

A MATRIX MINIMUM APPROXIMATION METHOD TO OBTAIN INITIAL BASIC FEASIBLE SOLUTION OF TRANSPORTATION PROBLEM

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Abstract

A transportation problem is a special case of linear programming problem which mainly aims to find the minimum transportation cost for supplying commodities from given source points to different demand points. The popular traditional methods to find initial basic feasible solution are NWCM, LCM, VAM. In this paper a new method called Matrix Minimum Approximation method is proposed to find an initial basic feasible solution of transportation problem. The method thus evolved gives an initial basic feasible solution which is very closed to the result obtained from MODI method. Also, it provides better solution in terms of transportation cost than that of the cost obtained under Karagul-Sahin Approximation Method. This proposed method is explained using some examples.

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Keywords: *Transportation Problem, IBFS, NWCM, LCM, MODI Method, Karagul-Sahin Approximation Method.*

I Introduction

A certain class of linear programming problems, known as transportation problems, arise very frequently as practical applications. A transportation problem is one of the earliest and most important application of linear programming problem. It was first studied by F.L. Hitchcock [5] and finally placed in framework of linear programming and solved by simplex method by G. B. Dantzing[4]. The Simplex method is not suitable for the transportation problem especially for large scale transportation problem due to its special structure of model which was studied by Charnes and Cooper[3]. In last few years Abdual Quddoos et.al[2] proposed two different methods for finding the optimal solution. Reena G. Patel et. al[8, 9] and A. Amaravathy et.al [1] developed a very helpful method as it includes less computation and shorter time of period for getting the optimal solution. Besides the conventional methods, a large number of researchers have provided various methods for finding better solutions of transportation problem. Urvashi kumari D. Patel et.al [10] obtained several results related to transportation problem. An alternative method to North West Corner Method was given by Neetu M. Sharma et.al[7] for solving transportation problem which provides a different notion.

Transportation problem is one of the important problem in the field of optimization. In mathematics and economics transportation theory is given to study the optimal transportation cost and allocation of resources. A transportation problem being special case of linear programming problem finds the minimum cost of transportation to send commodities from a set of sources to set of destinations. In this paper, a new method called Matrix Minimum Approximation method is proposed to obtain initial basic feasible solutions to different transportation problems and the outcomes have been compared with the existing methods such as NWCM, LCM, VAM. Initial basic feasible solutions thus obtained are much better than the results obtained under Karagul-Sahin Approximation method in terms of cost and time. In addition, it is very closed to results of MODI method. To explain this new proposed method, various examples are illustrated.

II Preliminaries

2.1 Mathematical formulation of transportation problem

Let there be m origins, i^{th} origin possessing x_{ij} units of a certain product, whereas there be n destinations with destination j^{th} requiring b_j units. Let c_{ij} be the cost of shipping one unit product from i^{th} origin to j^{th} destination. Let x_{ij} be the number of units shipped from i^{th} origin to j^{th} destination. The problem is to determine non-negative values of x_{ij} satisfying the constraints

$$\sum_{j=1}^n x_{ij} = a_i, i = 1, 2, 3, \dots, m \text{ (availability constraints)}$$

$$\sum_{i=1}^m x_{ij} = b_j, j = 1, 2, 3, \dots, n \text{ (requirement constraints)}$$

$$\text{and minimizing the total transportation cost } Z = \sum_{i=1}^m \sum_{j=1}^n x_{ij} c_{ij}$$

Algorithm of the new method

1. Evaluate the r_{ij} (pdm) and r_{ji} (psm) values for matrix A (wcd) and B (wcs).

r_{ij} : Proportional demand matrix (pdm)

$$r_{ij} = \frac{d_j}{s_i}, i = 1, 2, 3, \dots, m \text{ and } j = 1, 2, 3, \dots, n$$

r_{ji} : Proportional supply matrix (psm)

$$r_{ji} = \frac{s_i}{d_j}, j = 1, 2, 3, \dots, n \text{ and } i = 1, 2, 3, \dots, m$$

2. Evaluate the weighted transportation cost matrix by multiplying the rates and the cost values and form A (wcd) and B (wcs) matrices.

A : Weighted transportation cost matrix by demand (wcd)

B : Weighted transportation cost matrix by supply (wcs)

3. Always select the first row in matrix A (wcd) and in the matrix B (wcs). Then look for the least weighted cost in the first row and first column. Subtract the least weighted cost in the first column. Repeat the process for m number of columns without changing the first row. When all columns are used make new wcd or wcs taking into account the demand and supply constraints.
4. After that, apply LCM in new wcd and new wcs.
5. If all demands are met, finish the algorithm. Otherwise, go back to Step 3.
6. Now transfer these allocations to the original transportation table.
7. Finally, calculate the total transportation cost of the transportation table. This calculation is the sum of the product of cost and corresponding allocated value of the transportation table.

III Numerical Illustrations

Example 1.

Table 1: Consider the Transportation problem presented in the following table

Source	D_1	D_2	D_3	D_4	D_5	Supply (S)
S_1	73	40	9	79	20	8
S_2	62	93	96	8	13	7
S_3	96	65	80	50	65	9
S_4	57	58	29	12	87	3
S_5	56	23	87	18	12	5
Demand (D)	6	8	10	4	4	32

Table 2: r_{ij} : Proportional demand matrix (pdm)

Source	D_1	D_2	D_3	D_4	D_5	Supply (S)
S_1	0.75	1.00	1.25	1.25	0.50	8
S_2	0.86	1.14	1.43	0.57	0.57	7
S_3	0.67	0.89	1.11	0.44	0.44	9
S_4	2.00	2.67	3.33	1.33	1.33	3
S_5	1.20	1.60	2.00	0.80	0.80	5
Demand (D)	6	8	10	4	4	

Table 3: r_{ji} : Proportional supply matrix (psm)

Source	D_1	D_2	D_3	D_4	D_5	Supply (S)
S_1	1.33	1.00	0.80	2.00	2.00	8
S_2	1.17	0.88	0.70	1.75	1.75	7
S_3	1.50	1.13	0.90	2.25	2.25	9
S_4	0.50	0.38	0.30	0.75	0.75	3
S_5	0.83	0.63	0.50	1.25	1.25	5
Demand (D)	6	8	10	4	4	

Table 4: Calculate the weighted transportation cost matrix by multiplying the corresponding rates and the cost values using Table:1 and Table:2 respectively to form the matrix A (wcd)

Source	D_1	D_2	D_3	D_4	D_5	Supply (S)
S_1	54.75	40.00	11.25	39.50	10.00	8
S_2	53.14	106.29	137.14	4.57	7.43	7
S_3	64.00	57.78	88.89	22.22	28.89	9
S_4	114.00	154.67	96.67	16.00	116.00	3
S_5	67.20	36.80	174.00	14.40	9.60	5
Demand (D)	6	8	10	4	4	

Table 5: Calculate the weighted transportation cost matrix by multiplying the corresponding rates and the cost values using Table:1 and Table:3 respectively to form the matrix B (wcs)

Source	D_1	D_2	D_3	D_4	D_5	Supply (S)
S_1	97.33	40.00	7.20	158.00	40.00	8
S_2	72.33	81.38	67.20	14.00	22.75	7
S_3	144.00	73.13	72.00	112.50	146.25	9
S_4	28.50	21.75	8.70	9.00	65.25	3
S_5	46.67	14.38	43.50	22.50	15.00	5
Demand (D)	6	8	10	4	4	

Table 6: Consider the matrix A (wcd) using Table : 4

Source	D_1	D_2	D_3	D_4	D_5	Supply (S)
S_1	54.75	40.00	11.25	39.50	10.00	8
S_2	53.14	106.29	137.14	4.57	7.43	7
S_3	64.00	57.78	88.89	22.22	28.89	9
S_4	114.00	154.67	96.67	16.00	116.00	3
S_5	67.20	36.80	174.00	14.40	9.60	5
Demand (D)	6	8	10	4	4	

Table 7: The least weighted cost in first row and first column is 10.00. Subtract it from each entry of the first column

Source	D_1	D_2	D_3	D_4	D_5	Supply (S)
S_1	54.75	40.00	11.25	39.50	10.00	8
S_2	53.14	106.29	137.14	4.57	7.43	7
S_3	64.00	57.78	88.89	22.22	28.89	9
S_4	114.00	154.67	96.67	16.00	116.00	3
S_5	67.20	36.80	174.00	14.40	9.60	5
Demand (D)	6	8	10	4	4	

Source	D_1	D_2	D_3	D_4	D_5	Supply (S)
S_1	44.75	40.00	11.25	39.50	10.00	8
S_2	43.14	106.29	137.14	4.57	7.43	7
S_3	54.00	57.78	88.89	22.22	28.89	9
S_4	104.00	154.67	96.67	16.00	116.00	3
S_5	57.20	36.80	174.00	14.40	9.60	5
Demand (D)	6	8	10	4	4	

Table 8: The least weighted cost in first row and second column is 10.00. Subtract it from each entry of the second column

Source	D_1	D_2	D_3	D_4	D_5	Supply (S)
S_1	54.75	40.00	11.25	39.50	10.00	8
S_2	53.14	106.29	137.14	4.57	7.43	7
S_3	64.00	57.78	88.89	22.22	28.89	9
S_4	114.00	154.67	96.67	16.00	116.00	3
S_5	67.20	36.80	174.00	14.40	9.60	5
Demand (D)	6	8	10	4	4	

Source	D_1	D_2	D_3	D_4	D_5	Supply (S)
S_1	44.75	30.00	11.25	39.50	10.00	8
S_2	43.14	96.29	137.14	4.57	7.43	7
S_3	54.00	47.78	88.89	22.22	28.89	9
S_4	104.00	144.67	96.67	16.00	116.00	3
S_5	57.20	26.80	174.00	14.40	9.60	5
Demand (D)	6	8	10	4	4	

Table 9: The least weighted cost in first row and third column is 10.00. Subtract it from each entry of the third column

Source	D_1	D_2	D_3	D_4	D_5	Supply (S)
S_1	54.75	40.00	11.25	39.50	10.00	8
S_2	53.14	106.29	137.14	4.57	7.43	7
S_3	64.00	57.78	88.89	22.22	28.89	9
S_4	114.00	154.67	96.67	16.00	116.00	3
S_5	67.20	36.80	174.00	14.40	9.60	5
Demand (D)	6	8	10	4	4	

Source	D_1	D_2	D_3	D_4	D_5	Supply (S)
S_1	44.75	30.00	1.25	39.50	10.00	8
S_2	43.14	96.29	127.14	4.57	7.43	7
S_3	54.00	47.78	78.89	22.22	28.89	9
S_4	104.00	144.67	86.67	16.00	116.00	3
S_5	57.20	26.80	164.00	14.40	9.60	5
Demand (D)	6	8	10	4	4	

Table 10: The least weighted cost in first row and fourth column is 4.57. Subtract it from each entry of the fourth column

Source	D_1	D_2	D_3	D_4	D_5	Supply (S)
S_1	54.75	40.00	11.25	39.50	10.00	8
S_2	53.14	106.29	137.14	4.57	7.43	7
S_3	64.00	57.78	88.89	22.22	28.89	9
S_4	114.00	154.67	96.67	16.00	116.00	3
S_5	67.20	36.80	174.00	14.40	9.60	5
Demand (D)	6	8	10	4	4	

Source	D_1	D_2	D_3	D_4	D_5	Supply (S)
S_1	44.75	30.00	1.25	34.93	10.00	8
S_2	43.14	96.29	127.14	0	7.43	7
S_3	54.00	47.78	78.89	17.65	28.89	9
S_4	104.00	144.67	86.67	11.43	116.00	3
S_5	57.20	26.80	164.00	9.83	9.60	5
Demand (D)	6	8	10	4	4	

Table 11: The least weighted cost in first row and fifth column is 7.43. Subtract it from each entry of the fifth column

Source	D_1	D_2	D_3	D_4	D_5	Supply (S)
S_1	54.75	40.00	11.25	39.50	10.00	8
S_2	53.14	106.29	137.14	4.57	7.43	7
S_3	64.00	57.78	88.89	22.22	28.89	9
S_4	114.00	154.67	96.67	16.00	116.00	3
S_5	67.20	36.80	174.00	14.40	9.60	5
Demand (D)	6	8	10	4	4	

Source	D_1	D_2	D_3	D_4	D_5	Supply (S)
S_1	44.75	30.00	1.25	34.93	2.57	8
S_2	43.14	96.29	127.14	0	0	7
S_3	54.00	47.78	78.89	17.65	2.46	9
S_4	104.00	144.67	86.67	11.43	108.57	3
S_5	57.20	26.80	164.00	9.83	2.17	5
Demand (D)	6	8	10	4	4	

Table 12: After that, apply LCM in new wcd

Source	D_1	D_2	D_3	D_4	D_5	Supply (S)
S_1	44.75	30.00	1.25	34.93	2.57	8
S_2	43.14	96.29	127.14	0	0	7
S_3	54.00	47.78	78.89	17.65	2.46	9
S_4	104.00	144.67	86.67	11.43	108.57	3
S_5	57.20	26.80	164.00	9.83	2.17	5
Demand (D)	6	8	10	4	4	

Source	D_1	D_2	D_3	D_4	D_5	Supply (S)
S_1	44.75	30.00	1.25(8)	34.93	2.57	8/0
S_2	43.14	96.29	127.14	0(4)	0(3)	7/3/0
S_3	54.00(5)	47.78(4)	78.89	17.65	2.46	9/5/0
S_4	104.00(1)	144.67	86.67(2)	11.43	108.57	3/1/0
S_5	57.20	26.80(4)	164.00	9.83	2.17(1)	5/4/0
Demand (D)	6/1/0	8/4/0	10/2/0	4/0	4/1/0	

Now transfer these allocations to the original transportation table.

Source	D_1	D_2	D_3	D_4	D_5	Supply (S)
S_1	73	40	9(8)	79	20	8
S_2	62	93	96	8(4)	13(3)	7
S_3	96(5)	65(4)	80	50	65	9
S_4	57(1)	58	29(2)	12	87	3
S_5	56	23(4)	87	18	12(1)	5
Demand (D)	6	8	10	4	4	32

Finally, calculate the total transportation cost corresponding to above transportation table. This calculation includes the sum of the product of cost and respective allocated value in the transportation table.

$$TC = 9 \times 8 + 8 \times 4 + 13 \times 3 + 96 \times 5 + 65 \times 4 + 57 \times 1 + 29 \times 2 + 23 \times 4 + 12 \times 1 = 1102.$$

Table 13: B (wcs) matrices

The least weighted cost in first row and first column is 7.20. Subtract it from each entry of the first column.

Source	D_1	D_2	D_3	D_4	D_5	Supply (S)
S_1	97.33	40.00	7.20	158.00	40.00	8
S_2	72.33	81.38	67.20	14.00	22.75	7
S_3	144.00	73.13	72.00	112.50	146.25	9
S_4	28.50	21.75	8.70	9.00	65.25	3
S_5	46.67	14.38	43.50	22.50	15.00	5
Demand (D)	6	8	10	4	4	

Source	D_1	D_2	D_3	D_4	D_5	Supply (S)
S_1	90.13	40.00	7.20	158.00	40.00	8
S_2	65.13	81.38	67.20	14.00	22.75	7
S_3	136.80	73.13	72.00	112.50	146.25	9
S_4	21.30	21.75	8.70	9.00	65.25	3
S_5	39.47	14.38	43.50	22.50	15.00	5
Demand (D)	6	8	10	4	4	

Table 14: The least weighted cost in first row and second column is 7.20. Subtract it from each entry of the second column

Source	D_1	D_2	D_3	D_4	D_5	Supply (S)
S_1	97.33	40.00	7.20	158.00	40.00	8
S_2	72.33	81.38	67.20	14.00	22.75	7
S_3	144.00	73.13	72.00	112.50	146.25	9
S_4	28.50	21.75	8.70	9.00	65.25	3
S_5	46.67	14.38	43.50	22.50	15.00	5
Demand (D)	6	8	10	4	4	

Source	D_1	D_2	D_3	D_4	D_5	Supply (S)
S_1	90.13	32.80	7.20	158.00	40.00	8
S_2	65.13	74.18	67.20	14.00	22.75	7
S_3	136.8	65.93	72.00	112.50	146.25	9
S_4	21.30	14.55	8.70	9.00	65.25	3
S_5	39.47	7.18	43.50	22.50	15.00	5
Demand (D)	6	8	10	4	4	

Table 15: The least weighted cost in first row and third column is 7.20. Subtract it from each entry of the third column

Source	D_1	D_2	D_3	D_4	D_5	Supply (S)
S_1	97.33	40.00	7.20	158.00	40.00	8
S_2	72.33	81.38	67.20	14.00	22.75	7
S_3	144.00	73.13	72.00	112.50	146.25	9
S_4	28.50	21.75	8.70	9.00	65.25	3
S_5	46.67	14.38	43.50	22.50	15.00	5
Demand (D)	6	8	10	4	4	

Source	D_1	D_2	D_3	D_4	D_5	Supply (S)
S_1	90.13	32.80	0	158.00	40.00	8
S_2	65.13	74.18	60.00	14.00	22.75	7
S_3	136.8	65.93	64.8	112.50	146.25	9
S_4	21.30	14.55	1.5	9.00	65.25	3
S_5	39.47	7.18	36.3	22.50	15.00	5
Demand (D)	6	8	10	4	4	

Table 16: The least weighted cost in first row and fourth column is 7.20. Subtract it from each entry of the fourth column

Source	D_1	D_2	D_3	D_4	D_5	Supply (S)
S_1	97.33	40.00	7.20	158.00	40.00	8
S_2	72.33	81.38	67.20	14.00	22.75	7
S_3	144.00	73.13	72.00	112.50	146.25	9
S_4	28.50	21.75	8.70	9.00	65.25	3
S_5	46.67	14.38	43.50	22.50	15.00	5
Demand (D)	6	8	10	4	4	

Source	D_1	D_2	D_3	D_4	D_5	Supply (S)
S_1	90.13	32.8	0	150.8	40.00	8
S_2	65.13	74.18	60	6.8	22.75	7
S_3	136.8	65.93	64.8	105.3	146.25	9
S_4	21.3	14.55	1.5	1.8	65.25	3
S_5	39.47	7.18	36.3	15.3	15.00	5
Demand (D)	6	8	10	4	4	

Table 17: The least weighted cost in first row and fifth column is 7.20. Subtract it from each entry of the fifth column

Source	D_1	D_2	D_3	D_4	D_5	Supply (S)
S_1	97.33	40.00	7.20	158.00	40.00	8
S_2	72.33	81.38	67.20	14.00	22.75	7
S_3	144.00	73.13	72.00	112.50	146.25	9
S_4	28.50	21.75	8.70	9.00	65.25	3
S_5	46.67	14.38	43.50	22.50	15.00	5
Demand (D)	6	8	10	4	4	

Source	D_1	D_2	D_3	D_4	D_5	Supply (S)
S_1	90.13	32.8	0	150.8	32.8	8
S_2	65.13	74.18	60	6.8	15.55	7
S_3	136.8	65.93	64.8	105.3	139.05	9
S_4	21.3	14.55	1.5	1.8	58.05	3
S_5	39.47	7.18	36.3	15.3	7.8	5
Demand (D)	6	8	10	4	4	

Table 18: After that, apply LCM in new wcs

Source	D_1	D_2	D_3	D_4	D_5	Supply (S)
S_1	90.13	32.8	0	150.8	32.8	8
S_2	65.13	74.18	60	6.8	15.55	7
S_3	136.8	65.93	64.8	105.3	139.05	9
S_4	21.3	14.55	1.5	1.8	58.05	3
S_5	39.47	7.18	36.3	15.3	7.8	5
Demand (D)	6	8	10	4	4	

Source	D_1	D_2	D_3	D_4	D_5	Supply (S)
S_1	90.13	32.8	0(8)	150.8	32.8	8/0
S_2	65.13(2)	74.18	60	6.8(1)	15.55(4)	7/6/2/0
S_3	136.8(4)	65.93(5)	64.8	105.3	139.05	9/4/0
S_4	21.3	14.55	1.5(2)	1.8(3)	58.05	3/0
S_5	39.47	7.18(3)	36.3	15.3	7.8	5/3/0
Demand (D)	6/4/0	8/5/0	10/2/0	4/1/0	4/0	

Now transfer these allocations to the original transportation table.

Source	D_1	D_2	D_3	D_4	D_5	Supply (S)
S_1	73	40	9(8)	79	20	8
S_2	62(2)	93	96	8(1)	13(4)	7
S_3	96(4)	65(5)	80	50	65	9
S_4	57	58	29(2)	12(3)	87	3
S_5	56	23(3)	87	18	12	5
Demand (D)	6	8	10	4	4	32

Finally, calculate the total transportation cost corresponding to above transportation table. This calculation includes the sum of the product of cost and respective allocated value in the transportation table.

$$TC = 9 \times 8 + 62 \times 2 + 8 \times 1 + 13 \times 4 + 96 \times 4 + 65 \times 5 + 29 \times 2 + 12 \times 3 + 23 \times 3 = 1128.$$

Table 19: Solutions obtained from different methods

Solution Methods	Value	Deviation from optimal solution (%)
Optimal	1102	0.00
MMAM	1102	0.00
KSAM	1102	0.00
NWC	1994	44.73
VAM	1104	0.18

IV Results and Discussions

Under this section, to check the efficiency of the new proposed method (Matrix Minima Approximation Method), 11 transportation problems have been taken in account. It consists of balanced as well as unbalanced problems. The optimal values [6] of corresponding transportation problems are considered for suitable comparison. In addition, the outcomes of different transportation problems thus obtained using the proposed method, are compared to commonly known methods such as North West Corner Method (NWC), Vogel’s Approximation Method (VAM) along with Karagul-Sahin approximation method (KSAM). Ultimately, the results obtained that is the transportation costs have been analyzed for deviation from the optimal transportation costs of both balanced and unbalanced transportation problems.

Table 20: Details of the problem

Number	Name	Problem Size	Optimal Solution	Status
1	Pro1	5 × 5	1102	Balanced
2	Pro2	3 × 4	4010	Balanced
3	Pro3	3 × 3	1390	Balanced
4	Pro4	3 × 5	1580	Balanced
5	Pro5	3 × 4	2850	Balanced
6	Pro6	4 × 4	410	Balanced
7	Pro7	3 × 4	1580	Balanced
8	Pro8	3 × 4	640	Unbalanced
9	Pro9	4 × 6	71	Balanced
10	Pro10	3 × 3	530	Balanced
11	Pro11	3 × 3	1669	Unbalanced

Table 21: Solution and deviation values of the method

Name	Optimal	MMAM	KSAM	NWCM	VAM
Pro 1 Sol. (dev %)	1102(0.00)	1102(0.00)	1102(0.00)	1994(80.94)	1104(0.18)
Pro 2 Sol. (dev %)	4010(0.00)	4010(0.00)	4010(0.00)	6580(64.09)	4010(0.00)
Pro 3 Sol. (dev %)	1390(0.00)	1390(0.00)	1390(0.00)	1500(7.91)	1500(7.91)
Pro 4 Sol. (dev %)	1580(0.00)	1580(0.00)	1600(1.26)	1950(23.42)	1690(6.96)
Pro 5 Sol. (dev %)	2850(0.00)	2850(0.00)	2850(0.00)	4400(54.39)	2850(0.00)
Pro 6 Sol. (dev %)	410(0.00)	415(1.22)	415(1.22)	540(31.71)	470(14.63)
Pro 7 Sol. (dev %)	1580(0.00)	1580(0.00)	1600(1.27)	1950(23.42)	1580(0.00)
Pro 8 Sol. (dev %)	640(0.00)	640(0.00)	640(0.00)	670(4.69)	650(1.56)
Pro 9 Sol. (dev %)	71(0.00)	71(0.00)	77(8.45)	109(53.52)	77(8.45)
Pro 10 Sol. (dev %)	530(0.00)	530(0.00)	555(4.72)	560(5.66)	530(0.00)
Pro 11 Sol. (dev %)	1669(0.00)	1670(0.06)	1690(1.26)	1786(7.01)	1705(2.16)
Number of best Sol.	-	9	5	0	4
Mean deviation %	-	0.116	1.65	32.43	3.73

Table 20 and Table 21 show the solutions of different problems obtained by using different methods and their percentage of deviation from the optimal solution respectively.

It is obvious from the table 21 that the proposed method is quite efficient to find the best initial feasible solution for the 9 transportation problems out of 11 problems as compared to methods KSAM (5), VAM (4) and NWCM (0). Additionally, it is noticed that the proposed method produces a result very closed to the optimal solution.

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