Union and one point union of some strongly multiplicative graphs

M. Simaringa[1*] and P. Sudar oli [2]

[1, 2] PG and Research department of Mathematics, Thiru Kolanjiappar Government Arts College, Vriddhachalam-606001, Tamil Nadu, India.

Abstract A graph G with *p* vertices is said to be *strongly multiplicative* if the vertices of G can be labeled with p consecutive positive integers 1, 2 , . . . , *p* such that label induced on the edges by the product of labels of end vertices are all distinct. In this paper, we analyze some strongly multiplicative labeling of some one point union of h copies of triangular snake graph, wheel graph , flower graph, helm graph and the union of *h* copies of barycentric graph , crown graph.

Keywords : Strongly multiplicative graph, Union , one point union.

AMS Subject Classification

05C78.

1. Introduction

In this investigating article, all the graphs assumed as finite, connected, undirected, simple graph. For basic terminology and notations , we refer to Balakrishnan and Ranganathan [2]. Multiplicative labeling was introduced by Beineke and Hegde [3] and evidenced that every cycle C_n, wheel W_n , the complete graphs, spanning subgraph , an induced subgraph are strongly multiplicative and defined as, a graph G with p vertices is said to be ***strongly multiplicative,*** if the vertices of G can be labeled with p consecutive positive integers 1, 2 , . . . , *p* such that label induced on the edges by the product of labels of end vertices are all distinct. In 2017 , K.K. Kanani et. al., [5] have explained some standard graphs and proved that the flower graph, the helm graph, and some graphs are strongly multiplicative. In 2017, K.K. Kanani et. al.,[6] have discussed some path related graphs and proved that the total graph, the splitting graph , the shadow graph, the triangular snake TS_n are strongly multiplicative. In 2018, K.K. Kanani et. al., [5] explained about barycentric graph. In 2020, Mehul Chaurasiya et. al., [8] explained some union of graphs. For comprehensive survey on labeling we refer to Gallian [4] .

If the vertices are assigned values subject to certain conditions, then it is known as ***graph*** labeling. A *triangular Snake* TS_n is resulted from a path P_n by altering each edge path by the triangle C_3 . The *helm* (H_n, $n \geq 3$) graph is drawn from a wheel graph W_n in which a pendant edge is pairing with each rim node. The *flower graph* (Fl_n) is resulted from the graph helm H_n in which every pendant vertices are paired with the central node of the helm. The *barycentric subdivision* of graph *G* is obtained by subdividing every edge of G. A *crown graph* on 2n nodes is an undirected graph with two sets of vertices $\{u_1, u_2, . . . , u_n\}$ and $\{v_1, v_2, . . . , v_n\}$ and with an edge from u_i to v_j whenever

$i \neq j$. A graph G in which a node is distinguished from other vertices is called a *rooted graph* and the vertex is called the *root of G*. The graph G(n) is obtained by identifying the roots of n copies of G is called the *one-point union* of n copies of G. The *union* of $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ so that $V_1 \cap V_2 = Ø$ is mentioned by $G_1 \cup G_2$ which contains all the vertices of G_1 or G_2 (or in both) and edge set contains all edges which are in G_1 or in G_2 (or in both).

2. MAIN RESULTS

In this section, we investigate the one point union of k copies of triangular snake , flower graph, helm graph and the union of k copies of barycentric graph , crown graph.

THEOREM 2.1. The one point union of k copies of triangular snake is strongly multiplicative graph.

Proof. Let TS_n^k be the one point union of triangular snake TS_n , $n \geq 3$. Then the vertex set

$V(TS_n^k) = \{ u_1, u_i^k : 2 \leq i \leq n\} \cup \{ v_i^k : 1 \leq i \leq n -1 \}$ and edge set

$E(TS_n^k) = \{ u_1u_2^k , u_i^k u_{i+1}^k : 2 \leq i \leq n -1 \} \cup \{ u_1 v_1^k, u_i^k v_i^k, v_i^k u_{i+1}^k : 1 \leq i \leq n-1\}$ and so

$| V(TS_n^k)| = k(2n-2) +1$ and $| E(TS_n^k)| = k(3n-3)$.

Define a labeling g: $V(TS_n^k) \rightarrow \{1, 2, \dots , k(2n-2)+1 \}$ then we label the vertices sequentially in the clockwise direction by

$g(u_1) = 1$,

$g(u_i^k) = (2n-2)(k -1) + 2i-1$ for $2 \leq i \leq n$,

$g(v_i^k) = (2n-2)(k-1) + 2i$ for $1 \le i \le n-1$,

Thus the labeling structure are all distinct. Therefore the one point union of k copies of Triangular snake is strongly multiplicative graph.

Illustration 2.2. The one point union of 4 copies of Triangular snake is given below.

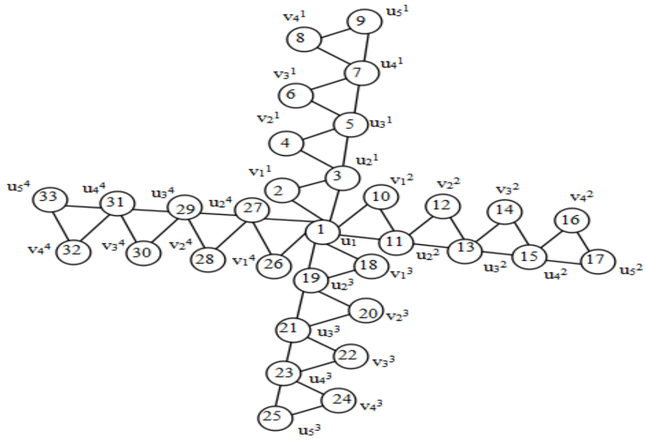

Figure 2.1. Strongly multiplicative labeling of TS_5^4 .

THEOREM 2.3. The one point union of k copies of flower graph is strongly multiplicative.

Proof. Let Fl_n^k be the one point union of k copies of flower graph.Then vertex set

$V(Fl_n^k) = \{ u_o, u_i^k, v_i^k, 1 \le i \le n \}$ and the edge set

$E(Fl_n^k) = \{ u_o u_i^k, u_o v_i^k, u_i^k v_i^k : 1 \le i \le n \} U \{ u_i^k u_{i+1}^k : 1 \le i \le n-1 \} U \{u_1^k u_n^k\}$

And so $|V(Fl_n^k)| = 2kn+1$ and $|E(Fl_n^k)| = 4kn$

Define $g : V(Fl_n^k) \to \{1,2, \dots , 2kn+1\}$ and mark the vertices sequentially in the clockwise direction by regarding the two cases.

Case 1. $n = \dfrac{m(m+2)+1}{2}$, where m is odd and $m \neq 1$, Consider the following two subcases.

Subcase 1a. 2kn+1 is a prime number.

$g(u_0) = p$,

$g(u_i^k) = (k-1)2n + 2i-1$ for $1 \le i \le n$,

$g(v_i^k) = (k-1)2n + 2i$ for $1 \le i \le n$.

If the labeling value of $u_i u_j = 2n-1$ ($i < j$), then interchange the labeling values of u_j and u_{j+1}. Further by proceeding in this same manner if any of the labeling is not distinct.

Subcase 1b. 2kn+1 is not a prime number.

Choose a biggest prime between 2n(k-1)+1 and 2kn.

Let us set the vertex labeling upto k-1 copies by

$g(u_0) = p$,

$g(u_i^{k-1}) = (k-2)2n+2i-1$ for $1 \le i \le n$,

$g(v_i^{k-1}) = (k-2)2n+2i$ for $1 \le i \le n$.

If the labeling value is $u_i u_j = 2n-1$ ($i < j$), then interchange the labeling values of u_j and u_{j+1} .

Further by proceeding in this same manner if any of the labeling is not distinct.

Now place the labeling for the k^{th} copy by

$g(u_0) = p$,

If $p > 2n(k-1) + 2t-1$ for $1 \le t \le i$, then

$g(u_i^k) = 2n(k-1) + 2i-1$ for $1 \le i \le n$,

$g(v_i^k) = (k-1)2n + 2i$ for $1 \le i \le n$.

If $p \le 2n(k-1) + 2t-1$ for $i+1 \le t \le n+1$, then

$g(u_i^k) = 2n(k-1) + 2i+1$ for $1 \le i \le n$.

$g(v_i^k) = (k-1)2n + 2i$ for $1 \le i \le n$.

Illustration. 2.4. The following figure 2.2 shows that the one point union of flower graph . Here $2kn+1 = 33$ (not a prime number).

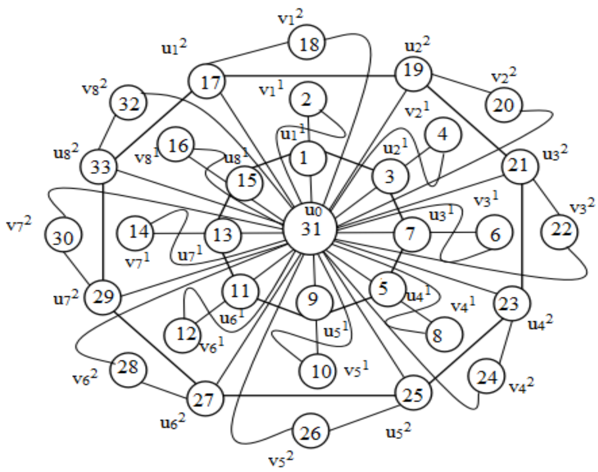

Figure 2.2 . Strongly multiplicative labeling of Fl_8^2.

Case 2. $n \neq \dfrac{m(m+2)+1}{2}$, where m is odd and $m \neq 1$. Consider the following two subcases

Subcase 2a. 2kn+1 is a prime number.

$g(u_0) = p$,

$g(u_i^k) = 2n(k-1) + 2i-1$ for $1 \le i \le n$,

$g(v_i^k) = 2n(k-1) +2i$ for $1 \le i \le n$.

Illustration2.5. The following figure 2.3 shows that the one point union of 2 copies of flower graph satisfies strongly multiplicative. Here $2kn+1= 17$(a prime number).

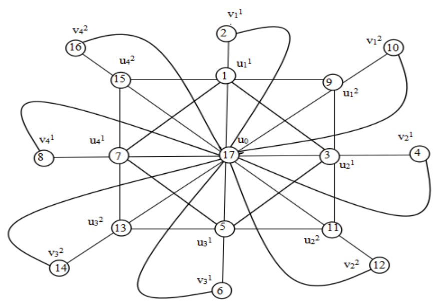

Figure 2.3. Strongly multiplicative labeling of Fl_4^2.

Subcase 2b. 2kn+1 is not a prime number.

Choose a largest prime between $2n(k-1)+1$ and $2kn$.

Let us set the vertex labeling upto k-1 copies by

$g(u_0) = p$,

$g(u_i^{k-1}) = 2n(k-2) + 2i-1$ for $1 \le i \le n$,

$g(v_i^{k-1}) = 2n(k-2) + 2i$ for $1 \le i \le n$.

Now place the labeling for the k th copy by

$g(u_0) = p$,

If $p > 2n(k-1) + 2t-1$ for $1 \le t \le i$, then

$g (u_i^k) = 2n(k -1) + 2i-1$ for $1 \le i \le n$,

$g (v_i^k) = (k-1)2n +2i$ for $1 \le i \le n$.

If $p \le 2n(k-1) + 2t-1$ for $i +1 \le t \le n +1$, then

$g (u_i^k) = 2n(k -1) + 2i +1$ for $1 \le i \le n$.

$g (v_i^k) = (k-1)2n +2i$ for $1 \le i \le n$.

Illustration 2.6. The following figure 2.4 shows that the one point union of 2 copies of flower graph. Here 2kn+1= 21 (not a prime number).

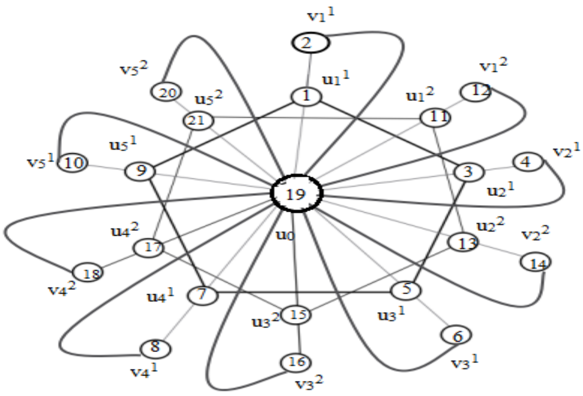

Figure 2.4 Strongly multiplicative labeling of Fl_5^2.

So the labeling are all varied from each other. This proves that the one point union of k copies of flower graph admits strongly multiplicative ■

Corollary 2.2.1 The one point union of k copies of helm graph is strongly multiplicative. ■

THEOREM 2.7. The one point union of k copies wheel graph is strongly multiplicative.

Proof. Let W_n^k be the one point union of k copies of wheel graph. Then vertex set

$V(W_n^k) =\{ u_o , u_i^k :1 \le i \le n \}$ and edge set

$E(W_n^k) =\{ u_o u_i^k : 1 \le i \le n \} U \{ u_i^k u_{i+1}^k : 1 \le i \le n-1\} U \{ u_1^k u_n^k \}$ Also ,

$|V(W_n^k) | = kn +1$ and $|E(W_n^k) | = 2kn$

Define a labeling $g : V(W_n^k) \to \{ 1 , 2 , . . . , kn +1\}$ and mark the vertices continuously in the clockwise direction by regard the succeeding two cases.

Case 1. For $n \ne m (m+1)$, where $m = 2,3,\ldots$ and $n \ne 3$. Consider the following two subcases.

Subcase 1a. kn+1 is a prime number.

$g(u_0) = p$,

$g(u_i^k) = n(k-1) + i$ for $1 \le i \le n$.

Illustration 2.8. The following figure 2.5 illustrates that the one point union of 3 copies of wheel graph . Here 2kn+1= 13(a prime number).

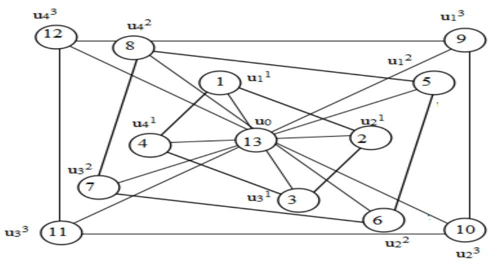

Figure 2.5 Strongly multiplicative labeling of W_4^3.

Subcase 1b. kn +1 is not a prime number.

Choose a largest prime between n(k-1)+1 and kn.

Let us set the vertex labeling upto k-1 copies by

$g(u_i^{k-1}) = (k-2)\,n + i$ for $1 \leq i \leq n$.

Now place the labeling for the k^{th} copy by

$g(u_0) = p$,

If $p > n(k-1) + t$ for $1 \leq t \leq i$, then

$g(u_i^k) = n(k-1) + i$ for $1 \leq i \leq n$,

If $p \leq n(k-1) + 2t-1$ for $i+1 \leq t \leq n+1$, then

$g(u_i^k) = n(k-1) + i + 1$ for $1 \leq i \leq n$.

Illustration 2.9. The following figure 2.6 gives the one point union of 3 copies of wheel graph . Here kn+1= 28 (not a prime number).

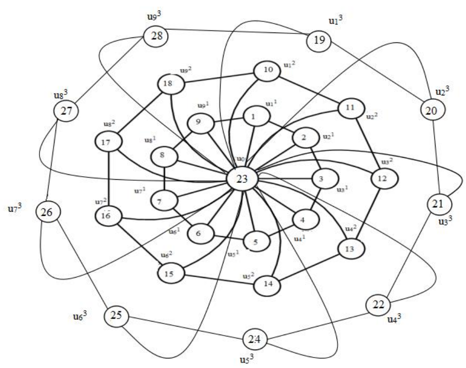

Figure 2.6 Strongly multiplicative labeling of W_9^3.

Case 2. For n = 3,

Proceeding in the same manner of subcases 1 and 2 as in case 1 . But the exception is in the k^{th} copy, If u_1^k is p+1 then label the values as

$g(u_0) = p$,

$g(u_i^k) = n(k-1) + i + 1$ for $1 \leq i \leq n$.

Illustration 2.10. The following figure 2.7 gives the one point union of 3 copies of wheel graph . Here kn+1= 10 (not a prime number).

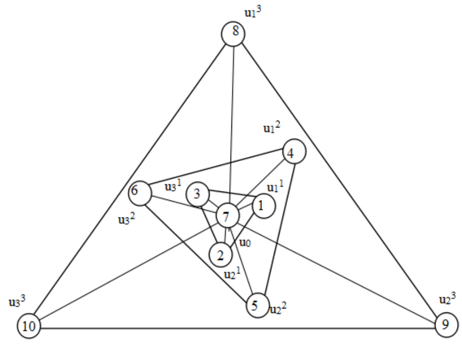

Figure 2.7 Strongly multiplicative labeling of W_3^3.

Case 3. For n = m (m+1) , where m = 2,3, …. Consider the following two subcases.

Subcase 2a. kn+1 is a prime number.

$g(u_0) = p$,

$g(u_i^k) = n(k-1) + i$ for $1 \leq i \leq n$.

If the labeling value of $u_i u_j = n$ ($i < j$) , then interchange the labeling values of u_j and u_{j+1} . Further by proceeding in this same manner if any of the labeling is not distinct.

Illustration 2.11. The following figure 2.8 gives the one point union of 3 copies of wheel graph . Here kn+1= 19 (a prime number).

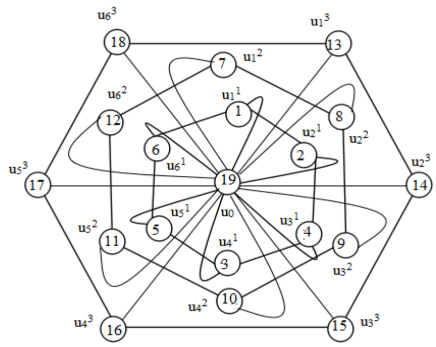

Figure 2.8. Strongly multiplicative labeling of W_6^3.

Subcase 2b. kn+1 is not a prime number

Choose a highest prime between n(k-1)+1 and kn.

Let us set the labeling upto k-1 copies by

$g(u_i^{k-1}) = (k-2)\, n + i$ for $1 \leq i \leq n$.

Now place the labeling for the k th copy by

$g(u_0) = p$,

If $p > n(k-1) + t$ for $1 \leq t \leq i$, then

$g(u_i^k) = n(k-1) + i$ for $1 \leq i \leq n$,

If $p \leq n(k-1) + 2t-1$ for $i+1 \leq t \leq n+1$, then

$g(u_i^k) = n(k-1) + i + 1$ for $1 \leq i \leq n$.

Therefore all the labeling structure are different from one another. Thus the one point union of k copies of wheel graph is strongly multiplicative. ■

Theorem 2.12. The union of k copies of barycentric subdivision of cycle C_n , $n \geq 3$ is strongly multiplicative.

Proof. Let $H = C_n(C_n) \cup C_n(C_n) \cup , \ldots , \cup C_n(C_n)$ be the union of k copies of barycentric subdivision of cycle $C_n(C_n)$. Then vertex set $V(H) = \{u_i^k, v_i^k, 1 \leq i \leq n\}$ and edge set

$E(k) = \{ u_1 u_n , u_i^k u_{i+1}^k : 1 \le i \le n\} \cup \{ u_i v_i , u_1 v_n , v_i^k u_{i+1}^k : 1 \le i \le n\}$

And then $|V(H)| = 2kn$ and $|E(H)| = 3kn$.

Define a labeling as $g : V(H) \rightarrow \{ 1,2, \dots , 2kn\}$ and mark the vertices continuously in the clockwise direction by regard the pursuing two cases.

Case 1. $n \neq \dfrac{m(m+2)}{2}$, where $m = 2, 3, 4, \dots$

$g(u_i^k) = 2n(k-1) + 2i-1$ for $1 \le i \le n$,

$g(v_i^k) = 2n(k-1) + 2i$ for $1 \le i \le n$.

Illustration 2.13. The following figure 2.9 gives the union of 4 copies of barycentric graph $C_5 (C_5)$.

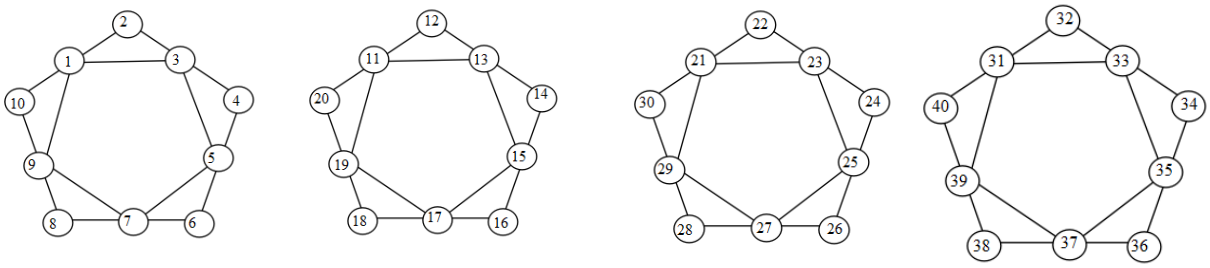

Figure 2.9 Strongly multiplicative labeling of union of 4 copies of $C_5 (C_5)$.

Case 2. $n = \dfrac{m(m+1)}{2}$, where $m = 2, 3, 4, \dots$

By giving same labeling as in case 1, suppose if the labeling are either $u_i v_j = 2n$ ($i \le j$) ,

or $v_i u_j = 2n$ ($i < j$) , then interchange the labeling values of v_j and u_{i+1} or u_j and v_{i+1} respectively. Further by proceeding in this same manner if any of the labeling is not distinct.

Illustration2.14. The following figure 2.10 shows that the union of 3 copies of barycentric graph $C_3 (C_3)$.

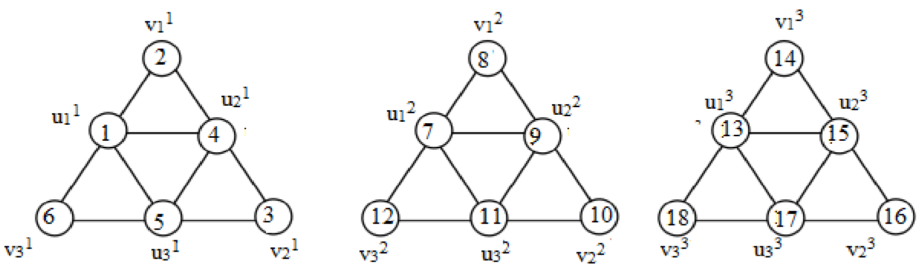

Figure 2.10 Strongly multiplicative labeling of union of 2 copies of barycentric graph $C_3 (C_3)$.

Case 3. $n = \dfrac{m(m+2)+1}{2}$, where m is odd and $m \neq 1$.

By giving same labeling as in case 1 suppose , If the labeling are either $u_i v_j = 2n-1$ ($i \le j$) , or $v_i u_j = 2n-1$ ($i < j$) , then interchange the labeling values of v_j and u_{i+1} or u_j and v_{i+1} respectively. Further by proceeding in this same manner if any of the labeling is not distinct. The vertex labeling designs are all varied . Thence the union of k copies of barycentric graph is strongly multiplicative.

Illustration2.15. The following figure 2.11 shows that the union of 3 copies of barycentric graph $C_8 (C_8)$.

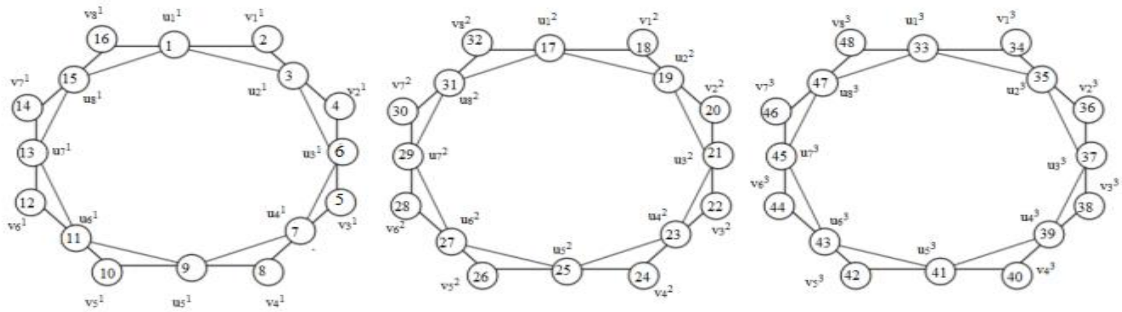

Figure 2.11. Strongly multiplicative labeling of union of 3 copies of barycentric graph $C_8(C_8)$.

Theorem 2.16. The Union of k copies of crown graph C_n^+ is strongly multiplicative graph.

Proof. Let $H = C_n^+ \cup C_n^+ \cup C_n^+ \ldots \cup C_n^+$ be the union of k copies of cnown C_n^+ graph. Then vertex set of $V(H) = \{ u_i^k, v_i^k ; 1 \le i \le n \}$ and the edge set

$E(H) = \{ u_i^k v_i^k ; 1 \le i \le n \} \cup \{ u_i^k u_{i+1}^k ; 1 \le i \le n-1 \} \cup \{u_1^k u_n^k\}$.

So that $| V(H) | = 2kn$ and $| E(H) | = 3kn$.

Define a labeling as $g : | V(H) | \rightarrow \{ 1,2,\ldots 2kn\}$ and mark the vertices continuously in the clockwise direction by regard the succeeding two cases.

Case 1. $n = \dfrac{m(m+2)+1}{2}$, where m is odd and $m \neq 1$.

$g(u_i^k) = (k-1)2n + 2i-1$ for $1 \le i \le n$

$g(v_i^k) = (k-1)2n + 2i$ for $1 \le i \le n$.

If the labeling value is $u_i u_j = 2n-1$ ($i < j$), then interchange the labeling values of u_j and u_{j+1} . Further by proceeding in this same manner if any of the labeling is not distinct.

Illustration 2.17. The following figure 2.12 shows that union of 2 copies of crown C_8^+ .

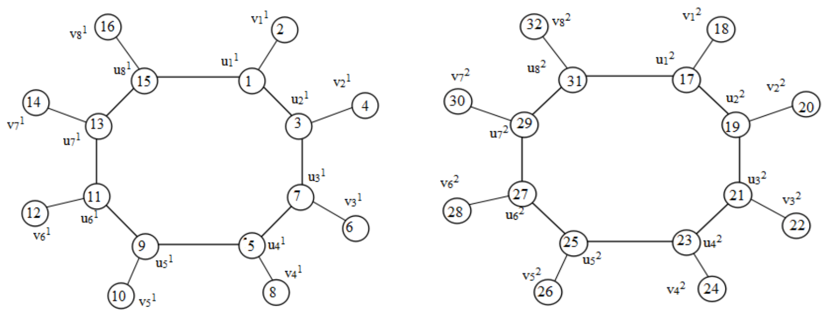

Figure 2.12 Strongly multiplicative labeling of union of 2 copies of crown C_8^+ .

Case 2. $n \neq \dfrac{m(m+2)+1}{2}$, where m is odd and $m \neq 1$.

By giving same labeling as in case 1 the strongly multiplicative condition is satisfied.

Illustration 2.18. The following figure 2.13 shows that the union of 3 copies of crown C_5^+ satisfies strongly multiplicative.

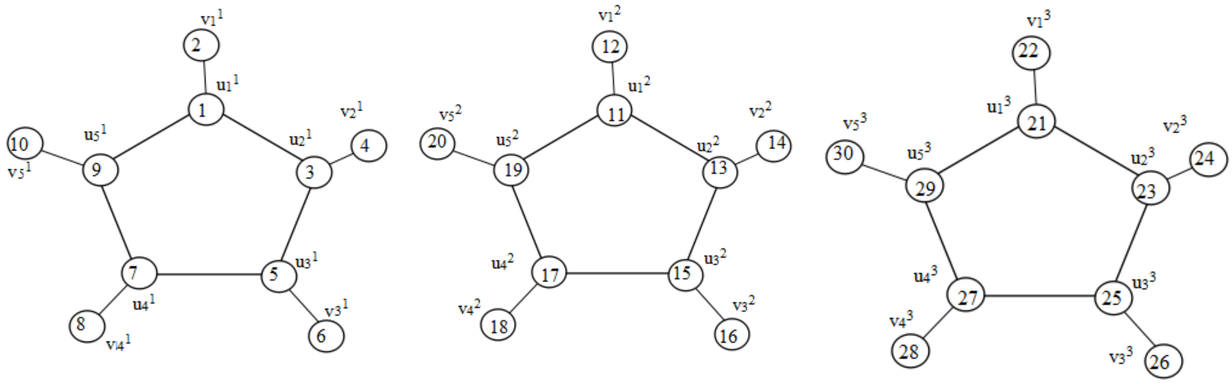

Figure 2.13 Union of 3 copies of crown C_5^+ .

The labeling are all distinct. Thence the union of k copies of crown C_n^+ is strongly multiplicative.■

REFERENCES

[1] Agasthi. P, Parvathi.N. and Thirusangu. K, " *On Some Labelings of Subdivision of Snake Graphs* ", Annals of Pure and Applied Mathematics Vol. 16, page no.1, 245-254, (2018).

[2] Balakrishnan. R and Ranganathan. K, " *A text book of graph theory* ", Springer-Verlag, New York, (2000).

[3] Beineke. L.W. and Hegde. S.M, " *Strongly multiplicative graphs* ", Discuss. Math. Graph Theory, 21, 63-75, (2001).

[4] Gallian,J.A., "*A Dynamic Survey of Graph Labeling*", Electronic Journal of Combinatorics (2010).

[5] Kanani, K,K. and Chhaya. T.M , " *Strongly multiplicative labeling of some standard graphs* ", International Journal of Mathematics and Soft Computing Vol.7, No.1 , 13 - 21, ISSN Print : 2249 –3328, ISSN Online :2319 - 5215 (2017).

[6] Kanani, K,K. and Chhaya. T.M , " *Strongly Multiplicative Labeling of Some Snake Related Graphs* ", International Journal of Mathematics Trends and Technology (IJMTT) - Volume 45 page number 1- May (2017).

[7] Kanani, K,K. and Chhaya. T.M , " *Strongly Multiplicative labeling in the context of various graph operations* " , International Journal of Applied Engineering Research ISSN 0973-4562 Volume 13, Number 20 pp. 14488-14494(2018).

[8] Mehul Chaurasiya, Yash Kandar, Mehul Rupani, Parth Thakar , " *Some new families of minimization of Multipicative graphs"* Journal of Interdiscipinary Cycle Research ISSN N0:0022-1945 Volume XII, number 165, (2020).