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Dispersion of Oil Spilled with and without Porosity Variation between Two Parallel Plates

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Abstract

This paper deals with an analysis of the dispersion of oil spilled with and without porosity variation between two parallel plates in the presence of chemical reaction. The flow consisting of two regions first region filled with topsoil and second region filled with oil. The Taylor's dispersion model is utilized to obtain the volumetric flow rate and effective dispersion coefficients are numerically calculated. The effects of various parameters are entering into the problems and discussed withgraphs.

Keywords: Taylor's dispersion, Porosity variation, Horizontal channel andChemical reaction.

1 Introduction

Oil spill on topsoil can be contained and recovered or left to degrade throughnature processes. oil is the main source of energy in the industrial world, and oil spills may be regarded as an inexorable consequence of the ever increasing demand for delving, manufacturing and use of oil. The concept of spread of oilspilled in ice channel was analysed by several authors Yapa and Chowdhury(1990), Bellino et.al., (2013) and Nirmala Ratchagar and Hemalatha (2014). An enormous variety of extensions of two phase model has been developed by many authors. Prathap kumar et.al., (2012), (2013) have discussed theanalytical solution of two equivalent plates containing porous and viscous fluidlayers. Linga Raju and Gowri Sankara Rao (2015) studied two layer fluid flow between two parallel porous walls. Study of homogenous and heterogeneous reactions on the dispersion of solute has been demonstrated by Gupta and Gupta (1972) and Prathap kumar et.al., (2012). Meenapriya (2015) and Nirmala Ratchagar and Vijayakumar (2019) have discussed dispersion of solutewith the chemical reaction.

Porosity has been known to be the most significant property describing a porous medium. It controls fluid storage in aquifers, oil and gas fields. Several authors Arzhang Khalili et.al.(2014) and Colin Sayers (2021) havebeen investigated porosity variation on the fluid porous interface. Nirmala Ratchagar and Senthamilselvi (2019) analysed the porosity variation on groundwater with and without chemical reaction. Zhigang Zhan et.al., (2006) have presented the same variation on the liquid water flux through gas diffusion layers.

In this paper we study the dispersion of oil spilled in presence of chemical reaction between two parallel plates using Taylor's (1953) model. The work reported here covers two cases, the first case deals with the porosity variation on the oil spilled in topsoil. The second case describes without porosity.

2 Problem Formulation

The physical geometry is exhibited in Figure.1. We use a rectangular coordinates system (x, y) through two horizontal plates. where x and y denotethe horizontal and vertical coordinates. Region 1 $(0 \le y \le h)$ is consider

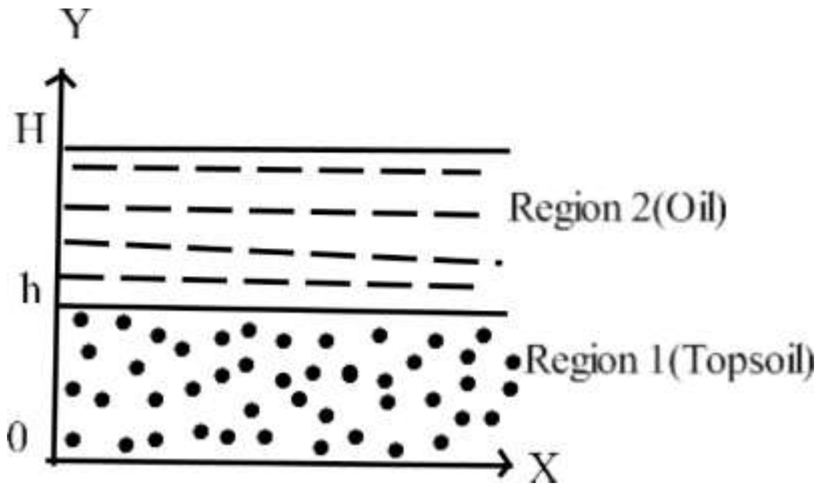

Figure 1: Physical configuration.

to be topsoil with density ρ_1 , viscosity μ_1 and uniform pressure gradient $\frac{\partial p_1}{\partial x}$ $with$ permeability k_p. Region 2 $(h \leq y \leq H)$ is containing with oil with density $\rho 2$, viscosity $\mu 2$ and uniform pressure gradient $\frac{\partial p_2}{\partial x}$. The fluid is assumed to be laminar, incompressible and steady flow.

Under these assumptions, the governing equations are rendered as follows.

Case 1:with porosity variation

Region: 1

$$\mu_1 \left(\frac{1}{\Theta} \frac{\partial^2 u_1}{\partial y^2} - \frac{1}{k_p} u_1 \right) = \frac{\partial p_1}{\partial x} \tag{1}$$

Region: 2

$$\mu_2 \left(\frac{\partial^2 u_2}{\partial y^2} \right) = \frac{\partial p_2}{\partial x} \tag{2}$$

where, u_1 is the velocity of oil in region 1, u_2 is the velocity of oil in region2 along the x direction respectively, Amiri and Vafai (1994) has presented variable porosity can be expressed as in the form $\Theta = \Theta_s(1 + a_1 e^{\frac{-a_2 y}{dp}})$. Where Θ_s is the mean porosity, a_1 and a_2 are empirical constants and dp is the particle diameter.

Under these assumption and appropriate boundary conditions on velocity becomes,

$$u_1 = 0 \; at \; y = 0,$$

$$u_1 = u_2, \; \mu_2 \frac{\partial u_2}{\partial y} = \mu_1 \frac{\partial u_1}{\partial y} \quad \text{at } y = \text{h},$$

$$\frac{\partial u_2}{\partial y} = 0 \;\; at \;\; y = h + H \tag{3}$$

Accordingly the non dimensional quantities are:

$$\eta = \frac{y}{H}, \; x^* = \frac{x}{H}, u_i{}^* = \frac{u_i}{u_0}, p_i{}^* = \frac{p_i}{\frac{\mu_i u_0}{H}}, \text{(i=1,2)}, h^* = \frac{h}{H}, dp^* = \frac{dp}{H},$$

where H , u_0 are the characteristic height and velocity respectively, the dimensionless form of the governing equations (1) and (2) and omitting asterisk symbols.

$$\frac{\partial^2 u_1}{\partial \eta^2} - \Theta\, \sigma^2 \mathrm{u}_1 = \Theta \mathrm{g}_1 \tag{4}$$

$$\frac{\partial^2 u_2}{\partial \eta^2} = g_2 \qquad (5)$$

where, $\sigma^2 = \frac{H}{k_p}$ is the porous parameter, $g_1 = \frac{\partial p_1}{\partial x}$ *and* $g_2 = \frac{\partial p_2}{\partial x}$

with boundary conditions:

$$u_1 = 0 \; at \; \eta = 0,$$

$$u_1 = u_2, \mu_2 \frac{\partial u_2}{\partial y} = \mu_1 \frac{\partial u_1}{\partial y} \; at \; \eta = h,$$

$$\frac{\partial u_2}{\partial y} = 0 \; at \; \eta = h + 1. \qquad (6)$$

Case 2:without porosity

The non dimensional equation of motion is given by

$$\frac{\partial^2 u_1}{\partial \eta^2} - \sigma^2 u_1 = g_1 \qquad (7)$$

$$\frac{\partial^2 u_2}{\partial \eta^2} = g_2 \qquad (8)$$

3 Method of solution

3.1 Velocity Disribution:

Case 1:with porosity variation

Equation (4) by making use of ”changing the independent variable method” we obtained,

$$u_1(\eta) = e^{\frac{2}{B_2}\sqrt{B_1 \sigma^2 e^{B_2 \eta}}} f_1 + e^{\frac{-2}{B_2}\sqrt{B_1 \sigma^2 e^{B_2 \eta}}} f_2 - \frac{g_1}{\sigma^2} \qquad (9)$$

$$u_2(\eta) = f_3 \eta + f_4 - \frac{g_2 \eta^2}{2} \qquad (10)$$

Where $B_1 = \Theta_s a_1$,$B_2 = \frac{-a_2}{dp}$,$f_1, \; f_2, \; f_3$ *and* f_4 *are integrating constant then ,*
then substituting the boundary conditions (6) *in equations* (9)*and* (10) *we get*

$$f_1 = \frac{1}{B_1 (e^{\frac{4\sqrt{B_1 \sigma^2}}{B_2}} + e^{\frac{4\sqrt{B_1 e^{B_2 h} \sigma^2}}{B_2}}) \sigma^2 \mu_1} \left(B_1 e^{\frac{2\sqrt{B_1 \sigma^2}}{B_2}} g_1 \mu_1 - \frac{B_1 e^{\frac{2\sqrt{B_1 e^{B_2 h} \sigma^2}}{B_2}} g_2 \sigma^2 \mu_2}{\sqrt{B_1 e^{B_2 h} \sigma^2}} \right),$$

$$f_2 = \frac{e^{-B_2 h + \frac{2\left(\sqrt{B_1 \sigma^2} + \sqrt{B_1 e^{B_2 h} \sigma^2}\right)}{B_2}} (B_1 e^{B_2 h + \frac{2\sqrt{B_1 e^{B_2 h}}}{B_2}} g_1 \mu_1 + e^{\frac{2\sqrt{B_1 \sigma^2}}{B_2}} g_2 \sqrt{B_1 e^{B_2 h} \sigma^2} \mu_2)}{B_1 (e^{\frac{4\sqrt{B_1 \sigma^2}}{B_2}} + e^{\frac{4\sqrt{B_1 e^{B_2 h} \sigma^2}}{B_2}}) \sigma^2 \mu_1}, f_3 = -g_2(1 + h),$$

$$f_4 = \frac{1}{2 (e^{\frac{4\sqrt{B_1 \sigma^2}}{B_2}} + e^{\frac{4\sqrt{B_1 e^{B_2 h} \sigma^2}}{B_2}}) \sigma^2 \sqrt{B_1 e^{B_2 h} \sigma^2} \mu_1} (\sqrt{B_1 e^{B_2 h} \sigma^2} (4 e^{\frac{2\left(\sqrt{B_1 \sigma^2} + \sqrt{B_1 e^{B_2 h} \sigma^2}\right)}{B_2}} g_1 + e^{\frac{4\sqrt{B_1 \sigma^2}}{B_2}} (-2 g_1 +$$

$g_2\, h(2+h)\, \sigma^2) + \; e^{\frac{4\sqrt{B_1 e^{B_2\, h}\, \sigma^2}}{B_2}} \; (-2\, g_1 + g_2\, h(2+h)\, \sigma^2))\, \mu_1 \; + 2\, (e^{\frac{4\sqrt{B_1 \sigma^2}}{B_2}} - \; e^{\frac{4\sqrt{B_1 e^{B_2\, h}\, \sigma^2}}{B_2}})\; g_2\, \sigma^2\, \mu_2\,)$.

The average velocity is given by,

$\overline{u_1(\eta)} = \frac{1}{2} \int_0^h u_1(\eta)\; d\eta$

$\overline{u_1(\eta)} = \frac{1}{2} \left(\frac{-g_1\, h}{\sigma^2} - \frac{2\, f_2\, (g_3 - g_4)}{B_2} - \frac{2\, f_1\, (g_5 - g_6)}{B_2} \right)$ (11)

$\overline{u_2(\eta)} = \frac{1}{2} \int_h^1 u_2(\eta)\; d\eta$

$\overline{u_2(\eta)} = \frac{-1}{12}\, (-1+h)(6\, f_4 \; + 3\, f_3\, (1+h) + g_2 (1+h+h^2))$ (12)

where $g_3 = \; E_i \left(\frac{-2\sqrt{B_1\; \sigma^2}}{B_2}\right)$, $g_4 = \; E_i \left(\frac{-2\sqrt{B_1\; e^{B_2\, h}\, \sigma^2}}{B_2}\right)$, $g_5 = \; E_i \left(\frac{2\sqrt{B_1\; \sigma^2}}{B_2}\right)$ *and* $g_6 = \; E_i \left(\frac{2\sqrt{B_1\; e^{B_2\, h}\, \sigma^2}}{B_2}\right)$.

Where, $E_i(x)$ is the Exponential integral function of x and is defined as $E_i(x) = \int_{-\infty}^{x} \frac{e^t}{t}\; dt$.

Case 2:without porosity

Solving equations (7) and (8) we get

$u_1(\eta) = e^{\sigma\, \eta}\, A_1 + e^{-\sigma\, \eta}\, A_2 - \frac{g_1}{\sigma^2}$ (13)

$u_2(\eta) = A_3 \eta + \; A_4 - \frac{g_2\, \eta^2}{2}$ (14)

where A_1, A_2, A_3 and A_4 are integrating constant then substituting theboundary conditions (6) in equations (13) and (14)

$$A_1 = \frac{g_1\, \mu_1 - e^{h\sigma} g_2 \sigma\, \mu_2}{(1+e^{2h\sigma})\sigma^2 \mu_1},$$

$$A_2 = \frac{e^{h\sigma}(e^{h\sigma} g_1\, \mu_1 + g_2 \sigma\, \mu_2)}{(1+e^{2h\sigma})\sigma^2 \mu_1},$$

$$A_3 = -g_2(1+h),$$

$A_4 = \frac{(-2(-1+e^{h\sigma})^2 g_1 + (1+e^{2h\sigma}) g_2 h(2+h)\sigma^2 \mu_1 - 2(-1+e^{2h\sigma}) g_2 \sigma \mu_2}{2(1+e^{2h\sigma})\sigma^2 \mu_1}$.

The average velocity is given by,

$\overline{u_1(\eta)} = \frac{1}{2} \int_0^h u_1(\eta)\; d\eta$

$\overline{u_1(\eta)} = \frac{-g_1 h + e^{-h\sigma}(-1+e^{h\sigma})(A_2 + A_1 e^{h\sigma})\sigma}{2\sigma^2}$ (15)

$\overline{u_2(\eta)} = \frac{1}{2} \int_h^1 u_2(\eta)\; d\eta$

$\overline{u_2(\eta)} = \frac{-1}{12}(-1+\mathrm{h})(6\mathrm{A}_4 + 3\mathrm{A}_3(1+\mathrm{h}) + \mathrm{g}_2(1+\mathrm{h}+\mathrm{h}^2))$ (16)

3.2 Concentration Distribution:

The concentration of c_1 with chemical reaction k_1 of the solute for the region1 expressed as follows

$\frac{\partial c_1}{\partial t} \; + u_1\, \frac{\partial c_1}{\partial x} = D_1 \left(\frac{\partial^2 c_1}{\partial x^2} + \frac{\partial^2 c_2}{\partial y^2}\right) - k_1 c_1$ (17)

Similarly, the concentration of c_2 with chemical reaction k_2 of the solute forthe region 2 satisfies $\frac{\partial c_2}{\partial t} + u_2 \frac{\partial c_2}{\partial x} = D_2 \left(\frac{\partial^2 c_2}{\partial x^2} + \frac{\partial^2 c_2}{\partial y^2}\right) - k_2 c_2$ (18)

where D_1 and D_2 are the molecular diffusion coefficients (assumed constant)for the region 1 and region 2, respectively. The longitudinal diffusion is very much less than the transverse diffusion which implies $\frac{\partial^2 c_1}{\partial x^2} \ll \frac{\partial^2 c_1}{\partial y^2}$, $\frac{\partial^2 c_2}{\partial x^2} \ll \frac{\partial^2 c_2}{\partial y^2}$.

Equations (17) and (18) becomes,

$$\frac{\partial c_1}{\partial t} + u_1 \frac{\partial c_1}{\partial x} = D_1 \frac{\partial^2 c_1}{\partial y^2} - k_1 c_1, \tag{19}$$

$$\frac{\partial c_2}{\partial t} + u_2 \frac{\partial c_2}{\partial x} = D_2 \frac{\partial^2 c_2}{\partial y^2} - k_2 c_2. \tag{20}$$

The dimensionless quantities are:

$$\theta_1 = \frac{t_1}{\overline{t_1}},\ \overline{t_1} = \frac{L_1}{\overline{u_1}},\ \xi_1 = \frac{x_1 - \overline{u_1} t}{L},\ \theta_2 = \frac{t_2}{\overline{t_2}},\ \overline{t_2} = \frac{L_2}{\overline{u_2}}, \xi_2 = \frac{x_2 - \overline{u_2} t}{L}.$$

Equations (19) and (20) becomes

Region: 1

$$\frac{1}{t_1}\frac{\partial c_1}{\partial \theta_1} + \frac{u_{1x}}{L}\frac{\partial c_1}{\partial \xi_1} = \frac{D_1}{H^2}\frac{\partial^2 c_1}{\partial \eta^2} - k_1 c_1. \tag{21}$$

Region: 2

$$\frac{1}{t_2}\frac{\partial c_2}{\partial \theta_2} + \frac{u_{2x}}{L}\frac{\partial c_2}{\partial \xi_2} = \frac{D_2}{H^2}\frac{\partial^2 c_2}{\partial \eta^2} - k_2 c_2. \tag{22}$$

where L is the normal length along direction of the flow.

The dimensionless boundary conditions on concentration is given by:

$$\frac{\partial c_1}{\partial \eta} = 0\ at\ \eta = 0,$$

$$c_1 = c_2, \frac{\partial c_1}{\partial \eta} = \frac{D_2}{D_1}\frac{\partial c_2}{\partial \eta}\ at\ \eta = h,$$

$$\frac{\partial c_2}{\partial \eta} = 0\ at\ \eta = 1. \tag{23}$$

To obtain c_1 and c_2 as the variation of η by approximating equations (21) and (22).

Region: 1

$$\frac{\partial^2 c_1}{\partial \eta^2} - \alpha_1^2 c_1 = z_1 u_{1x} \tag{24}$$

Region: 2

$$\frac{\partial^2 c_2}{\partial \eta^2} - \alpha_2^2 c_2 = z_2 u_{2x} \tag{25}$$

$$where, \alpha_1 = H\sqrt{\frac{k_1}{D_1}},\ \alpha_2 = H\sqrt{\frac{k_2}{D_2}},\ z_1 = \frac{H^2}{D_1 L}\frac{\partial c_1}{\partial \xi_1}, z_2 = \frac{H^2}{D_2 L}\frac{\partial c_2}{\partial \xi_2}.$$

Case 1 :with porosity variation

The relative velocities are given by.

Region: 1

$$u_{1x} = u_1 - \overline{u} = e^{\frac{2}{B_2}\sqrt{B_1 \sigma^2 e^{B_2 \eta}}} f_1 + e^{\frac{-2}{B_2}\sqrt{B_1 \sigma^2 e^{B_2 \eta}}} f_2 + L_1 \tag{26}$$

Region: 2

$$u_{2x} = u_2 - \overline{u} = f_3 \eta + \frac{g_2 \eta^2}{2} + L_2 \tag{27}$$

$$where\ L_1 = \frac{-g_1}{\sigma^2} + L_3,\ L_2 = f_4 + L_3\ and\ L_3 = \frac{1}{12}(-1+h)$$

$$(6 f_4 + 3 f_3 (1+h) + g_2(1+h+h^2)) + \frac{g_1 h}{2\sigma^2} + \frac{f_2 g_3 - f_2 g_4 + f_1 g_5 - f_1 g_6}{B_2}$$

and $\overline{u}$ is the sum of average velocities of region 1 and 2. Using the equations (26) and (27) and satisfying the boundary condition (23), the solution ofequations (24) and (25) we get

Region:1

$$c_1 = z_1 c_{11} + z_2 c_{12} \tag{28}$$

Region:2

$$c_2 = z_1 c_{21} + z_2 c_{22} \tag{29}$$

From equations (28) and (29), the lengthy expression of c_{11}, c_{12}, c_{21} and c_{22} are computed and the results are using in the graph.

Case 2:without porosity

Region: 1

$$u_{1x} = u_1 - \overline{u} = e^{\sigma\eta} A_1 + e^{-\sigma\eta} A_2 + l_1 \tag{30}$$

Region: 2

$$u_{2x} = u_2 - \overline{u} = A_3\eta + \frac{g_2\eta^2}{2} + l_2 \tag{31}$$

$$where\ l_1 = \frac{-g_1}{\sigma^2} + lR1,\ l_2 = A_4 + lR1\ and\ lR1 = \frac{1}{12}(-1+h)$$

$$(6A_4 + 3A_3(1+h) + g_2(1+h+h^2)) + \frac{g_1 h + A_1\sigma - A_2\sigma + A_2 e^{-h\sigma}\sigma - A_1 e^{h\sigma}\sigma}{2\sigma^2}.$$

The solution of equations (24) and (25) with satisfying the boundary condition (23) using the equations (30) and (31). The expression for c_1 and c_2 can be written as.

Region:1

$$c_1 = z_1 c_{11}{}^* + z_2 c_{12}{}^* \tag{32}$$

Region:2

$$c_2 = z_1 c_{21}{}^* + z_2 c_{22}{}^* \tag{33}$$

Where

$c_{11}^* =$

$$\left(\frac{1}{\alpha_1^2\,(e^{2h(\alpha_1+\alpha_2)}s_2 - e^{2\alpha_2}s_2 - e^{2h\alpha_2}s_7 + e^{2(h\alpha_1+\alpha_2)}s_{7)s_1}}\right)$$

$$\begin{aligned}(e^{-h(\alpha_1+\sigma)-\eta(2\alpha_1+\sigma)}(e^{h+\eta)(\alpha_1+\sigma)}l_1\big(&-de^{h(\alpha_1+2\alpha_2)}\alpha_2 + de^{h\alpha_1+2\alpha_2}\alpha_2 + de^{h\alpha_1+2\alpha_2+2\alpha_1\eta}\alpha_2\\ &- de^{h\alpha_1+2h\alpha_2+2\alpha_1\eta}\alpha_2 + e^{2\alpha_2+\alpha_1\eta}s_2 - e^{2h(\alpha_1+\alpha_2)+\alpha_1\eta}s_2 - e^{2h\alpha_1+2\alpha_2+\alpha_1\eta}s_2 + e^{2h\alpha_1+\alpha_1\eta}s_2\big)s_1\\ &+ A_1\alpha_1\big(e^{2\alpha_2+h(\alpha_1+\sigma)+2\eta(\alpha_1+\sigma)}s_3 - e^{2\eta(\alpha_1+\sigma)+h(3\alpha_1+2\alpha_2+\sigma)}s_3 - e^{2\alpha_2+2\eta(\alpha_1+\sigma)+h(3\alpha_1+\sigma)}s_4\\ &+ e^{2\eta(\alpha_1+\sigma)+h(\alpha_1+2\alpha_2+\sigma)}s_4 - e^{2\alpha_2+3\alpha_1\eta+\eta\sigma+h(\alpha_1+\sigma)}s_5 - e^{\eta(\alpha_1+\sigma)+h(3\alpha_1+2\alpha_2+\sigma)}s_5\\ &- e^{2\alpha_2+\eta(\alpha_1+\sigma)+h(3\alpha_1+\sigma)}s_6 - e^{\eta(3\alpha_1+\sigma)+h(\alpha_1+2\alpha_2+\sigma)}s_6 + e^{\eta(\alpha_1+\sigma)+2h(\alpha_1+\alpha_2+\sigma)}s_5\\ &+ e^{\eta(3\alpha_1+\sigma)+2h(\alpha_1+\alpha_2+\sigma)}s_5 + e^{2\alpha_2+3\alpha_1\eta+\eta\sigma+2h(\alpha_1+\sigma)}s_6 + e^{2\alpha_2+2h(\alpha_1+\sigma)+\eta(\alpha_1+\sigma)}s_6\big)\\ &+ A_2\alpha_1(e^{2(\alpha_2+\alpha_1\eta)+h(\alpha_1+\sigma)}s_3 - e^{2\alpha_1\eta+h(3\alpha_1+2\alpha_2+\sigma)}s_3 - e^{3h\alpha_1+2\alpha_2+2\alpha_1\eta+h\sigma}s_4\\ &+ e^{2\alpha_1\eta+h(\alpha_1+2\alpha_2+\sigma)}s_4 + e^{2\alpha_2+3\alpha_1\eta+\eta\sigma+h(\alpha_1+\sigma)}s_5 + e^{\eta(\alpha_1+\sigma)+h(3\alpha_1+2\alpha_2+\sigma)}s_5\\ &+ e^{2\alpha_2+\eta(\alpha_1+\sigma)+h(3\alpha_1+\sigma)}s_6 + e^{\eta(3\alpha_1+\sigma)+h(\alpha_1+2\alpha_2+\sigma)}s_6 - e^{2h\alpha_1+2\alpha_2+3\alpha_1\eta+\eta\sigma}s_5\\ &- e^{2h\alpha_1+2\alpha_2+\eta(\alpha_1+\sigma)}s_5 - e^{2h(\alpha_1+\alpha_2)+\eta(\alpha_1+\sigma)}s_6 - e^{2h(\alpha_1+\alpha_2)+\eta(3\alpha_1+\sigma)}s_4)))\end{aligned}$$

$$c_{12}^* = \frac{de^{\alpha_1(h-\eta)}(1+e^{\alpha_1\eta})(-4e^{\alpha_2+h\alpha_2}s_8 - e^{2h\alpha_2}s_9 + e^{2\alpha_2}s_{10})}{2\alpha_2^3(e^{2\alpha_2}s_2 - e^{2h(\alpha_1+\alpha_2)}s_2 + e^{2h\alpha_2}s_7 - e^{2(h\alpha_1+\alpha_2)}s_7)};$$

$$c_{21}^* = \frac{1}{\alpha_1(e^{2h(\alpha_1+\alpha_2)}s_2 - e^{2\alpha_2}s_2 - e^{2h\alpha_2}s_7 + e^{2(h\alpha_1+\alpha_2)}s_7)s_1}(-(e^{h\alpha_2-\alpha_2\eta-h\sigma}(e^{2\alpha_2}+e^{2\alpha_2\eta})$$

$$\begin{aligned}(A_2\alpha_1((-1+e^{2h\alpha_1})\alpha_1 + (1+e^{2h\alpha_1} - 2e^{h(\alpha_1+\sigma)})\sigma) + e^{h\sigma}((-1+e^{2h\alpha_1})l_1 s_1 -\\ A_1\alpha_1(-2e^{h\alpha_1}\sigma + e^{h(2\alpha_1+\sigma)}(-\alpha_1+\sigma)e^{h\sigma}(\alpha_1+\sigma))))));\end{aligned}$$

$c_{22}^{*} = \frac{1}{2{\alpha_2}^4(e^{2h(\alpha_1+\alpha_2)}s_2 - e^{2\alpha_2}s_2 - e^{2h\alpha_2}s_7 + e^{2(h\alpha_1+\alpha_2)}s_7)}(e^{-\alpha_2\eta}(-2e^{\alpha_2+2h(\alpha_1+\alpha_2)}s_8 s_2 -$

$2e^{\alpha_2+2\alpha_2\eta}s_8 s_2 + 2e^{\alpha_2+2h\alpha_2}s_8 s_7 + 2e^{2h\alpha_1+\alpha_2+2\alpha_2\eta}s_8 s_7 + e^{2h\alpha_1+2\alpha_2+h\alpha_2}s_{11} +$

$e^{2h\alpha_1+h\alpha_2+2\alpha_2\eta}s_{11} - e^{(2+h)\alpha_2}s_{12} - e^{\alpha_2(h+2\eta)}s_{12} + e^{\alpha_2(2+\eta)}s_2(2\alpha_2^2(l2 + A_3\eta) +$

$g_2(2 + \alpha_2^2\,\eta^2)) - e^{2h(\alpha_1+\alpha_2)+\alpha_2\eta}s_2(2\alpha_2^2\,(l2{+}A_3\eta) + g_2(2 + \alpha_2^2\eta^2)) + e^{\alpha_2(2h+\eta)}s_7$

$\left(2\alpha_2^2\,(l2 + A_3\eta) + g_2(2 + \alpha_2^2\eta^2)\right) - e^{2h\alpha_1+\alpha_2(2+\eta)}s_7(2\alpha_2^2\,(l2{+}A_3\eta) + \ g_2(2 + \alpha_2^2\eta^2)))).$

$Where\ \ s_1 = \alpha_1^2 - \sigma^2,\ s_2 = \alpha_1 - d\alpha_2\ ,\ s_3 = \alpha_1(\alpha_1 - d\alpha_2),\ s_4 = \alpha_1(\alpha_1 + d\alpha_2),$

$s_5 = \sigma(\alpha_1 - d\alpha_2),\ s_6 = \sigma(\alpha_1 + d\alpha_2),\ s_7 = \alpha_1 + d\alpha_2,\ s_8 = (A_3 + g_2)\alpha_2,$

$s_9 = 2\alpha_2(-A_3 + A_3 h\alpha_2 + l2\alpha_2) + g_2(2 - \alpha_2 + h^2\alpha_2^2),$

$s_{10} = 2\alpha_2(A_3 + A_3 h\alpha_2 + l2\alpha_2) + g_2(2 + 2h\alpha_2 + h^2\alpha_2^2),$

$s_{11} = 2(l2\alpha_1 + A_3(\text{-d+h}\alpha_1))\alpha_2^2 + g_2(\text{-2dh}\alpha_2^2 + \alpha_1(2{+}h^2\alpha_2^2)),$

$s_{12} = 2(l2\alpha_1 + A_3(\text{d+h}\alpha_1))\alpha_2^2 + g_2(\text{2dh}\alpha_2^2 + \alpha_1(2{+}h^2\alpha_2^2))$ and $d=\frac{D_2}{D_1}$.

3.3 Dispersion coefficient:

Case 1:with porosity variation

The fluid is transported across the section of layer per unit breadth then thevolumetric rate of the fluid $Q_1$and Q_2 are given by.

Region: 1

$Q_1 = H\int_0^h c_1 u_{1x} d\eta = -(Q_{11} + Q_{12}).$ (34)

Region: 2

$Q_2 = H\int_0^h c_2 u_{2x} d\eta = -(Q_{21} + Q_{22}).$ (35)

Where $Q_{11} = -z_1 H\int_0^h c_{11}u_{1x}d\eta,\ Q_{12} = -z_2 H\int_0^h c_{12}u_{2x}d\eta,$

$$Q_{21} = -z_1 H\int_h^1 c_{21}u_{2x}d\eta,\ Q_{22} = -z_2 H\int_h^1 c_{22}u_{2x}d\eta.$$

We assume that the variations of $c_1$and c_2 with η are small compared to thelongitudinal direction, and if c_{m1} and c_{m2} is the mean concentration over a

Section, then$\frac{\partial c_1}{\partial \xi_1}$ and $\frac{\partial c_2}{\partial \xi_2}$ are indistinguishable from $\frac{\partial c_{m1}}{\partial \xi_1}$ and $\frac{\partial c_{m2}}{\partial \xi_2}$

(Taylor's (1953)) so that equations (34) and (35) can be written as.

Region: 1

$Q_{11} = -D_{11}^{*}\frac{\partial c_{m1}}{\partial \xi_1}$; $Q_{12} = -D_{12}^{*}\frac{\partial c_{m2}}{\partial \xi_2}$ (36)

Region: 2

$Q_{21} = -D_{21}^{*}\frac{\partial c_{m1}}{\partial \xi_1}$; $Q_{22} = -D_{22}^{*}\frac{\partial c_{m2}}{\partial \xi_2}$ (37)

No material is lost in the process which is expressed by the continuity equationfor c_{m1} and c_{m2} namely,

Region: 1

$$\frac{\partial Q_{11}}{\partial \xi_1} = -2\frac{\partial c_{m1}}{\partial t};\ \frac{\partial Q_{12}}{\partial \xi_2} = -2\frac{\partial c_{m2}}{\partial t} \quad (38)$$

Region: 2

$$\frac{\partial Q_{21}}{\partial \xi_1} = -2\frac{\partial c_{m1}}{\partial t};\ \frac{\partial Q_{22}}{\partial \xi_2} = -2\frac{\partial c_{m2}}{\partial t} \quad (39)$$

Equations (36) and (37) using (38) and (39) we get

Region: 1

$$\frac{\partial c_{m1}}{\partial t} = \frac{D_{11}^*}{2}\frac{\partial^2 c_{m1}}{\partial {\xi_1}^2}\ ;\ \frac{\partial c_{m2}}{\partial t} = \frac{D_{12}^*}{2}\frac{\partial^2 c_{m2}}{\partial {\xi_2}^2} \qquad (40)$$

Region: 2

$$\frac{\partial c_{m1}}{\partial t} = \frac{D_{21}^*}{2}\frac{\partial^2 c_{m1}}{\partial {\xi_1}^2}\ ;\ \frac{\partial c_{m2}}{\partial t} = \frac{D_{22}^*}{2}\frac{\partial^2 c_{m2}}{\partial {\xi_2}^2} \qquad (41)$$

We obtain an effective dispersion coefficient as follows

$$D_{11}^* = \frac{H^2}{2D_1}\int_0^h c_{11}u_{1x}d\eta = \frac{H^2}{2D_1}\ F_{11}(\sigma, g_1, g_2, \alpha_1, \alpha_2) \qquad (42)$$

$$D_{12}^* = \frac{H^2}{2D_2}\int_0^h c_{12}u_{1x}d\eta = \frac{H^2}{2D_2}\ F_{12}(\sigma, g_1, g_2, \alpha_1, \alpha_2) \qquad (43)$$

$$D_{21}^* = \frac{H^2}{2D_1}\int_h^1 c_{21}u_{2x}d\eta = \frac{H^2}{2D_1}\ F_{21}(\sigma, g_1, g_2, \alpha_1, \alpha_2) \qquad (44)$$

$$D_{22}^* = \frac{H^2}{2D_2}\int_h^1 c_{22}u_{2x}d\eta = \frac{H^2}{2D_2}\ F_{22}(\sigma, g_1, g_2, \alpha_1, \alpha_2) \qquad (45)$$

Case 2:without porosity

Following the same procedure in case (1) .From equations (40) and (41) we get, Region: 1

$$\frac{\partial c_{m1}}{\partial t} = \frac{\overline{D_{11}}}{2}\frac{\partial^2 c_{m1}}{\partial {\xi_1}^2}\ ;\ \frac{\partial c_{m2}}{\partial t} = \frac{\overline{D_{12}}}{2}\frac{\partial^2 c_{m2}}{\partial {\xi_2}^2} \qquad (46)$$

Region: 2

$$\frac{\partial c_{m1}}{\partial t} = \frac{\overline{D_{21}}}{2}\frac{\partial^2 c_{m1}}{\partial {\xi_1}^2}\ ;\ \frac{\partial c_{m2}}{\partial t} = \frac{\overline{D_{22}}}{2}\frac{\partial^2 c_{m2}}{\partial {\xi_2}^2} \qquad (47)$$

We obtain an effective dispersion coefficient as follows

$$\overline{D_{11}} = \frac{H^2}{2D_1}\int_0^h {c_{11}}^* u_{1x}d\eta = \frac{H^2}{2D_1}\ {F_{11}}^*(\sigma, g_1, g_2, \alpha_1, \alpha_2) \qquad (48)$$

$$\overline{D_{12}} = \frac{H^2}{2D_2}\int_0^h {c_{12}}^* u_{1x}d\eta = \frac{H^2}{2D_2}\ {F_{12}}^*(\sigma, g_1, g_2, \alpha_1, \alpha_2) \qquad (49)$$

$$\overline{D_{21}} = \frac{H^2}{2D_1}\int_h^1 {c_{21}}^* u_{2x}d\eta = \frac{H^2}{2D_1}\ {F_{21}}^*(\sigma, g_1, g_2, \alpha_1, \alpha_2) \qquad (50)$$

$$\overline{D_{22}} = \frac{H^2}{2D_2}\int_h^1 {c_{22}}^* u_{2x}d\eta = \frac{H^2}{2D_2}\ {F_{22}}^*(\sigma, g_1, g_2, \alpha_1, \alpha_2) \qquad (51)$$

The values of F_{ii}, F_{ii}^* are computed for various values of dimensionless parameter

Porous σ, pressure gradient g_1 *and* g_2, *reaction rate parameter* α_1 *and* α_2.

4 Discussion of the Results

The dispersion of oil flow between two parallel plates is discussed. The resultsof the analysis for different values of porous parameters, chemical reactions and particle diameter for velocity and dispersion coefficient are obtained usedby mathematica software.

Figures 2 and 3 displays the effects of porous parameter σ and particle diameter dp on the velocity field with porosity variation in region: 1 . It is reveal that the velocity reduces as the porous parameter increases. This is dueto frictional drag resistance against the flow in the porous region. Figures 4 and 5 represents the dispersion coefficient D^*_{11} with σ for different values of k_1 and dp. Figure 4 we observe that the parameters increases as the * increases. In Figure 5 depicts that increasing the parameters values enhances dispersion coefficient. Figures 6 and 7 the effects of chemical reaction k_1 and particle diameter dp on the dispersion coefficient D^*_{21} . Figure 6 displayed that the values of parameters are increasing with increasing the dispersion coefficient. In Figure 7 shows that the parameters increases as decreasing the dispersion. From figure 8 and 9 signify the chemical reaction k_2 and dp on dispersion coefficient D^*_{12} . They indicates that D^*decreases with increasing the parameters. From Figure 10 we see that whenthe k_2 increases the dispersion coefficient increases. The effective dispersion coefficients $\overline{D11}$, $\overline{D12}$, $\overline{D21}$ and $\overline{D22}$ with σ for variation of chemical reaction are shown in figure 11 , 12 , 13 and 14 . The figures indicates that increasing the chemical reaction parameters with decreases the dispersion coefficients.The above results are very useful for analysis the effect of flow in the oil spilled affect the topsoil is carried out.

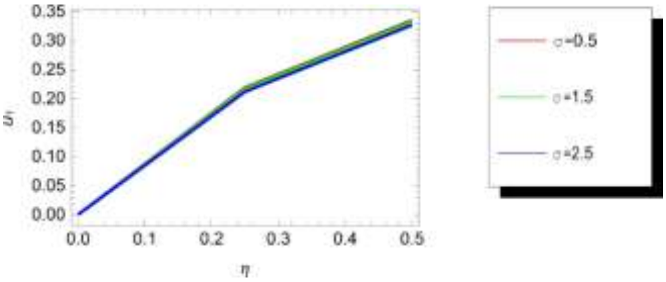

Figure 2: Velocity profiles for various values of porous in region 1

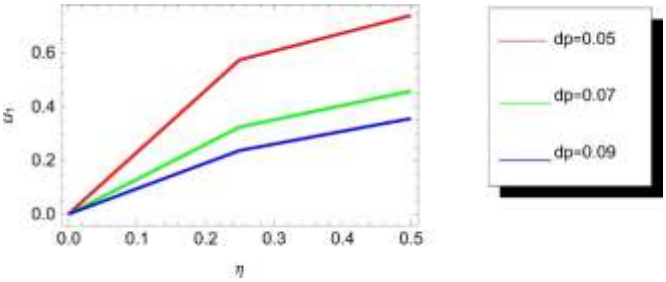

Figure 3: Velocity profiles for various values of particle diameter in region 1

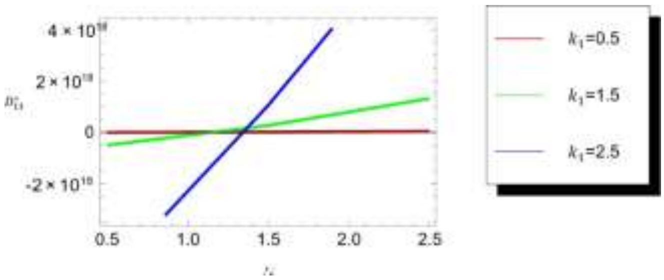

Figure 4: Dispersion coefficient D^* on distinct values of chemical reaction

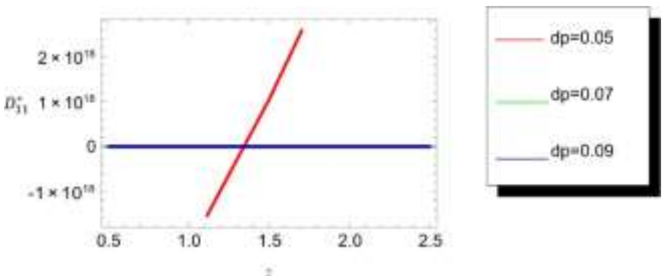

Figure 5: Dispersion coefficient D^* on distinct values of particle diameter

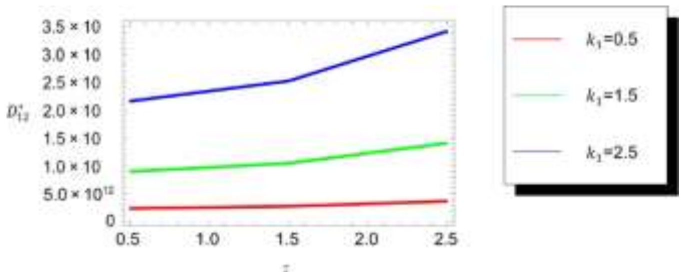

Figure 6: Dispersion coefficient D^*on distinct values of chemical reaction

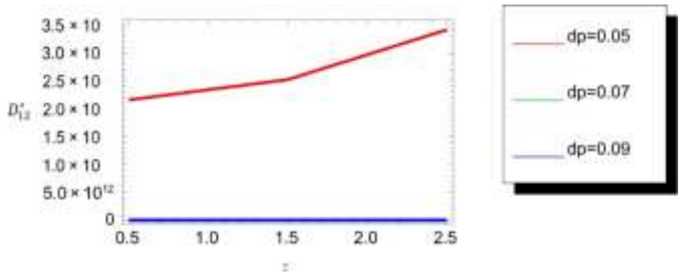

Figure 7: Dispersion coefficient D^* on distinct values of particle diameter

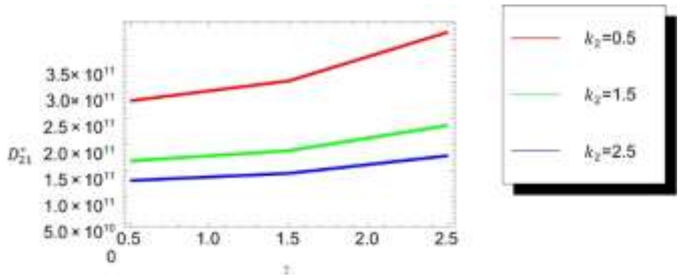

Figure 8: Dispersion coefficient D^* on distinct values of chemical reaction

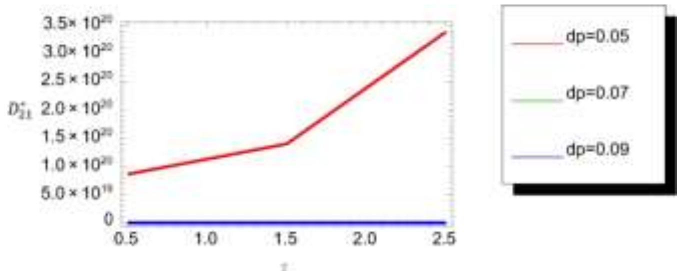

Figure 9: Dispersion coefficient D^*on distinct values of particle diameter

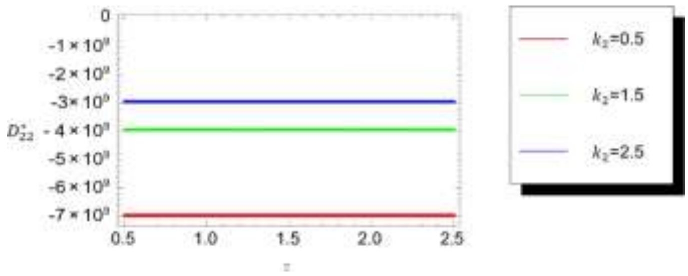

Figure 10: Dispersion coefficient D^*on distinct values of chemical reaction

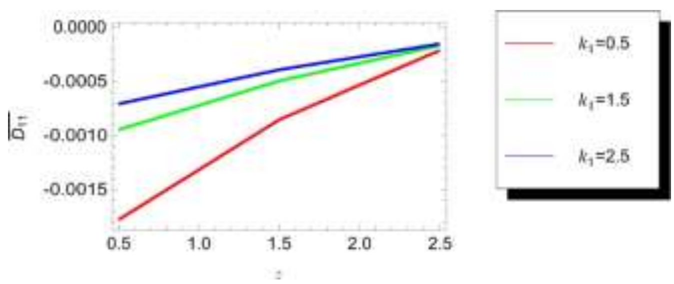

Figure 11: Dispersion coefficient D_{11} on distinct values of chemical reaction

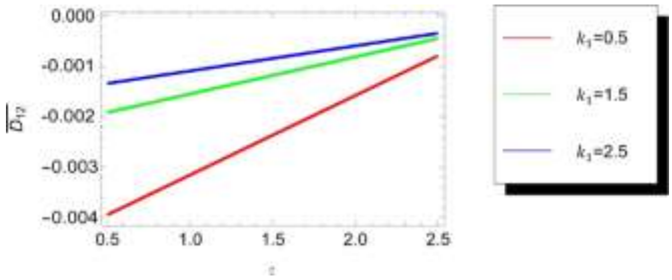

Figure 12: Dispersion coefficient D_{12} on distinct values of chemical reaction

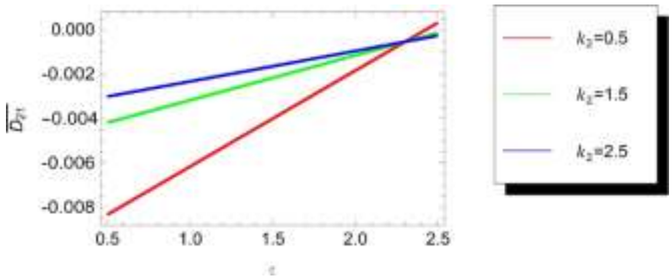

Figure 13: Dispersion coefficient D_{21} on distinct values of chemical reaction

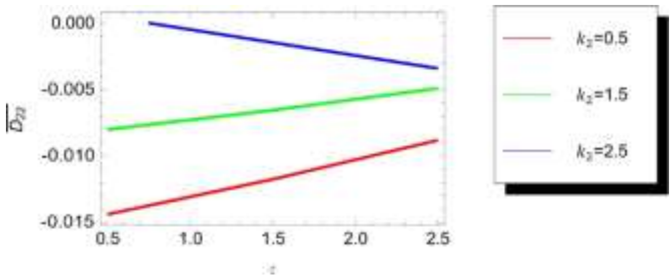

Figure 14: Dispersion coefficient D_{22} on distinct values of chemical reaction

5 Conclusion

The dispersion of oil spilled with the chemical reaction to obtained by Taylor's diffusion model. To evaluate the average velocities, volume flow rateand effective dispersion coefficient in each region. It is concluded that the dispersion coefficient of case 1 increases with increases in chemical reaction parameters but dispersion coefficient of case 2 decreases with increases chemical reaction. In this context, the main aim of the present research was observed that soil contamination due to oil spilled can affect soil health.

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