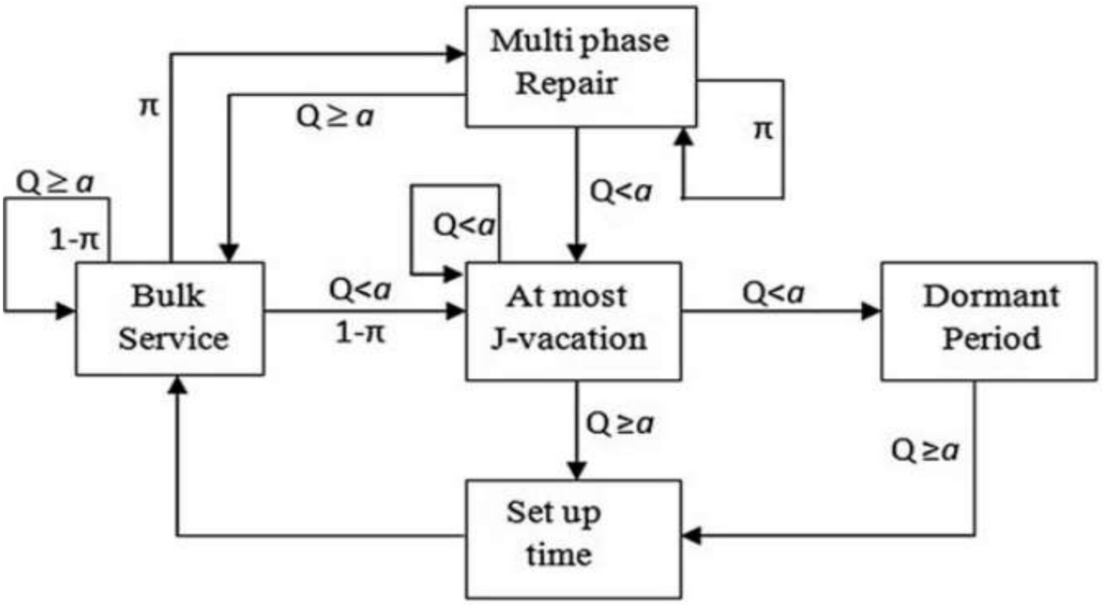

Fig. 1 Schematic representation of the model: Q, Queue length

Let L(x) [l(x)] {L (θ) ˜}(L0(t)) denotes the cumulative distribution function (CDF) [probability density function(PDF)] {Laplace–Stieltjes transform (LST)}(remaining time) of the server repair. Let M(x) [m(x)] {M (θ) ˜}(M0(t)) denotes the CDF [PDF] {LST} (remaining time) of batch service. Let N(x) [n(x)] {N (θ) ˜}(N0(t)) denotes the CDF[PDF]{LST} (remaining time) of the server vacation. Let H(x) [h(x)] {H (θ) ˜ }(H0(t)) denotes the CDF [PDF]{LST} (remaining time) of the setup time.

$$C(t) = \begin{cases} [0] - & iftheserverisonsingleservice \\ [1] - & iftheserverisonbatchservice \\ [2] - & iftheserverisonfastvacation \\ [3] - & iftheserverisonslowvacation \\ [4] - & iftheserverisondormantperiod \end{cases}$$

$$Z(t) = \begin{cases} [j] - & iftheserverisonj-thvacation \\ [k] - & iftheserverisonk-threpair \end{cases}$$

Now, the state probabilities are established as follows:

(i) $G_{ij}(x, t)\delta t = Pr\{Ns(t) = i, Nq(t) = j, x \le M0(t) \le x + \delta t, C(t) = 0\}, a \le i \le b, j \ge 0$

(ii) $T_{nj}(x, t)\delta t = Pr\{, Nq(t) = j, x \le L0(t) \le x + \delta t, C(t) = 1, Z(t) = k\}, n = 1, 2, 3, \ldots, k, j \ge 0$

(iii) $F_{ki}(x, t)\delta t = Pr\{Nq(t) = j, x \le N0(t) \le x + \delta t, C(t) = 2, Z(t) = j\}, k = 1, 2, 3, \ldots, j, 1 \le i \le a - 1$

(iv) $S_j(x, t)\delta t = Pr\{Nq(t) = j, x \le H0(t) \le x + \delta t, C(t) = 3\}, j \ge a$ $D_j(x, t)\delta t = Pr\{Nq(t) = j, C(t) = 4\}, 0 \le j \le a - 1$

M/M (a,b) /(2,1) QUEUEING SYSTEMS WITH SERVERS REPEATED AND DELAYED VACATION

Nadarajan& Subramanian (1984) have discussed M/M(a,b)/1 queueing system with single and multiple vacations using matrix-geometric method. A bulk service queue with accessible and non-accessible batches has been analyzed by Sivasamy (1990). Batch service queueing systems with single and multiple vacation for a=1 is studied by Lee et al (1992,1997). Afthab Begum (1996) has obtained analytic solution for M/M(a,b)/1 queues, Ek/M(a,b)/1 queue with servers single and multiple vacation in her Ph.D thesis. An M/M(a,b)/1 queueing model with server's delayed vacation has been analyzed by Anitha (1997). Gupta & Vijayalakshmi (2001) have studied MAP/G(a,b)/1/N queueing system. A bulk queueing model M/M(a,b,c)/2 for non-identical servers with vacation has been discussed by Mishra & Pandey (2002). Performance measures of the bulk service queues with vacation have been analyzed by Lakshmi Srinivasan et al (2001,2003b). Queueing model 106 for a bulk service queue with single vacation and feedback facility has been discussed by Lakshmi Srinivasan &Kalyanaraman (2003a).

Queueing models are very useful to provide basic framework for efficient design and analysis of several practical situations including various technical systems also predictions the behavior of system such as waiting times of customers, various vacations for servers and so forth. The range of applications has grown in manufacturing, air traffic control, military logistics, design of theme parks, and many other areas that involve service systems whose demands are random. Such queueing 2 situations may arise in many real time systems such as telecommunication, data/voice transmission, manufacturing system, etc. In computer communication systems, messages which are to be transmitted could consist of a random number of packets. Queueing systems with server vacations have also found wide applicability in computer and communication network and several other engineering systems. Vacation models are explained by their scheduling disciplines, according to which when a service stops, a vacation starts.

These predictions help us to anticipate situations of the system and to take appropriate measures to shorten the queue. In most of the queueing models, service begins immediately when the customers arrives. But some of the physical systems in which idle servers will leave the system for some other uninterrupted task referred as vacation. Most of the bulk service Queueing models with server vacation have been analyzed by many authors. S.Palaniammal has studied M/M(a,b)/(2,1) queueing model and derived analytic solutions for servers repeated and single vacation also presented the steady state results in terms of characteristic equation of a difference equation. M.I.Afthabbegamhas tried analytic solution for M/M(a,b)/1 queues, Ek/M(a,b)/1 queue with servers single and multiple vacations.

The queueing models with vacations have been studied due to their wide applications in flexible manufacturing or computer communication systems over more than two decades. Several surveys on server vacation models have been done by Doshi (4), Takagi.H(13) analyzed the M/G/1/N queues with server vacation and exhaustive service., Medhi.J and Borthakur.A(7) have introduced a general bulk service rule with two server. Also a bulk queueing model M/M(a,b,c)/2 with servers vacation has been studied by Mishra.S.S and Pandey.N.K (8). The Ek/M(a,b)/1 queueing system and its numerical results are analyzed by Chaudry.M.C and Easton.G.D (3).

The transient of Ek/M(a,b)/1/N derived by Anjanasolanki and Srivastava.P.N(2). In many waiting line systems, the role of server is played by mechanical/ electronic device, such as computer, pallets, ATM, Traffic light, etc., which is subject to accidental waiting of customers, it may solved by the servers vacation due to batch criteria. Ke (5) studied the control policy of the N-Policy M/G/1 queue with server vacations, startup and breakdowns, where arrival forms a Poisson and service times are generally distributed also Mx /G/1 queue with feedback and optional server vacations based on a single vacation policy studied by Madan.K.C and AIRawwash.M(6). SreeParimala.R and Palaniammal.S(12) has analyzed M/M(a,d,b)/(2,1) bulk service queueing model with servers repeated vacation. 3 In the literature described above, customer inter-arrival times and customer service times are required to follow certain probability distributions with fixed parameters. The present investigation an attempt has been made to analyze the server's delayed and single vacation of M/M(a,d,b)/(2,1) Queueing model.

CONCLUSION

Future research should look at a number of variations on the queueing model presented in this article. In the introduction, we mentioned two possible future study areas that are closely relevant to this article. During the idle interval, many queueing models only consider one sort of secondary task (holiday). To save money, the server must be able to do a variety of secondary tasks with varying threshold rules. The unique contribution is the suggested queueing paradigm, which addresses this issue. Consideration is given to a real-world example from a manufacturing system. Managers may utilise this paper's findings to make better-informed decisions on the overall cost and operating policy of a waiting line system. Furthermore, a comparison research was conducted and demonstrated that the suggested model is beneficial in terms of cost analysis.

REFERENCES

[1] Banerjee, A.; Gupta, U.C.; Chakravarthy, S.R. Analysis of a finite-buffer bulk-service queue under Markovian arrival process with batch-size dependent service. Comput. Oper. Res. 2015, 60, 138–149.

[2] Arienzo, D.; M.P.; Dudin, A.N.; Dudin, S.A.; Manzo, R. Analysis of a retrial queue with group service of impatient customers. J. Ambient. Intell. Humaniz. Comput. 2019.

[3] Neuts, M.F. A General Class of Bulk Queues with Poisson Input. Ann. Math. Stat. 1967, 38, 759–770.

[4] Ayappan, G.; Renganathan, N. A preemptive priority queue with accessible batch services and heterogeneous arrivals. Inf. Manag. Sci. 1997, 8, 63–72.

[5] Ayappan, G.; Sridevi, S. Bulk service Markovian queue with service batch size dependent and accessible and non accessible batches and with vacation. J. Comput. Model. 2012, 2, 123–135.

[6] Jeeva, M & Fernandes Jayashree Felix 2011, Recruitment model with pre- emptive repeat priority service, Proceedings of the international conference on Stochastic modeling and simulation, Vel Tech Dr. RR & Dr.SR technical university, Chennai, India, pp.171-178.

[7] Choudhury, G& Madan, KC2005, A two-stage batch arrivalqueueing system with a modified Bernoulli schedule vacation under *N*-policy, Mathematical and computer modelling‟, vol.42, no.2, pp.71-85

[8] Agrawal, S & Kapoor, S 2012, Analysis of queueing model with pre-emptive priority service, International Journal of Applied Mathematics and Mechanics, vol.8, no.7, pp.52-92

[9] Choudhury, G& Madan, KC2005, A two-stage batch arrivalqueueing system with a modified Bernoulli schedule vacation under *N*-policy, Mathematical and computer modelling‟, vol.42, no.2, pp.71-85

[10] Komal Madan &Sukhbir Singh 2012, An Efficient Probability BasedBroadcast Scheme for AODV Mobile ad-hoc Network Protocol, International Journal of Innovations in Engineering and Technology, vol.1, no.1. pp.64-71

[11] Ayyappan, G & Muthu Ganapathi Subramanian, A 2010, M/M/1 Retrial Queueing System with Loss and Feedback under Non-pre-emptive Priority Service by Matrix Geometric Method, Applied Mathematical Sciences, vol. 4, no. 48, pp.2379 – 2389

[12] Kashyap, BRK & Chaudhry, ML 1988, An introduction to Queueing theory, Kingston, Ontario

[13] Imran Ali Khan, Sajjad Ahmad Madani, Waqas Anwar &Khizar Hayat 2011, Location Based Dynamic Probabilistic Broadcasting for MANETs, World Applied Sciences Journal, vol. 13,no.11, pp. 2296-2305

[14] Balamani, N 2012, A single server M/G/1 queue with service interruption and bernoulli schedule server vacation having general vacation time distribution, International Journal of MathematicalArchive-3(2), pp.347-353

[15] Al-khedhairi, A &Tadj, L 2007, A Bulk Service Queue with a Choice of Service and Re-Service under Bernoulli Schedule, International Journal of Contemporary Mathematical Sciences, vol. 2, no. 23, pp.1107 – 1120