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A New Wavelet Thresholding Technique for Denoising of Digital Images by using Wavelet Packet Transformation

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Abstract: During the acquisition process, digital images are corrupted by noisy signals. From decades different important techniques are introduced in the area of digital image noise reduction. Wavelet Transform technique has its own beauty of de-noising all types of noisy digital images. In this paper, an algorithm has been introduced for de-noising of noisy images of a cameraman with size 256×256 by applying the wavelet based thresholding technique at standard deviation (σ) obtained from the synthesized image, followed by wavelet packet transform. Finally, a comparative study between original and synthesised digital images for image denoising is performed on the bases of mean square error, noise signal ratio, and peak signal noise ratio values. We find under the proposed thresholding technique and wavelet packet transformation, wavelet functions sym4, db4, bior3.3, bior2.4, bior2.6, sym6, sym8, coif2, bior2.8, haar and coif5, sequentially performs better in terms of noise signal ratio (SNR) values for image denoising. Also the wavelet functions like, bior3.3, followed by sym4, bior2.8, bior2.4, bior2.6, sym6, haar coif2, Sym8, db4 and coif5 also sequentially performs well in terms of PSNR for the same.

Keywords: **Wavelet Transformation, Wavelet Packets Transform, Thresholding, MSE, SNR, PSNR**

1. Introduction

Noise enters into the digital images during acquisition, processing, and transmission process and it is one of the big challenge for researchers to remove it [1][2]. Some of the noises like Poisson Noise, Gaussian Noise, Additive noise Speckle Noise, Salt & Pepper noise, etc. may affect some important features, parts, or the whole image [3]. Therefore, digital image de-noising is one of the important area in the modern research. Different techniques are adopted for digital image de-noising form several decades. One of the important features of a successful method (with the aim of de-noising digital images) is that it should reduce noise as much as possible, preserve edges and essential image information, and also bring out better image visual quality [4].

Here are some of research works related to my paper. A new method for digital image denoising based on discrete wavelet transform at the best level of decomposition. The optimal threshold is obtained by the techniques like the crow search algorithm and social spider optimization techniques. Here thresholding function is taken from the detail coefficients[5]. The author applied the thresholding technique for speckle noise reduction from ultrasound images and is based on wavelet transform. The wavelet transform performs multi-scale analysis of the signal by choosing various frequency components of the signal one by one.[6]. An image enhancement technique to extract and highlight the significant features of suspected ribs fractures in X-ray images. The integration of wavelet denoising and image contrast enhancement removed noises, improving the readability of the images. Number of Wavelets like daubechies, symlets, coiflets, and biorthogonal were applied at multiple thresholds of “horizontal, diagonal and vertical coefficients” at different decomposition wavelet levels[7]. The author collectively applied three methods to improve the image denoising quality in the wavelet domain. First discrete wavelet transformation was used to enable image processing in the frequency domain. For the images denoising, soft thresholding technique was combined with the median filter also to preserve the image sharpness, edge coefficients were kept and not affected by the image denoising process[8]. An image denoising based method on wavelet thresholding, with a new nonlinear thresholding function differentiated by a shape parameter and basic properties, which makes the new method able to achieve a compromise between techniques such as Hard and Soft thresholding[9]. Due to deficiencies in traditional thresholding, i.e. hard threshold function is discontinuous, and the soft threshold function causes constant deviation. A new method for removing image noise is proposed, which decomposes the noise image to determine the wavelet coefficients also the wavelet coefficient is applied to the high-frequency part of the threshold processing by using the improved threshold function[10]. The author proposes novel approach that controls the threshold (T) adjustably to remove noise. The fact-finding results of the proposed method are superior than the existing image denoising algorithm such as NSTISM(19), SPBIDM(18), BayesShrink, Normal Shrink, and Modified Bayes Shrink[11]. A simple method is applied to denoise digital image which have been affected by “Additive White Gaussian Noise (AWGN)”.This method also uses Wiener filter before and after the wavelet transform. All three tools 2-D DWT, threshold techniques and Wiener filter is used to remove noise from pixels. Finally 2-D IDWT is applied to find noise free image and complete the denoising technique[12].

Therefore, reviewing and investigating several research articles, they were mainly focused on producing optimal threshold value, because threshold value plays important role in digital image denoising. Different techniques and methods were applied for digital image denoising, almost all are focused on the right selection threshold value. On analyses bases and findings we proposed a new algorithm based on the thresholding technique (threshold value) for digital image denoising. Finally, we observe this technique acts much better in terms of digital image de-noising.

2. Wavelet Theory

The wavelet transform could be generalized to different dimensions. The concept of wavelet was first introduced by Morlet and Grossmann in beginning of 1980s[13][14]. The theory of wavelet transform has great applications in digital image processing. In this research paper, a two-dimensional discrete wavelet transform is taken into consideration.

3. Wavelets

A wavelet is small wave that helps to break down a big signal into different frequency components (wavelets) and study all these components with a resolution that matches to its scale. It is defined as[15][16],

$$\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}}\psi\left(\frac{t-b}{a}\right) \qquad \text{...... (1)}$$

Here $\psi \in L^2(R)$ is called mother wavelet, **a** and **b** are translation and scale parameters. Therefore, ψ generates a family of function (Daughter Wavelets) by translations and scaling[13].

If a function satisfies below conditions should be considered as mother wavelet

a). Wavelet must have finite energy[15],

$$E = \int |\psi(t)|^2 dx < \infty \qquad \text{...... (2)}$$

b). Wavelet should be a zero average function,

$$\int_{-\infty}^{+\infty} \psi(t)\ dt = 0 \qquad \text{...... (3)}$$

c). Have an admissibility condition

$$\int_{-\infty}^{+\infty} \frac{|\psi\ (\omega)\ |^2}{|\omega|}\ d\omega < \infty \qquad \text{...... (4)}$$

Here, $\psi(\omega)$ is fourier transform of $\psi(t)$

4. Wavelet Transformation

In the wavelet transform we utilize a family of functions (wavelets) which are localized in both time and frequency (scale) domains. The wavelet transforms in digital image processing simply represent the image as a set of wavelets with varied locations and scales. . In the process of decomposition of digital images into wavelets through wavelet transformation literally applies two waveforms. One represents its “high frequency” matching to its detailed part is known as wavelet function. Other represent the low frequency corresponding to the approximation part of an image is known as scaling function. To do the same transformation the mother wavelet (Transform Function) is modified by translations and scaling.

5. Wavelet signal decomposition Process

At beginning the given image or signal is filtered through low and high pass filter along the row and the result of each filter are down sampled by two[17]. Each sub-signal shows the low and high frequency component along the rows and each have a size $N \times N/2$ .Then each sub signal is again passed through high and low pass filter along the columns, again the dub-data is down sampled by two. This process can be shown by below diagram,

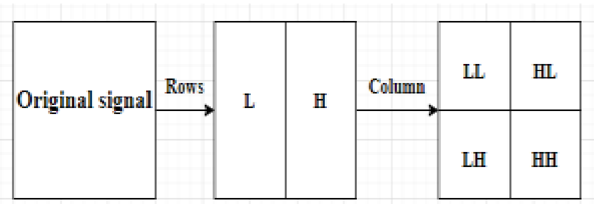

Fig.1: Diagram of wavelet decomposition of the digital image at level -2

In way the image is sub-divided into four images taking each size of $N/_2 \times N/_2$ and also taking information of various frequency components. Here LL part of the signal is called the approximation coefficient, and is result of the low-pass filter both to the rows and columns and it contains rough description of the image. The HL, LH, and HH part of the signal is called, horizontal, vertical and diagonal detailed coefficients. The HL and LH sub signals are result of low pass filtering along rows and high pass filtering along columns. The HH sub signal is result obtained by high pass filter in both direction and also taking high frequency along the diagonals[17]. These whole four parts are visualized below fig,

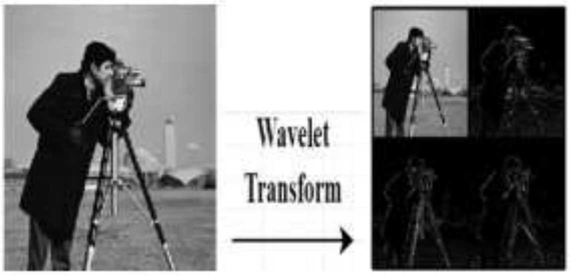

Fig.2: The digital image 2nd level wavelet decomposition.

6. Continuous wavelet Transform

The CWT of digital image is defined as,

$$W_f(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(t) \overset{*}{\psi}\left(\frac{t-a}{b}\right) \quad ...... (5)$$

Here a, b and ψ already defined in (1), $f(t)$ is a two dimensional function (Image) and $*$ is known as complex conjugate used if and only if wavelet function is complex. Through CWT the original function (image) is broke down into wavelets (inner product) i.e. inner product of original function $f(t)$ with wavelet (family) $\psi_{a,b}(t)$ defined below;

$$W_f(a,b) = \langle f, \psi_{a,b} \rangle = \frac{1}{\sqrt{a}} \int f(t) \overset{*}{\psi}\left(\frac{t-a}{b}\right) \ ...\ (7)$$

Here, $1/\sqrt{a}$ is normalization constant, **a** is always greater than zero. If **a** is greater than one, wavelets are dilated and if less than one, wavelets are contracted[18][15].

7. Discrete wavelet transformation

The DWT is in fact CWT but with discrete scales and translations. In other words we can say through DWT wavelets are discretely sampled[14][15]. The scale and translation values are $a = a_0^i$ & $b = kb_0 a_0^i$ here, $i, k \ \varepsilon Z$, $a_0 > 1$.The discrete wavelets are given as;

$\psi_{i,k}(t) = (a_0)^{\frac{-i}{2}} \psi(a_0^{-i}(t - kb_0 a_0^i))$ ...... (8) Therefore it clearly shows that discrete wavelets depends on a_0 and b_0. And also if b_0 is small and a_0 is close to one, in that case discrete wavelets are alike continuous wavelets. Therefore mathematically we write DWT as;

$$W_f(i,k) = \langle f(t), \psi_{i,k} \rangle = \int f(t) \overset{*}{\psi}_{i,k}(t) dt \ ....\ (9)$$

Where $f(t)$ can be reconstructed by below formula;

$$f(t) = A \sum \sum_k W_f(i,k) h_{i,k}(t) \quad ...... (10)$$

Here A is a constant, and is independent to a function $f(t)$.

8. Wavelet Packet Transform (WPT)

The theory of wavelet packet was first introduced by Meyer, Coifman and wickerhauser [19][20]. While doing WT on digital images, at first level it decomposes image into both approximation and detailed part. At further levels only approximation part is further decomposed into both approximation and details part. For WT of a signal there maybe probability of loosing information related to signal included to its detail part. So there is a need of further investigation for analyzing the original signal. But in WPT both coefficients “approximation and detailed” where further decomposed into approximation and detail coefficients. That is why it is so called a generalization of a wavelet transformation [13]. The difference between the application of WT and WPT can be shown below figure

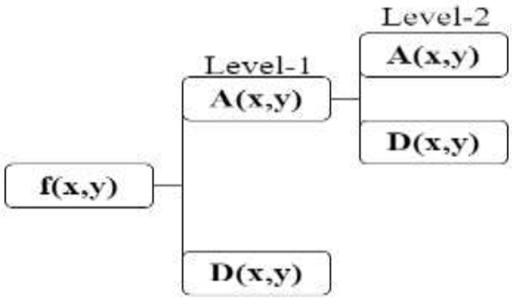

Fig.3. WT with $f(x, y)$ original signals, $A(x, y)$ and $D(x, y)$ are approximation and detail coefficients.

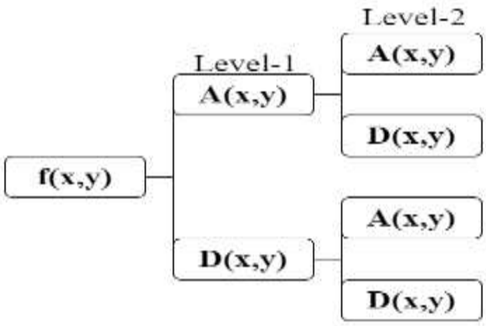

Fig.4. WPT with $f(x, y)$ original signals, $A(x, y)$ and $D(x, y)$ are approximation and detail coefficients

9. Thresholding

Digital images catch different noises during capturing, processing, transmission. Therefore removal of these noises requires different techniques and methods. One of the well known techniques which help us in de-noising such noisy images is thresholding technique[21][14]. This technique letting small wavelet coefficient to zero and remove them (i.e. the noise available in these images). Thresholding technique depends on thresholding value T called threshold value. Selection of threshold value is not easy task[22]. As we choose small threshold value still noise will remain in synthesised image because some of the coefficients which represent noise will not remove and if we choose large threshold value it may be remove a few coefficients, those not carry any noise. Therefore here is a risk of losing signal information as well as creating problem in getting de-noised image from noisy one. This problem is usually denoted by SNR, MSE and PSNR. Higher the SNR, superior the accuracy of de-noising. To maximize the SNR we must to choose optimal threshold value T. The PSNR is usually applied to check the result of image resolution enhancement. Therefore, if PSNR is high, distortion in enhanced image is less, and if PSNR is less, distortion may be high. Usually we call PSNR is a tool used to measure the quality of the image[1][23].

$$SNR = 20\log_{10}\frac{\sum_{x=0}^{m-1}\sum_{y=0}^{n-1}[f(x,y)]^2}{\sum_{x=0}^{m-1}\sum_{y=0}^{n-1}[f(x,y)-f(x,y)]^2}$$ Here, $f(x, y)$ an original is image and $f(x, y)$ is a synthesized image.

$$PSNR = 10\log_{10}\frac{(255)^2}{MSE}$$

Here, 255 is maximum gray level of the image (because it has 0 to 255 gray levels)

$$MSE = \frac{1}{M \times N}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left[f(x,y)-f(x,y)\right]^2$$

here $f(x, y)$ an original is image and $f(x, y)$ is a synthesized image

As per the literature different thresholding techniques were applied for denoising digital images, like Soft, Hard, Hybrid, VisuShrink, SureShrik etc thresholdings[17].

Threshold function is defined as;

$$\lambda(n) = \sigma\sqrt{n\log(M)} \quad , n\, \varepsilon\, N \qquad \text{...... (11)}$$

Here, M is size of function (image), σ is standard deviation of detailed coefficients at best level.

For, n=2

$$\lambda = \sigma\sqrt{2\log(M)} = T \qquad \text{...... (12)}$$

Then T is called universal threshold, M and σ are mentioned above.

Similarly;

For, n=1, 3, we have

$$\lambda(1) = \sigma\sqrt{\log(M)} = T \qquad \text{...... (13)}$$

$$\lambda(3) = \sigma\sqrt{3\log(M)} = T \qquad \text{...... (14)}$$

Here σ^2 is noise variance of detailed coefficients and M are already defined above. In the present article we use all three, (12), (13).and (14) separately, compare result to each other, and their average value.

In this paper whole work is done through MATLAB 2020a, by below proposed method.

10. Proposed Method

In this paper whole work is done through MATLAB 2020a, under these steps

1. The image of cameraman with size 256×256 is first converted to gray scale image.
2. This noisy image is then decomposed by wavelet transformation at level 3.
3. At the higher level of decomposition of signal, determine threshold at the detailed part of coefficients by given formula and their average for comparison purposes in next step.
4. Apply WPT at threshold value obtained in step 3 for de-noising digital image.
5. Then apply inverse wavelet packet transformation (IWPT) for reconstruct the de-noised digital image.
6. Finally compares the results of MSE, SNR and PSNR between original and synthesized images. Whole process of this approach is shown below,

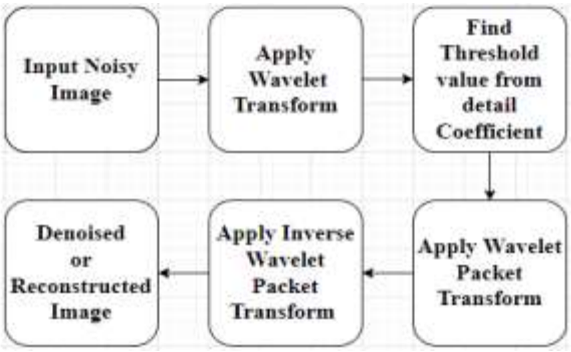

Fig.5**:** Diagram representation of the above proposed method

11. Results

In this piece of research we have used the image of **cameraman** as a test image, with size **256×256** and applied above proposed approach.

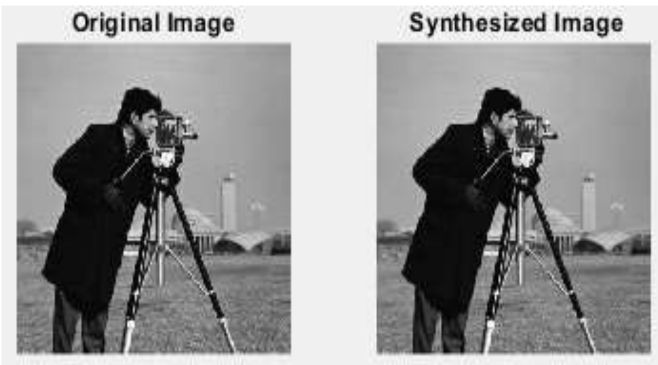

Fig.6**.** Pictures of original and synthesized image of cameraman

12. Table Data Analyses

Size of image M=256×256	Wavelet Transform (Haar)			
Threshold value	λ_1	λ_2	λ_3	Average threshold
σ =0.227	0.499	1.549	1.897	1.315

Table-1

Wavelet Packet Transform (haar) at $\lambda = 1.315$		
MSE	Between original and synthesized image	90.9403
SNR		18.1690
PSNR		28.5895

Table-2

Size of image M=256×256	Wavelet Transform (sym4)			
Threshold value	λ_1	λ_2	λ_3	Average threshold
σ =0.2604	0.571	1.773	2.172	1.505

Table-3

Wavelet Packet Transform(sym4) at $\lambda = 1.505$		
MSE	Between original and synthesized image	90.3907
SNR		18.2440
PSNR		28.6759

Table-4

Size of image M=256×256	Wavelet Transform (sym6)			
Threshold value	λ_1	λ_2	λ_3	Average threshold
σ =0.2463	0.540	1.677	2.054	1.423

Table-5

Wavelet Packet Transform(sym6) at $\lambda = 1.423$		
MSE	Between original and synthesized image	90.8933
SNR		18.2161
PSNR		28.6036

Table-6

Size of image M=256×256	Wavelet Transform (sym7)			
Threshold value	λ_1	λ_2	λ_2	Average threshold
σ =0.2371	0.520	1.615	1.977	1.370

Table-7

Wavelet Packet Transform(sym7) at $\lambda = 1.370$		
MSE	Between original and synthesized image	91.1485
SNR		18.2047
PSNR		28.5679

Table-8

Size of image M=256×256	Wavelet transform (db4)			
Threshold value	λ_1	λ_2	λ_3	Average threshold
σ=0.2604	0.571	1.733	2.172	1.505

Table-9

Wavelet Packet Transform(db4) at $\lambda = 1.505$		
MSE	Between original and synthesized image	90.6363
SNR		18.2357
PSNR		28.5098

Table-10

Size of image M=256×256	Wavelet transform (coif2)			
Threshold value	λ_1	λ_2	λ_3	Average threshold
σ =0.2463	0.540	1.677	2.054	1.423

Table-11

Wavelet Packet Transform(coif2) at $\lambda = 1.423$		
MSE	Between original and synthesized image	90.8420
SNR		18.1911
PSNR		28.5888

Table-12

Size of image M=256×256	Wavelet transform (coif5)			
Threshold value	λ_1	λ_2	λ_3	Average threshold
σ=0.1869	0.410	1.273	1.559	1.081

Table-13

Wavelet Packet Transform(coif5) at $\lambda = 1.081$		
MSE	Between original and synthesized image	93.2874
SNR		18.0836
PSNR		28.4653

Table-14

Size of image M=256×256	Wavelet transform (bior2.4)			
Threshold value	λ_1	λ_2	λ_3	Average threshold
σ=0.2561	0.562	1.744	2.136	1.480

Table-15

Wavelet Packet Transform(bior2.4) $\lambda = 1.480$		
MSE	Between original and synthesized image	89.6341
SNR		18.2308
PSNR		28.6617

Table-16

Size of image M=256×256	Wavelet transform (bior2.6)			
Threshold value	λ_1	λ_2	λ_3	Average threshold
σ=0.2371	0.520	1.615	1.977	1.370

Table-17

Wavelet Packet Transform(bior2.6) $\lambda = 1.370$		
MSE	Between original and synthesized image	90.8283
SNR		18.2237
PSNR		28.6197

Table-18

Size of image M=256×256	Wavelet transform (bior2.8)			
Threshold value	λ_1	λ_2	λ_3	Average threshold
σ=0.2250	0.493	1.532	1.877	1.301

Table-19

Wavelet Packet Transform(bior2.8) $\lambda = 1.301$		
MSE	Between original and synthesized image	90.0804
SNR		18.1869
PSNR		28.6725

Table-20

Size of image M=256×256	Wavelet transform (bior3.3)			
Threshold value	λ_1	λ_2	λ_3	Average threshold
σ=0.2604	0.571	1.773	2.172	1.505

Table-21

Wavelet Packet Transform(bior3.3) $\lambda = 1.505$		
MSE	Between original and synthesized image	88.0702
SNR		18.23801
PSNR		28.8607

Table-22

MSE,SNR and PSNR values between Original and Synthesised Images at Average Threshold (λ) Threshold WPT						
S.No.	Wavelet		Threshold value (λ)	MSE	SNR	PSNR
01	haar	Wavelet Packet Transform	1.315	90.9403	18.1690	28.5895
02	sym4		1.505	90.3907	18.2440	28.6759
03	sym6		1.423	90.8433	18.2161	28.6036
04	sym8		1.370	91.1485	18.2047	28.5679
05	db4		1.504	90.6363	18.2357	28.5098
06	coif2		1.423	90.8420	18.1911	28.5888
07	coif5		1.081	93.2874	18.0836	28.4653
08	bior2.4		1.480	89.6341	18.2308	28.6617
09	bior2.6		1.370	90.8283	18.2237	28.6197
10	bior2.8		1.301	90.0804	18.1869	28.6725
11	bior3.3		1.506	88.0702	18.2780	28.8607

Table-23

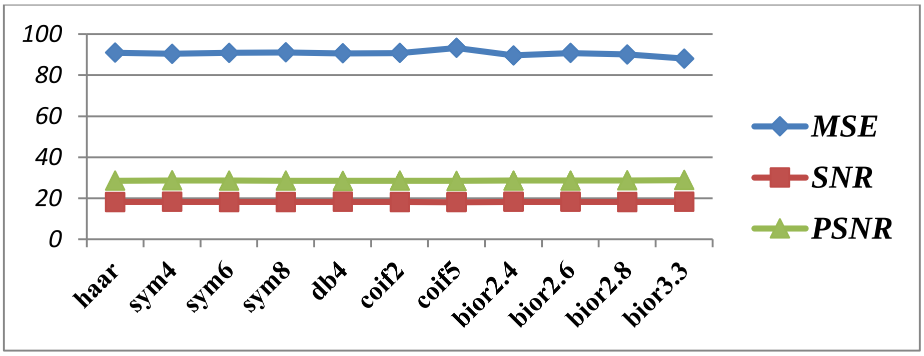

Fig.7.Graphical representation of MSE, SNR and PSNR with various wavelets

In above tables usually we observe bigger the threshold lesser the SNR and PSNR and lesser the threshold bigger the SNR and PSNR. As big as the SNR values shows good results in terms digital image de-noising. Therefore, from above Proposed Method, it shows better results in terms of de-noising of noisy image. The wavelet bior2.4, followed by db4, coif5, bior2.8, coif2, sym4, sym7, haar, sym6 bior2.8 and bior3.3 sequentially performs better in terms of SNR values. While on other side bior2.8, followed by bior2.4, sym6, bior2.6, sym7, coif2, db4, coif5, bior3.3, Sym4, and haar sequentially performs in terms of PSNR values. From above tables we can also observe that de-noising of noisy image is better when we choose standard deviation (σ) for threshold value from synthesized images than detailed coefficients. We also observe that the threshold value we get from average of (12), (13) and (14) is approximately near to the value of (12) i.e. $T=\sigma\sqrt{\log(N)}$ and almost lesser than it.

The haar wavelet performs lesser in terms of digital image de-noising. We cannot say which wavelet performs better every time. Because every wavelet has their own properties. Good performance in terms of signal de-noising through wavelets depends on signal which we consider for operation.

13. Conclusion

In this paper, we apply a digital image of a cameraman of size 256×256 as a test image. The de-noising performance of digital images is taken into consideration by choosing the threshold value provided by the above formula, which are applied in numerous research articles.

The proposed method performs better in terms of digital image denoising. The denoising performance of digital images by proposed technique are visualised by above tale-23 and figure-7.The results of WPT in terms of SNR and PSNR are better, when applying same wavelets and calculated threshold (Soft-thresholding) at level 3.

Finally in this paper we analyse wavelets sym4, db4, bior3.3, bior2.4,bior2.6, sym6, sym8, coif2, bior2.8, haar and coif5 performs better in terms of SNR values for digital image de-nosing on the other hand wavelets bior3.3, sym4, bior2.8, bior2.4, bior2.6, sym6, haar coif2, Sym8,db4 and coif5 one after another performs good in terms of PSNR for same purpose. The whole process for digital image denoising is performed through MATLAB (2020a) software.

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