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Nano α^{*}_{AS} - Compactness and Nano α^{*}_{AS} - Connectedness in Nano Topological Space

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Abstract: In this paper we introduce and study Nano α^{*}_{AS} - Compactness and Nano α^{*}_{AS} -Connectedness in Nano Topological Space. Also, Nano α^{*}_{AS} –Lindelof space and Nano α^{*}_{AS} –Hausdroff space are explained and discussed with other relative spaces.

Key Words: Nano α^{*}_{AS} – Compactness, Nano α^{*}_{AS} -Connectedness, Nano α^{*}_{AS} –Lindelof space, Nano α^{*}_{AS} –Hausdroff space.

1.INTRODUCTION

Connectedness and disconnectedness in topology are introduced by A.V.Arhangelskiiet al., Compactness, in general is an essential part of the topological spaces with regard to the property of closed and bounded subsets. The idea of compactness and connectedness are beneficial for basic ideas of general topology as well as for advanced branches of mathematics. LellisThivagar. M and Carmel Richard introduced the notion of Nano topology which was defined in terms of approximations and boundary region of a subset of a universe using an equivalence relation on it. In 2013, LellisThivagar. M and Carmel Richard studied a new class of function called nano continuous functions and their characterizations in nano topological spaces also make known about nano-closed sets, nano-interior, nano-closure and weak form of nano open sets namely nano semi-open sets, nano pre-open, nano α-open sets and Nβ-open sets. Nasef et.al. make known about some of nearly open sets in nano topological spaces. Anbarasi Rodrigo and Sahaya Dani gave the idea about $N\alpha^{*}_{AS}$ - closed sets,$N\alpha^{*}_{AS}$ – continuous,Contra $N\alpha^{*}_{AS}$ – continuous,Nano Totally α^{*}_{AS} – continuous,Nano Strongly α^{*}_{AS} – continuous and Nano Perfectly α^{*}_{AS} - continuous. Nano compactness and connectedness was first spoken by Krishna Prakash et.al. Followed by them many authors has derived nano compactness and connectedness for different nano closed sets in nano topological spaces. This paper also derives a different nano compact and nano connectedness in $N\alpha^{*}_{AS}$ - closed sets.

2. PRELIMINARIES

Definition 2.1:[4] A collection $\{C_d: d \in D\}$ of nano-open sets in a nano topological space ((U, τR(X))) is called a nano-open cover of a subset J of U if $J \subset \{C_d: d \in D\}$ holds.

Definition 2.2:[4] A subset J of a nano topological space (U, τR(X)) is said to be nano-compact relative to ((U, τR(X))), if for every collection $\{C_d: d \in D\}$of nano-open subsets of ((U, τR(X))) such that $J \subset \{C_d: d \in D\}$ there exists a finite subset D_0 of D such that $J \subset \{C_d: d \in D_0\}$

Definition 2.3:[4] A subset J of a nano topological space (U, τR(X)) is said to be nano-compact if J is nano-compact as a subspace of ((U, τR(X))).

Definition2.4:[4]A nano topological space (U, τR(X)) is countably nano-compact if every countable nano-open cover of (U, τR(X)) has a finite subcover.

Definition 2.5:[4] A space (U, τR(X)) is said to be nano-Hausdroff if whenever c and d are distinct points of (U, τR(X)), there exist disjoint nano-open sets J and K such that $c \in J$ and $d \in K$.

Definition 2.6:[4] A nano topological space (U, τR(X)) is said to be nano-Lindelof space if every nano-open cover of (U, τR(X)) has a countable subcover.

Definition 2.7:[4] A nano topological space (U, τR(X)) is said to be nano-connected if (U, τR(X)) cannot be expressed as a disjoint union of two non-empty nano-open sets. A subset of (U, τR(X)) is nano-connected as a subspace. A subset is said to be nano disconnected if and only if it is not nano-connected.

Definition 2.8:[1] A subset J of a nano topological space (U, τR(X)) is called nano α*AS (brieflyNα*AS) closed sets if $N\alpha cl(J) \subseteq Nint(V)$ whenever $J \subseteq V$ and V is nano open.

Definition 2.9:[2] A function f: (U, τR(X)) → (V, τR(Y)) is called Nano α*AS-continuous (briefly Nα*AS -continuous) if $f^{-1}(K)$ is Nα*AS-closed in (U, τR(X)) for every Nano closed set K in V, τR(Y)). That is, if the inverse image of every Nano closed set in (V, τR(Y)) is Nα*AS-closed in (U, τR(X)).

3. NANO α^*_{AS} – COMPACTNESS

Definition 3.1: A collection $\{C_d: d \in D\}$ of nano α^*_{AS}-open sets in a nano topological space ((U, τR(X))) is called a nano α^*_{AS}-open cover of a subset J of U if $J \subset \{C_d: d \in D\}$ holds.

Definition 3.2: A subset J of a nano topological space (U, τR(X)) is said to be nano α^*_{AS}-compact relative to ((U, τR(X))), if for every collection $\{C_d: d \in D\}$ of nano α^*_{AS}-open subsets of ((U, τR(X))) such that $J \subset \{C_d: d \in D\}$ there exists a finite subset D_0 of D such that $J \subset \{C_d: d \in D_0\}$.

Definition 3.3: A subset J of a nano topological space (U, τR(X)) is said to be nano α^*_{AS}-compact if J is nano α^*_{AS}-compact as a subspace of ((U, τR(X))).

Theorem 3.4: A nano α^*_{AS}-closed subset of nano α^*_{AS}-compact space (U, τR(X)) is nano α^*_{AS}-compact relative to (U, τR(X)).

Proof. Let C be a nano α^*_{AS}-compact subset of a nano topological space (U, τR(X)). Then C^c is nano α^*_{AS}-open in (U, τR(X)). Let $S = \{C_d: d \in D\}$ be a nano α^*_{AS}-open cover of C by nano α^*_{AS}-open subsets in (U, τR(X)). Then $S^* = S \cup C^c$ is a nano α^*_{AS}-open cover of (U, τR(X)). That is $U = (S\ C_d) \cup C^c$. By hypothesis (U, τR(X)) is also compact and hence $S^*$is reducible to a finite sub cover of (U, τR(X)) say $U = C_{d1} \cup C_{d2} ....\cup C_{dn} \cup C^c$, $C_{dk} \in S^*$. Thus, a nano α^*_{AS}open cover S of C contains a finite sub cover. Hence C is nano α^*_{AS}-compact relative to (U,τR(X)).

Theorem 3.5: A nano α^*_{AS}-continuous image of a nano α^*_{AS}-compact space is a nano α^*_{AS} compact space.

Proof. Consider a surjective mapping f: (U, τR(X)) −→ (V, τR(Y)) which is nano α^*_{AS}-continuous and U is nano α^*_{AS}-compact and Vis a nano topological space. Assume a nano α^*_{AS} open cover $P_i = \{C_d: d \in D\}$ in V. Since f is nano α^*_{AS}-continuous $\{f^{-1} S_d: d \in D\}$ is a nano α^*_{AS}-open cover in U. As U nano α^*_{AS}-compact, by definition $\{f^{-1} S_d: d \in D\}$ has a finite subcover$\{f^{-1}(S_1), f^{-1}(S_2), f^{-1}(S_3), ... f^{-1}(S_n)\}$ of nano α^*_{AS}-open sets for some finite n and $U = \cup f^{-1}(S_j)$, j = 1, 2, 3.... n. Since f is a surjective mapping $\{f^{-1}(S_1), f^{-1}(S_2), f^{-1}(S_3), ... f^{-1}(S_n)\} = \{S_1, S_2, S_3, ...Sn\} \in V$ is finite sub cover of nano α^*_{AS}- open sets in V. Therefore, V is a nano α^*_{AS}-compact space.

Theorem 3.6: A nanoα^*_{AS}-compact space (U, τR(X)) is always a nano compact space, but not the converse.

Proof. In a nano α^*_{AS}-compact space, consider a nano α^*_{AS}- open cover $S = \{ S_d: d \in D \}$ of nano open sets. Then can also be a nano α^*_{AS}-open cover, since every nano open set is a nano α^*_{AS}-open set. By definition of compactness S has a finite subcover $S_i = \{S_i: i = 1, 2, 3, ...n\}$ and so (U, τR(X)) has a finite sub cover of nano open sets, implying U is nano compact.

Converse part Let (U, τR(X)) be an indiscrete nano topological space of infinite sets U. Then U is nano compact. Every singleton set in U is nano α^*_{AS}-compact. But every nano α^*_{AS}-open cover of singleton sets does not have a finite sub cover. Therefore, U is not nano α^*_{AS}-compact.

Theorem 3.7: A nano topological space (U, τR(X)) is nano α^*_{AS} compact if and only if every family of nano α^*_{AS}-closed sets of (U, τR(X)) having finite intersection property has a non-empty intersection.

Proof. Suppose (U, τR(X)) is nano α^*_{AS}-compact. Let $\{C_d: d \in D\}$ be a family of nano α^*_{AS}-closed sets with finite intersection property. Suppose $\cap_{d\in D} C_d = \emptyset$. Then $U - \cap_{d\in D} C_d = U$. This implies $\cup_{d\in D}(U - C_d) = U$. Thus, the cover $\{U - C_d: d \in D\}$ is a nano-open cover of (U, τR(X)). Then the nano α^*_{AS}-open cover $\{U - C_d: d \in D\}$ has a finite sub cover say $\{U - C_d: d =1, 2, ... n\}$. This implies $U + \cup_{d\in D}(U - C_d)$ which implies $U - \cap_{d\in D} C_d$, which implies $U – U[U - \cap_{d\in D} C_d]$ which implies $\emptyset = \cap C_d$: d = 1, 2, ... n. This contradicts the assumption. Hence $\cap_{d\in D} C_d \neq \emptyset$.

Conversely, suppose (U, τR(X)) is not nano α^*_{AS}-compact. Then there exists a nano α^*_{AS}-open cover of (U, τR(X)) say $\{K_d: d\in D\}$ having no finite sub cover. This implies for any finite subfamily $\{K_d: d = 1, 2, ... n\}$ of $\{K_d: d\in D\}$, we have$\cup K_d \neq X$: i = 1, 2, ... n, which implies $U\text{-}\cup_{d\in D} S_d \neq U\ \text{-}U$, which implies $\cap_{d\in D} (U\text{-} K_d) \neq \emptyset$. Then the family $\{K_d: d\in D\}$ of nano α^*_{AS}-closed sets has a finite intersection property. Also, by assumption $\cap_{d\in D} (U\text{-} K_d) \neq \emptyset$which implies $U\text{-}\cup_{d\in D} K_d \neq \emptyset$. so that $\cup_{d\in D} K_d \neq U$. This implies $\{K_d: d\in D\}$is a cover for (U, τR(X)). This contradicts the fact that $\{K_d: d\in D\}$is a cover for (U, τR(X)). Therefore, a nano α^*_{AS}-open cover $\{K_d: d\in D\}$of (U, τR(X)) has a finite sub cover $\{ K_d: d = 1, 2, ... n\}$. Hence (U, τR(X)) is a nano α^*_{AS}-compact.

Definition 3.8: A nano topological space (U, τR(X)) is countably nano α^*_{AS}-compact if every countable nano α^*_{AS}-open cover of (U, τR(X)) has a finite sub cover.

Definition3.9: A space (U, τR(X)) is said to be nano α^*_{AS}-Hausdroff if whenever c and d are distinct points of (U, τR(X)), there exist disjoint nano α^*_{AS}-open sets J and K such that $c \in J$ and $d \in K$.

Theorem 3.10: Let (U, τR(X)) be a space and (V, τR(Y)) be a nano α^*_{AS}-Hausdroff. If f: (U, τR(X)) → (V,τR(Y)) is nano α^*_{AS}-continuous injective, then (U, τR(X)) is nano α^*_{AS}-Hausdroff.

Proof. Let c and d be any two distinct points of (U, τR(X)). Then f(c) and f(d) are distinct points of (V, τR(Y)), because f is injective. Since (V, τR(Y)) is nano α^*_{AS}-Hausdroff, there are disjoint nano α^*_{AS}-open sets J and K in (V, τR(Y)) containing f(c) and f(d) respectively. Since f is nano α^*_{AS}-continuous and $J \cap K = \emptyset$, we have $f^{-1}(J)$ and $f^{-1}(K)$ are disjoint nano-open sets in (U, τR(X)) such that $c \in f^{-1}(J)$ and $d \in f^{-1}(K)$. Hence (U, τR(X)) is nano α^*_{AS}Hausdroff.

Theorem 3.11: If f: (U, τR(X)) → (V, τR(Y)) is nano α^*_{AS}-continuous and bijective and if U is compact and V is Hausdroff, then f is a homeomorphism.

Proof. We have to show that the inverse g of f is nano α^*_{AS}-continuous. For this we show that if K is open in (U, τR(X)) then the pre-image $g^{-1}(K)$ is open in (V, τR(Y)). Since the open (or closed) sets are just the complements of closed (respectively open) subsets, and $g^{-1}(V - K) = U - g^{-1}(K)$. We see that the continuity of g is equivalent to: if J is closed in (U, τR(X)) then the pre-image $g^{-1}(J)$ is closed in V. To prove this, let J be a closed subset of U. Since g is the inverse off, we have $g^{-1}(J) = f(J)$, hence we have to show that f(J) is closed in V. By theorem 3.4, J is compact. Since V is Hausdroff space implies that f(K) is closed in (V, τR(Y)).

Definition 3.12: A nano topological space (U, τR(X)) is said to be nano α^*_{AS}-Lindelof space if every nano α^*_{AS}-open cover of (U, τR(X)) has a countable sub cover.

Theorem 3.13: Every nano α^*_{AS} -compact space is a nanoα^*_{AS} -Lindelof space.

Proof. Let (U, τR(X)) be nano α^*_{AS}-compact. Let $\{K_d: d\in D\}$ be a nano α^*_{AS}-open cover of (U, τR(X)). Then $\{K_d: d\in D\}$ has a finite sub cover $\{ K_d: d = 1, 2, 3, ...n\}$, since (U, τR(X)) is nano α^*_{AS}-compact. Since every finite sub cover is always a countable sub cover and therefore, $\{ K_d: d = 1, 2, ... n\}$ is countable sub cover of $\{K_d: d\in D\}$ for (U, τR(X)). Hence (U, τR(X)) is nanoα^*_{AS}-Lindelof space.

Theorem 3.14: The image of a nano α^*_{AS}-Lindelof space under a nano α^*_{AS}-continuous map is nano α^*_{AS}-compact.

Proof. f: (U, τR(X)) → (V, τR(Y)) be a nano α^*_{AS}-continuous map from a nano α^*_{AS}-Lindelof space (U, τR(X)) onto a nano topological space (V, τR(Y)). Let $\{K_d: d\in D\}$ be a nano-open cover of (V, τR(Y)), then $\{f^{-1}(K_d): d\in D\}$ is a nano α^*_{AS}-open cover of (U, τR(X)), since f is nano α^*_{AS}-continuous. As (U, τR(X)) is nano α^*_{AS}-Lindelof, the nanoα^*_{AS}-open cover $\{f^{-1}(K_d): d\in D\}$ of (U, τR(X)) has a countable subcover $\{f^{-1}(K_d): d = 1, 2...n\}$. Therefore $X = \cup_{d\in D} f^{-1}(K_d)$which implies $f(U) = V = \cup_{d\in D} K_d$, that is $\{K_1, K_2......K_n\}$ is a countable subfamily of $\{K_d: d\in D\}$ for (V, τR(Y)). Hence (V, τR(Y)) is nano α^*_{AS}-Lindelof space.

Theorem 3.15: If (U, τR(X)) is nano α^*_{AS}-Lindelof and countably nano α^*_{AS}-compact space, then (U, τR(X)) is nano α^*_{AS}-compact.

Proof. Suppose (U, τR(X)) is nano α^*_{AS}-Lindelof and countably nano α^*_{AS}compact space. Let $\{K_d: d\in D\}$ be an nano α^*_{AS}-open cover of (U, τR(X)). Since (U, τR(X)) is nano α^*_{AS}-Lindelof $\{K_d: d\in D\}$ has a countable subcover $\{K_{dn}: n \in N\}$. Therefore, $\{K_{dn}: n \in N\}$is a countable subcover of (U, τR(X)) and $\{K_{dn}: n \in N\}$is subfamily of $\{K_d: d\in D\}$ ans so $\{K_{dn}: n \in N\}$ is a countable nano-open cover of (U, τR(X)). Again since (U, τR(X)) is countably nano-compact, $\{K_{dn}: n \in N\}$ has a finite subcover $\{ K_{d\,k}: k = 1, 2, ...n\}$. Therefore $\{ K_{d\,k}: k = 1, 2, ...n\}$ is a finite sub cover of $\{K_d: d\in D\}$ for (U, τR(X)). Hence (U, τR(X)) is nano α^*_{AS}-compact space.

Theorem 3.16: A topological space (U, τR(X)) is nano α^*_{AS}-compact if and only if every basic nano α^*_{AS}-open cover of (U, τR(X)) has a finite subcover.

Proof. Let (U, τR(X)) be nano α^*_{AS}-compact. Then every nano α^*_{AS}-opencover of (U, τR(X)) have a finite sub cover. Conversely, suppose that every basic nano α^*_{AS}-open cover of (U, τR(X)) has a finite sub cover and let $J = \{G_\lambda: \lambda \in \Lambda\}$ be any nano α^*_{AS}-open cover of (U, τR(X)). If $K = \{H_\alpha: \alpha \in \Delta\}$ be any nanoα^*_{AS}-open base for (U, τR(X)), then each G_λ is union of some members of K and the totality of all such members of K evidently a basic nano α^*_{AS}-open cover of (U, τR(X)). By hypothesis this collection of members of K has a finite sub cover,$\{ H_{\alpha i}: i = 1, 2, ...n\}$. For each $H_{\alpha i}$ in this finite sub cover, we can select a G_λ from J such that $H_{\alpha i}\subset G_{\lambda i}$. It follows that the finite subcollection $\{G_{\lambda i}: i = 1, 2...n\}$. Which arises in this way is a sub cover of J. Hence (U, τR(X)) is nano α^*_{AS}-compact.

4. NANO α *$_{AS}$ - CONNECTEDNESS

Definition 4.1: A nano topological space (U, τR(X)) is said tobe nano α^*_{AS}-connected if (U, τR(X)) cannot be expressed as a disjoint union of two non-empty nano α^*_{AS}-open sets. A subset of (U, τR(X)) is nano α^*_{AS}-connected as a subspace. A subset is said to be nanoα^*_{AS} disconnected if and only if it is not nano α^*_{AS}-connected.

Example 4.2: Let U = {1, 2, 3, 4}, X = {1, 2} ⊂ U and U/R ={{1}, {3}, {2,4}}, with nano topology τR(X) = {U, ∅, {3},{1,3},{1,2,4}}, nanoα^*_{AS} closed set = {{U, ∅,{3}, {1,3}, {2,3}, {3,4}, {1,2,3}, {1,3, 4}, {2,3,4}}thenit is nano α^*_{AS}-connected.

Theorem 4.3: For a nano topological space (U, τR(X)) the following are equivalent.

(1) (U, τR(X)) is nanoα^*_{AS}-connected.

(2) (U, τR(X)) and ∅ are the only subsets of U which are both nano α^*_{AS}-open and nano α^*_{AS}-closed.

(3) Each nano α^*_{AS}-continuous map of (U, τR(X)) into a discrete space (V, τR(Y)) with atleast two points is a constant map.

Proof. (1) ⇒ 2 Let J be a nano α^*_{AS}-open and nano α^*_{AS}-closed subset of (U, τR(X)). Then X − J is also both nano α^*_{AS}-open and nano α^*_{AS}-closed. Then $X = J\cup(X-J)$ a disjoint union of two non-empty nano α^*_{AS}-open sets which contradicts the fact that (U, τR(X)) is nano α^*_{AS}-connected. Hence J = ∅ or X.

(2) ⇒ (1) Suppose that X = J ∪ K where J and K are disjointnon-empty nano α^*_{AS}-open subsets of (U, τR(X)). Since J = X −K, then J is both nano α^*_{AS}-open and nano α^*_{AS}-closed. By assumption A = ∅ or X, which is a contradiction. Hence (U, τR(X)) is nano α^*_{AS}-connected.

(2) ⇒ (3) Let f: (U, τR(X)) → (V, τR(Y)) be a nano-continuous map, where (V, τR(Y)) is discrete space with atleast two points. Then $f^{-1}(\{d\})$ is nano α^*_{AS}-closed and nano α^*_{AS}-open for each d ∈ Y. That is (U, τR(X)) is covered by nano α^*_{AS}-closed and nano α^*_{AS}-open covering $\{f^{-1}(\{d\})$: d ∈ Y}. By assumption, $f^{-1}(d)$ = ∅ or X for each d ∈ Y. If $f^{-1}(d)$ = ∅ for each d ∈ Y, then f fails to be map. Therefore, there exists atleast one point $f^{-1}(\{d_1\})$ = ∅, d_1∈ Y such thatf$^{-1}(\{d_1\})$ = X. This shows that f is a constant map.

(3) ⇒ (2) Let J be both nano α^*_{AS}-open and nano α^*_{AS}-closed in (U, τR(X)).Suppose J ≠∅. Let f: (U, τR(X)) → (V, τR(Y)) be a nano α^*_{AS} continuous map defined by f(J) = {c} and f(X − J) = {d} where c≠ d and c,d ∈ Y. By assumption, f is constant so J = X.

Theorem 4.4: If f: (U, τR(X)) → (V, τR(Y)) is nano-continuous surjection and X is nano-connected, then Y is nano-connected.

Proof. Suppose that Y is not nano α^*_{AS}-connected. Let Y = J∪K where J and K are disjoint non-empty open sets in (V, τR(Y)).Since f is nano α^*_{AS}-continuous and onto. X = $f^{-1}(J)$ ∪ $f^{-1}(K)$ wheref$^{-1}(J)$ and $f^{-1}(K)$ are disjoint non-empty nano α^*_{AS}-open subsets in(U, τR(X)). This contradicts the fact that (U, τR(X)) is nano α^*_{AS}-connected. Hence (V, τR(Y)) is nano α^*_{AS}-connected.

Theorem 4.5: If f is a nano-continuous mappings of a nano α^*_{AS}-connected space (U, τR(X)) onto an arbitrary nano topological space (V, τR(Y)), then (V, τR(Y)) is nano α^*_{AS}-connected.

Proof. Let (V, τR(Y)) be a nano α^*_{AS}-connected. Then there exists a non-empty proper subset J of (V, τR(Y)) which is both nanoα^*_{AS}-open and nanoα^*_{AS}-closed in (V, τR(Y)). Since f is nanoα^*_{AS}-continuous and onto (V, τR(Y)), f^{-1} (J) is a non-empty proper subset of (U, τR(X)) which is both nano α^*_{AS}-open and nano α^*_{AS}-closed in (U, τR(X)) and therefore (U, τR(X)) is disconnected which is a contradiction. Hence (V, τR(Y)) must be connected.

5. CONCLUSION

This paper explains the concepts nano α^*_{AS}-compact spaces in nano topological spaces.The work is further moved in defining nano α^*_{AS}-Lindelof spaces and nano α^*_{AS}-Hausdroff spaces. All discussed spaces under nano α^*_{AS}-continuous was explained. Future work is planned in proving a weaker form of Tychonoff's theorem under nano α^*_{AS}-compact sets in nano topological spaces

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