

Global Path Planning and Trajectory Design for Quadrotors Using Artificial Potential Field

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Abstract

Path planning is aimed to enable mobile robots with the capability of automatically deciding and executing a sequence of collision free and safety motions in static and dynamic environments. This paper presents an Artificial Potential Field (APF) based method for path planning of robots. In the process of path planning from the initial point to the destination, the collision free path has to be determined in workspace cluttered with obstacles. Resolving local minimum is one of the active research topics associated with artificial potential field method where an abrupt stop is experienced by the robot. A minimum potential is normally observed near the target. But due to the balance in opposite forces, a minimum potential is encountered even before reaching the target. This situation is known as local minima. From a survey on several methods used to resolve the local minima in the literature, all of the techniques are put into action only after the robot realizes being trapped into local minima. Hence, solutions that could predict and detect even before getting trapped would be more effective for real time implementation. One such method is proposed and implemented. This heuristic employs the concept of Verlet's integration method for prediction or local extrapolation. When the local minima are detected with the predicted position, the effect of force from the farthest obstacle is neglected so as to create a perturbation in the equilibrium and this predicted point would serve as way point to the current position vector. The path planning process is further extended for trajectory design of a quadrotor. The total path length from the start to the destination point is further observed.

Keyword - Artificial Potential Field, Path Planning, Verlet's integration.

I. Introduction

Autonomous motion planning is one of the important components for effective navigation of mobile robots and Unmanned Air Vehicles (UAV). Path planning is aimed to enable mobile robots with the capability of automatically deciding and executing a sequence of collision free and safety motions in static and dynamic environments. The main techniques involved in path planning process are road map, cell decomposition and potential field methods. In potential field method introduced by Khatib [12], the robot is assumed to move in a field of artificial forces towards the minimum goal position [1]. The main problem associated with this method is the entrapment of the robot in the local minima. Various solutions were proposed to overcome local minima while most of them had its base line on steepest descent and other optimization techniques [2]. A method based on simulated annealing is proposed in combination with solid modeling technique for potential fields [2]. On further implementation the computation time study is extended for 2D and 3D environments for mobile robots. [2] improvises the potential function by including the least distances between the obstacle and the robot in case of repulsive potential function and distance between the robot and the goal for attractive potential function. This introduction of the distance as a parameter of the navigation function enables target being seen as the global minimum ensuring the safe path. [3] extends the same concept for the three dimensional environment. The implementation is carried out as a MATLAB simulation. The concept of improved artificial potential function was further extended for 3D space [3] for mobile robot and in [6] the similar approach was demonstrated for Unmanned Air Vehicles in dynamic environments. The potential function in [4] was then combined with the artificial neural network for path planning in dynamic environments on the robotic system that was self-reconfigurable. The local minima problem was overcome by introducing the virtual obstacle concept for path planning in [7]. The virtual obstacle concept was implemented using

modeling method in [9] as an extension from [7]. [9] emphasizes on offline path planning by hybridizing all other algorithms with potential method. The implementation is carried out in stages namely the preprocessing phase and the search phase. Further a comparative analysis between the different algorithms was performed. In order to incorporate heuristics in path planning evolutionary algorithms were tested for UAVs in [5]. [8] and [10] propose a solution to resolve the problem of local minima through regression based method and virtual obstacle method respectively. B-Spline curves are used in order to model both 2-D trajectories and the velocity distribution along each way point. A comparison of three different approaches for collision avoidance in UAVs is described in [11] and [14] describes the usage of different potential functions and discusses their behavior in avoiding collision obstacle. The geometric and A* approaches are reflexive in nature while repulsive approach is quite reactive. On simulating the three approaches with the dynamic model of the quadrotor the repulsive approach yielded the advantage of lower complexity in execution. [8] presents the real time obstacle avoidance for mobile robots based on artificial potential approach which primarily focusses on high level path planning for low level real time control to perform real time operations while [13] describes the implementation of the Vector Field Histogram method for fast obstacle avoidance. [15] explains the autonomous navigation of mobile robots in unknown environments with the online sensor details. The SVM (Support Vector Machines) method in combination with k-Nearest Neighbors (k-NN) algorithm accomplishes both line compliance and obstacle avoidance. [16] describes the implementation of the Vector Field Histogram method for fast obstacle avoidance, while [17] discusses upon the different types of the heuristic methods. The implementation was carried out in a cluttered environment at an average speed of 0.6-0.7 m/s. Both of the algorithms, in this paper are implemented with the dynamic model developed in [18].

II. Artificial Potential Field Implementation

Artificial Potential Field was first proposed by Khatib [19] in late 1980's. The basic idea is constructing an artificial attractive field around the target and repulsive force field around the obstacles. A robot in the field is treated as analogous to a charge suspended in a potential field. The charge when in a potential field tends to move along the direction of the force being induced upon its current position. Hence the robot is represented as point in the field and is set to move under the influence of the artificial forces. The induced force makes the robot to acquire successive positions resulting in planning the path to reach the destination. In order to perform the attractive and repulsive actions, each of them is defined by potential function which is called as navigation function.

A) Attractive Force Function

The goal configuration is represented in the same shape as that of the robot. The potential field generated around the target

location is attractive in order to lead the robot to the goal location and create an energy minimum. The force is derived as a result of taking a negative gradient of the potential function U_{att} [2]. The attractive force equation is given by

$$F_{att}(x) = \begin{cases} AFC (x - x_d), & (x - x_d) < d_t \\ 0 & (x - x_d) \geq d_t \end{cases}$$

x denotes the current position vector

x_d denotes the position vector of the target

d_t the threshold distance

AFC denotes the attractive force constant

B) Repulsive Force Function

The obstacles in the workspace generate appropriate repulsive field in order to prevent collision. A nonlinear function of a repulsive field is used to derive the density proportional to the obstacle's position and enhance collision avoidance. The total potential at a point $P(x,y)$ is the sum of each potential provided at that point. The repulsive force at point is derived by the following equation

$$F_{rep}(x) = \begin{cases} -RFC \frac{x_0 - x}{\|x_0 - x\|^3} & x \leq x_d \\ 0 & x > x_d \end{cases}$$

x_0 denotes the position vector of the target

RFC denotes the repulsive force constant

C) Field Structure

The first step is to specify the area of the field and declare the initial conditions like initial position, velocity, goal position, attractive force constant, etc. The implementation is carried out only in the Global Coordinate System. The walls in the field are also considered as obstacles and they are modeled in order to exert force on the robot provided the robot is within specified distance of the wall. The robot and the goal position are modeled in the form of a circle at their respective positions. The details about the presence of the obstacles are made either known to the robot or it comes to know about the information while progressing in the field. Hence the obstacles are either static or dynamic in nature in a known environment. The trajectory construction occurs in the configuration that could be realized by the path travelled through the progressive points. This could be visualized by connecting the current position with the previous positions.

Table I: Environment details

Environment Details	Corresponding Measure
Length of the field	14 m
Width of the field	6 m
Number of Obstacles	3
Threshold Distance to realize the impact	0.5 m

Spacing between the equidistant points distributed in the obstacle	0.2 m
Range of coefficients used in the potential function	0.1-10
Minimum distance between obstacles	0.6 m

III. Quadrotor Dynamics

The generic control architecture of the quad rotor can be signified by the block diagram shown in figure 1. The control flow is distributed into inner and outer feedback loops for attitude and position control respectively.

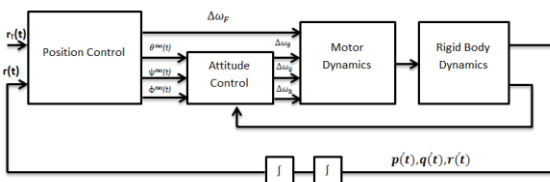


Figure 1 Nested Feedback Loops to Enable Position & Attitude Control

A) Dynamic Analysis & Modelling

The coordinated systems and the free body diagram of the UAV are shown in the figure 2. The world frame (W) denotes the fixed reference frame with respect to which all motion can be referred to the body frame (B) connected to the center of mass of the vehicle. The rotation of each of rotors creates a vertical force. In addition to the forces, each rotor produces a moment perpendicular to the plane of propeller rotation.

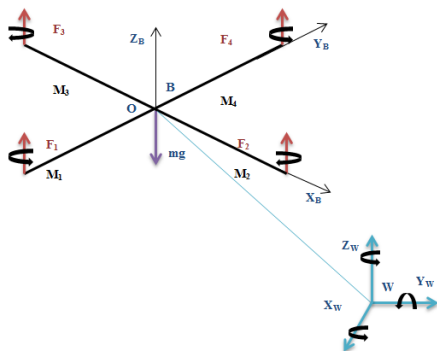


Figure 2 Free Body Diagram representing the Coordinate Systems

The Z-X-Y Euler angle convention is used to model the rotation of the quad rotor in the W frame. To get to the B frame, we first rotate through z_w by yaw angle, ψ then rotate about intermediate x-axis by roll angle, ϕ and then about the y_B by

pitch angle θ . Hence the rotation matrix R from W to B frame can be written as,

$$R = \begin{bmatrix} c\psi c\theta - s\phi s\psi s\theta & -c\phi s\psi & c\psi s\theta + c\theta s\phi s\psi \\ c\theta s\psi + c\psi s\phi s\theta & c\phi c\psi & s\psi s\theta - c\theta c\phi s\psi \\ -c\phi s\theta & s\phi & c\phi c\theta \end{bmatrix} \quad (1)$$

Where c indicates cos (angle) and s indicates sin (angle).

If r represents the position vector of center of mass in the world coordinate system, the acceleration of the center of mass is given by equation:

$$m\ddot{r} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + R \begin{bmatrix} 0 \\ 0 \\ \sum_{k=1}^4 F_k \end{bmatrix} \quad (2)$$

Where $\ddot{r} = \begin{bmatrix} \ddot{X} \\ \ddot{Y} \\ \ddot{Z} \end{bmatrix}$,

m - mass of the quadrotor

g - acceleration due to gravity.

In the body frame, the elements of angular velocity of the robot are p , q and r . The relationship between these components and the subordinates of the roll, pitch and yaw angles are

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \cos\phi \sin\theta \\ 0 & 1 & \sin\phi \\ \sin\theta & 0 & \cos\phi \cos\theta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \quad (3)$$

A) Motor Control

Each rotor produces a vertical force F_k due to the rotation, given by

$$F_k = k_F \omega_k^2 \quad (4)$$

Each rotor k produces a moment M_k which is perpendicular to the blades plane of rotation. Rotors 1 and 3 produce moments in the $-z_B$ direction. The moments created by rotors 2 and 4 are in direction z_B . The moment created on the quadrotor is opposite to the direction of the rotation of the blades thus M_1 and M_3 act in z_B direction whereas M_2 and M_4 act vice versa. The distance from the axis of rotation of the rotors to the center of the quadrotor is marked as L . Through weighing the individual components of the quad rotor, the moment of inertia matrix, I which is referenced to the center of mass along x_B - y_B - z_B axes is obtained. The angular acceleration is obtained by Euler equations as shown below

$$I \begin{bmatrix} \ddot{p} \\ \ddot{q} \\ \ddot{r} \end{bmatrix} = \begin{bmatrix} L(F_2 - F_4) \\ L(F_1 - F_3) \\ M_1 - M_2 + M_3 - M_4 \end{bmatrix} - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times I \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad (5)$$

Where p , q , and r are the components of angular velocities of the vehicle in the body frame. The rotor moments produced by angular velocity of the rotors and is given by

$$M = k_M \omega_k^2 \quad (6)$$

The quad rotor will levitate at a point once the nominal thrusts from propellers equals to gravity, so:

$$F_k = \frac{mg}{4} \quad (7)$$

And the motor speed will be:

$$\omega_{k,0} = \omega_h = \sqrt{\frac{mg}{4k_F}} \quad (8)$$

B) Attitude Control

The controller inputs are four independent speeds of propellers. The dynamic model of quadrotor is used to design the PD controller. On substituting (4) and (6) in (5), we get the vector of desired rotor speeds as a linear combination of four terms:

$$\begin{bmatrix} \omega_1^{des} \\ \omega_2^{des} \\ \omega_3^{des} \\ \omega_4^{des} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & 1 \\ 1 & 1 & 0 & -1 \\ 1 & 0 & 1 & 1 \\ 1 & -1 & 0 & -1 \end{bmatrix} + \begin{bmatrix} \omega_H + \Delta\omega_F \\ \Delta\omega_\phi \\ \Delta\omega_\theta \\ \Delta\omega_\psi \end{bmatrix} \quad (9)$$

where ω_k^{des} , (k=1,2,3,4) are the desired angular velocities of the respective rotors, and ω_H is the hovering speed. The proportional-derivative control laws are used to control $\Delta\omega_\phi, \Delta\omega_\theta, \Delta\omega_\psi$, and $\Delta\omega_F$, which are deviations that result into forces/moments causing roll, pitch, yaw and a net force along z_B axis, respectively and are calculated as

$$\begin{aligned} \Delta\omega_\phi &= k_{p,\phi}(\phi^{des} - \phi) + k_{d,\phi}(\dot{p}^{des} - \dot{p}) \\ \Delta\omega_\theta &= k_{p,\theta}(\theta^{des} - \theta) + k_{d,\theta}(\dot{q}^{des} - \dot{q}) \\ \Delta\omega_\psi &= k_{p,\psi}(\psi^{des} - \psi) + k_{d,\psi}(\dot{r}^{des} - \dot{r}) \\ \Delta\omega_F &= \frac{m}{8k_F\omega_H} \ddot{Z}^{des} \end{aligned} \quad (10)$$

C) Position Control

In order to have the quad rotor to track the desired trajectory $r_{i,T}$, the command acceleration $\ddot{r}_{i,T}^{des}$ is calculated from the proportional derivative controller based on the position error.

$$(\ddot{r}_{i,T} - \ddot{r}_i^{des}) + k_{d,i}(\dot{r}_{i,T} - \dot{r}_i^{des}) + k_{p,i}(r_{i,T} - r_i) = 0 \quad (11)$$

Where $r_{i,T}$, r_i (i=1, 2, 3) are the three dimensional position of the quad rotor and desired trajectory respectively. It may be noted that $r_{i,T} = r_i = 0$ for hover. During the flight, the orientation of the vehicle needs to be set close to zero. This can be obtained by linearizing the equation of motion that corresponds to the normal hover states. The nominal hover state is ($r=r_0$, $\phi=\theta=0$, $\psi=\psi_T$, $\dot{r}_i = 0$ and $\dot{\theta}_i = \dot{\psi}_i = \dot{\phi} = 0$). The change of pitch and roll angles are supposed to be small during the flight. By linearizing equation (11) about these nominal hovering states, desired pitch and roll angles to cause the motion can be derived as given by the following equations:

$$\phi^{des} = \frac{1}{g}(\ddot{X}^{des} \sin \psi_T - \ddot{Y}^{des} \cos \psi_T)$$

$$\theta^{des} = \frac{1}{g}(\ddot{X}^{des} \cos \psi_T - \ddot{Y}^{des} \sin \psi_T) \quad (12)$$

Where \ddot{X}^{des} and \ddot{Y}^{des} are the desired acceleration in X and Y directions respectively. ψ_T is the yaw angle to be tracked which is same as the desired yaw angle ψ^{des} .

D) Simulation Parameters

Parameters of the dynamic model used in [18] are provided in Table II.

Table II Parameters of the dynamic model

$k_{px}=1$	$k_{py}=1$	$k_{pz}=1$
$k_{dx}=1$	$k_{dy}=1$	$k_{dz}=1$
$k_F=6.11 \times 10^{-8}$ N/(r/min ²)	$k_M=1.5 \times 10^{-9}$ N*m/(r/min ²)	$k_m=20$
$m=1.08$ kg	$g=9.8$ m/s ²	$L=0.22$ m

Where k_d, k_p are the Controller gains, k_F denotes Propeller force constant, k_M denotes Propeller moment constant, m denotes mass of the quadrotor, g denotes acceleration due to gravity, L denotes Length from the axis of the rotors to the center of mass.

IV. Heuristic Method-Resolving Local Minima Problem

Heuristic methods have always outperformed the classical methods which suffer from inability to solving motion planning problem in real time and NP hardness [17]. Some of the heuristics include Neural Networks, Genetic Algorithms, Simulated Annealing, Ant Colony Optimization, Particle Swarm Optimization, Stigmergy, Wavelets, Tabu Search and Fuzzy Logic. A heuristic method based on local extrapolation using Verlet's Integration formula is proposed. The step by step procedure is proposed as described below:

Step 1: Using the initial values compute the force for the first time step.

Step 2: Get the next position vector (predicted value) using the Verlet's integration formula given in equation (13).

$$\overrightarrow{x_{n+1}} = 2\overrightarrow{x_n} - \overrightarrow{x_{n-1}} + \overrightarrow{F_n}\Delta t^2 \quad (13)$$

$\overrightarrow{x_{n+1}}$ is the predicted position vector, $\overrightarrow{x_n}$ is the current position vector, $\overrightarrow{x_{n-1}}$ is the previous position vector, $\overrightarrow{F_n}$ is the force vector at current position and Δt is the differential time.

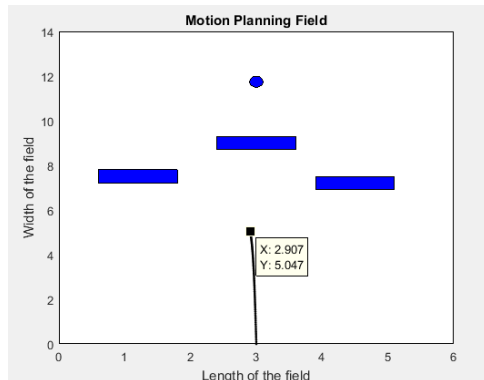
Step3: Check for local minima condition.

Step 3.1: If a local minimum is not detected assign the next position value as the way point and continue to reach the target. Step 3.2 if a local minimum is detected then neglect the value of the effect of force from the obstacle at larger distance from the current position. This creates a perturbation and disrupts the entrapment of the local minima. Make the new perturbed

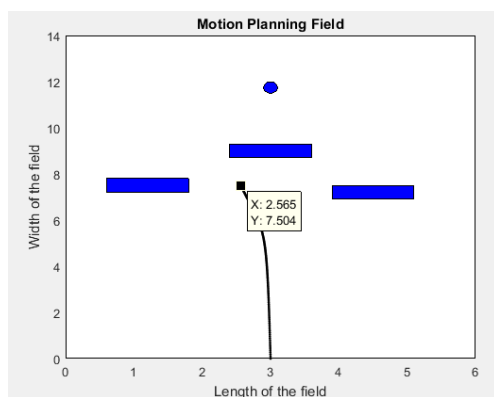
position as a way point and again check for the local minima condition. Keep repeating this procedure till local minima is overcome.

Step 4: Stop if the target is reached.

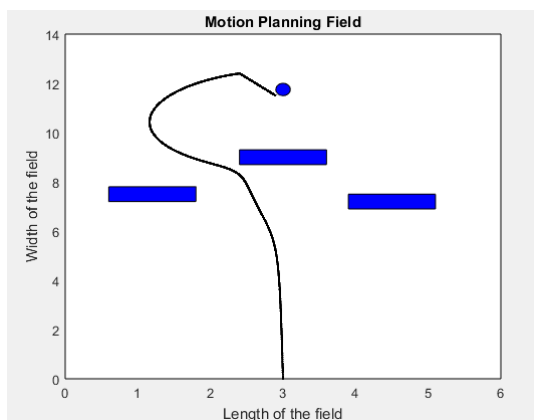
V. Results and Discussion



(a)



(b)



(c)

Figure 3 (a) Occurrence of the first local minima (b) Occurrence of the second local minima (c) Completion of the path

This configuration was implemented with the RFC= [0.6 2.4 5] and AFC= [0.8]. Figure 3(a) shows the position vector where

the occurrence of local minima is first realized [2.9 5.0]. This is the extrapolated position value. The distance between the extrapolated point and obstacles are as follows 2.4530, 1.3466 and 2.6113 m for the obstacles along the left, right and center positions respectively. The net force coming out of the obstacle from the center is neglected by making RFC=0 at point [2.9 5.0]. Hence the way point drifts along the left side due the effect of forces coming due to the obstacles in the center and right. The same condition continues with the case of the second local minima with the distances 1.9928, 1.5530, 1.0578 along the obstacles at left, center and right positions. Hence the forces due to right and center obstacle contribute to the motion planning process further and finally the target is also reached. The total path length from the start to the final point is 11.50 m and execution time was approximately 105 seconds.

VI. Conclusion

A survey of existing solutions to overcome local minima is studied. This method involves neglecting the actual repulsive force coming from the farthest obstacle when the extrapolated point gets trapped into local minima. The disturbance in equilibrium helps to avoid the abrupt stop serving as a waypoint for enabling the path planning process. As the actual force is employed in this method, the need for virtual obstacle is eliminated when the local minima is realized. This in turn reduces the computational cost. Also the method proposed eliminates one of the disadvantage associated with the random path planning by taking lesser time for execution. The main advantage of this method is that the local minima is detected and eliminated even before robot encounters the same. Hence automatic completion of path is achieved. The problems like collision, non-recognition of the target and incomplete path generation were still observed but they were overcome by tuning the repulsive and attractive force coefficients.

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