

Some importance measurement methods for units in reliability Systems

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Abstract: This research deals with one of the methods of measuring the importance of reliability, which is the method (Improvement Potential (1)) and an improvement on this method. And how to get rid of the disadvantages of this method. Then deduce method (Improvement Potential (2)) from the (Improvement Potential (1)) method and compare these methods with the methods of measuring importance in accurate reliability, which are (Birnbaum's Measure).

Keywords: Reliability, importance, Birnbaum's Measure, Improvement

1. Introduction

When a single or many components of a system fail or change state, importance measurements refer to the impact on system reliability, which is a function of component reliability characteristics and system structure. Importance measurements, as one of the essential branches and core ideas of reliability, pervade all stages of product development, including design, manufacture, inspection, sale, and maintenance. The most important thing is to figure out what influences system reliability [1,2]. Importance measures are used in the design process to discover flaws and help system improvement and optimization. During the functioning of a system, importance measures can be used to distribute enterprise resources to component parts of the system in a reasonable manner to guarantee that it runs smoothly. Importance measurements have been widely used in system reliability, decision making, and risk analysis by identifying and analyzing system flaws [3–7]. Kim and Song [14] introduced a generalized reliability importance measure that can handle numerous critical failure zones, huge curvatures of limit-state surfaces, and input random variable correlation. A power flow element importance metric introduced by Li et al. [15] can improve cascading failure prevention, system backup configuration, and overall resilience. Dui et al. [11] analyzed the applications of importance measures in the reliability The significance measure was extended to three-echelon inventory systems. Dui et al colleagues. Using copulas, Jia and Cui [8] investigated the reliability of supply chain systems. Supply chain uncertainty and reliability were theoretically conceptualized by Flynn et al. [10]. He et al. [7] developed a stochastic demand logistics service supply chain model that took non-storage and dependability into account.

2. Birnbaum's Measure

Birnbaum (1969) proposed the following measure of

the reliability importance of component i at time t :

$$I^B(i/t) = \frac{\partial R_s}{\partial R_i} \quad (1)$$

Birnbaum's measure is therefore obtained as the partial derivative of the system reliability R_s with respect to R_i . This approach is well known from classical sensitivity analysis. If $I^B(i/t)$ is large, a small change in the reliability of component i results in a comparatively large change in the system reliability at time t . When taking this derivative, the reliabilities of the other components remain constant – only the effect of varying R_i is studied. Birnbaum's measure measures the rate of change of the system reliability as a result of changes to the reliability of a single component. By fault tree notation, Birnbaum's measure may be written as

$$I^B(i/t) = \frac{\partial R_f}{\partial R_{fi}} \quad (2)$$

Birnbaum's measure is named after the Hungarian-American professor Zygmund William Birnbaum (1903-2000). In the definition of Birnbaum's measure, the system reliability is denoted R_s and the system reliability is therefore a function of the component reliabilities only This definition of Birnbaum's measure is therefore not useable when the components are dependent, e.g., when we have common-cause failures. By pivotal decomposition, we have

$$R_s = R_i R_s(1_i, R_i) - (1 - R_i) R_s(0_i, R_i) \quad (3)$$

$$= R_i [R_s(1_i, R_i) - R_s(0_i, R_i)] + R_s(0_i, R_i) \quad (4)$$

This shows that R_s is a linear function of R_i (when all the other reliabilities are kept constant) Birnbaum's measure can therefore be written as

$$I^B(i/t) = \frac{\partial R_s}{\partial R_i} = R_s(1_i, R_i) - R_s(0_i, R_i) \quad (5)$$

$$I^B(i/t) = R_s(1_i, R_i) - R_s(0_i, R_i) \quad (6)$$

where $R_s(1_i, R_i)$ is the system reliability when we know that component i is functioning and $R_s(0_i, R_i)$ is the system reliability when we know that component i is not functioning. This leads to a very simple way of calculating $I^B(i/t)$ as illustrated by the

example on the next slide. Most computer programs for fault tree analysis computes Birnbaum's measure by this approach. The same approach is sometimes used to determine $I^B(i/t)$ for systems exposed to common-cause failures.

3. Improvement Potential (1)

The improvement potential of component i at time t is defined as:

$$I^{ip}(i/t) = R_s(1_i, R_i) - R_s \quad (7)$$

$I^{ip}(i/t)$ is hence the difference between the system reliability with a perfect component i , and the system reliability with the actual component i . It tells us how much it is possible to improve the current system reliability if we could replace the current component i with a perfect component.

3.1. Series system

Consider a series system of two independent components, 1 and 2, with component reliabilities R_1 and R_2 , respectively. Assume that $R_1 > R_2$, i.e., component 1 is the most reliable of the two. The system reliability is therefore

$$R_s = R_1 R_2 \quad (8)$$

$I^{ip}(1) = (1) R_2 - R_1 R_2$, $I^{ip}(2) = R_1 (1) - R_1 R_2$. This means that $I^{ip}(2) > I^{ip}(1)$ and we can conclude that when using the Improvement Potential measure, the most important component in a series structure is the one with the lowest reliability. To improve a series structure, we should therefore improve the "weakest" component, i.e., the component with the lowest reliability.

3.2. Parallel system

Consider a parallel system of two independent components, 1 and 2, with component reliabilities R_1 and R_2 , respectively. Assume that $R_1 > R_2$, i.e., component 1 is the most reliable of the two. The system reliability is therefore

$$R_s = R_1 + R_2 - R_1 R_2 \quad (9)$$

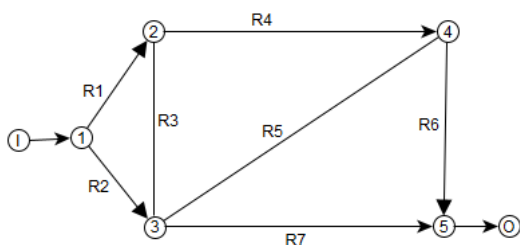
$I^{ip}(1) = 1 - [R_1 + R_2 - R_1 R_2]$, $I^{ip}(2) = 1 - [R_1 + R_2 - R_1 R_2]$. This means that $I^{ip}(1) = I^{ip}(2)$ and we can conclude that when using the Improvement Potential measure, all the components in a parallel structure are equally important. The improvement potential can therefore be expressed as

$$I^{ip}(i/t) = I^B(i/t) (1 - R_i) \quad (10)$$

One of the disadvantages of this method may not give accurate results, especially when the system is parallel, and here it is possible to improve this method as shown in the complex system.

3.3. Complex system

We will discuss the following complex system and the possibility of accessing the unit importance scale accurately and method of eliminating defects and errors and arriving at accurate results.



Figure(1) : Complex system

$$\begin{aligned}
 R_s = & R_2 R_7 + R_1 R_3 R_7 + R_1 R_4 R_6 + R_2 R_5 R_6 - R_1 R_2 R_3 R_7 + R_1 R_3 R_5 R_6 + \\
 & R_2 R_3 R_4 R_6 + R_1 R_4 R_5 R_7 - R_2 R_5 R_6 R_7 - R_1 R_2 R_3 R_4 R_6 - R_1 R_2 R_3 R_5 R_6 - \\
 & R_1 R_2 R_4 R_5 R_6 - R_1 R_2 R_4 R_5 R_7 - R_1 R_3 R_4 R_5 R_6 - R_1 R_2 R_4 R_6 R_7 - \\
 & R_1 R_3 R_4 R_5 R_7 - R_2 R_3 R_4 R_5 R_6 - R_1 R_3 R_4 R_6 R_7 - R_1 R_3 R_5 R_6 R_7 - \\
 & R_2 R_3 R_4 R_6 R_7 - R_1 R_4 R_5 R_6 R_7 + 2R_1 R_2 R_3 R_4 R_5 R_6 + R_1 R_2 R_3 R_4 R_5 R_7 + \\
 & 2R_1 R_2 R_3 R_4 R_6 R_7 + R_1 R_2 R_3 R_5 R_6 R_7 + 2R_1 R_2 R_4 R_5 R_6 R_7 + \\
 & 2R_1 R_3 R_4 R_5 R_6 R_7 + R_2 R_3 R_4 R_5 R_6 R_7 - 3R_1 R_2 R_3 R_4 R_5 R_6 R_7 \quad (11)
 \end{aligned}$$

If R_i , $i=1, \dots, 7$ have same reliability, put $R=0.8$, By $I^B(i/t) = \frac{\partial R_s}{\partial R_i}$ method we get the table

Table 1: determine the importance and level of units in figure(1) when have same reliability

Components	Importance	level
R ₂ ,R ₇	0.2235	Level 1
R ₁ , R ₆	0.1864	Level 2
R ₄	0.0584	Level 3
R ₃ ,R ₅	0.0379	Level 4

By Improvement Potential (1) method we get the table(2)

Table 2: determine the importance By Improvement Potential (1) method in figure (1) when have same reliability

Components	H(1,R)	R _s	Importance I ^p (i/t)=H(1,R)- R _s
R ₂ , R ₇	0.9526	0.9079	0.0447
R ₁ , R ₆	0.9452	0.9079	0.0373
R ₄	0.9196	0.9079	0.0117
R ₃ , R ₅	0.9155	0.9079	0.0076

These results can be converted into accurate results from the following relationship.

$$I^B(i/t) = \frac{Iip(i/t)}{(1-R_i)}$$

$$I^B(R_2/t) = I^B(R_7/t) = \frac{0.0447}{0.2} = 0.2235, I^B(R_1/t) = I^B(R_6/t) = \frac{0.0373}{0.2} = 0.1865$$

$$I^B(R_4/t) = \frac{0.0117}{0.2} = 0.0585, I^B(R_3/t) = I^B(R_5/t) = \frac{0.0076}{0.2} = 0.038$$

Conceder if R_i have different reliability units in figure (1)

Table 3: determine the importance and level of units in figure(1) when have different reliability

Components	Value reliability	Importance	Level
R ₇	0.8	0.4052	Level 1
R ₂	0.8	0.1946	Level 2
R ₆	0.6	0.1866	Level 3
R ₁	0.9	0.1618	Level 4
R ₃	0.7	0.0694	Level 5
R ₄	0.6	0.0588	Level 6
R ₅	0.9	0.0565	Level 7

Table 4: determine the importance By Improvement Potential (1) method in figure (1) have different reliability

Components	Value reliability	H(1,R)	R _s	Importance H(1,R) - R _s
R ₇	0.8	0.9571	0.8761	0.081
R ₆	0.6	0.9507	0.8761	0.0746
R ₂	0.8	0.9150	0.8761	0.0389
R ₄	0.6	0.8996	0.8761	0.0235
R ₃	0.7	0.8969	0.8761	0.0208
R ₁	0.9	0.8923	0.8761	0.0162
R ₅	0.9	0.8817	0.8761	0.0056

Through the above table ,it is not possible to rely on this measure when the values of the units are different , but it is possible to reach correct results and adopt it after making the following improvement

$$I^B(R_7/t) = \frac{0.081}{0.2} = 0.405 , I^B(R_2/t) = \frac{0.0389}{0.2} = 0.1945 , I^B(R_6/t) = \frac{0.0746}{0.4} = 0.1865 , I^B(R_1/t) = \frac{0.0162}{0.1} = 0.162 , I^B(R_3/t) = \frac{0.0208}{0.3} = 0.069 ,$$

$$I^B(R_4/t) = \frac{0.0235}{0.4} = 0.05875$$

$$I^B(R_5/t) = \frac{0.0056}{0.1} = 0.056$$

4. Improvement Potential (2)

The improvement potential of component i at time t is defined as:

$$I^{ip2}(i/t) = R_s - R_s(0, R_i) \quad (12)$$

4.1. Series system

Consider a series system of two independent components, 1 and 2, with component reliabilities R_1 and R_2 , respectively. Assume that $R_1 > R_2$, i.e., component 1 is the most reliable of the two. The system reliability is therefore $R_s = R_1 R_2$

$$I^{ip}(1) = R_1 R_2 - (0) R_2 = R_1 R_2$$

$$I^{ip}(2) = R_1 R_2 - (0) R_1 = R_1 R_2$$

This means that $I^{ip2}(1) = I^{ip2}(2)$ and we can conclude that when using the Improvement Potential(2) measure, all the components in a Series system are equally important.

4.2. Parallel system

Consider a parallel system of two independent components, 1 and 2, with component reliabilities R_1 and R_2 , respectively. Assume that $R_1 > R_2$, i.e., component 1 is the most reliable of the two .The system reliability is therefore $R_s = R_1 + R_2 - R_1 R_2$

$$I^{ip2}(1) = (R_1 + R_2 - R_1 R_2) - R_2 = R_1 - R_1 R_2 , I^{ip2}(2) = (R_1 + R_2 - R_1 R_2) - R_1 = R_2 - R_1 R_2$$

This means that $I^{ip2}(2) > I^{ip2}(1)$ and we can conclude that when using the Improvement Potential(2) measure, the most important component in a Parallel system is the one with the lowest reliability. To improve a Parallel system, we should therefore improve the “weakest” component, i.e., the component with the lowest reliability ,The improvement potential can therefore be expressed as

$$I^B(i/t) = \frac{I^{ip2}(i/t)}{R_i} \quad (13)$$

4.3. Complex system

If $R_i , i=1, \dots, 7$ have same reliability, put $R=0.8$ By Improvement Potential (2) method we get the table

Table 5: determine the importance By Improvement Potential (2) method in figure (1) when have same reliability

Components	H(0,R)	R_s	Importance $R_s - H(0,R) = I^{ip2}(i/t)$
R_2, R_7	0.7291	0.9079	0.1788
R_1, R_6	0.7588	0.9079	0.1491
R_4	0.8612	0.9079	0.0467
R_3, R_5	0.8776	0.9079	0.0303

These results can be converted into accurate results from the following relationship

$$I^B(2/t) = I^B(7/t) = \frac{0.1788}{0.8} = 0.2235 , I^B(1/t) = I^B(6/t) = \frac{0.1491}{0.8} = 0.186375$$

$$I^B(4/t) = \frac{0.0467}{0.8} = 0.058375 , I^B(3/t) = I^B(5/t) = \frac{0.0303}{0.8} = 0.037875 \quad \text{Conceder if } R_i \text{ have different reliability}$$

Table 6: determine the importance By Improvement Potential (2) method in figure (1) have different reliability

Components	Value reliability	H(0,R)	R_s	Importance $R_s - H(0,R)$
R_7	0.8	0.5520	0.8761	0.3241
R_2	0.8	0.7204	0.8761	0.1557
R_1	0.9	0.7304	0.8761	0.1457
R_6	0.6	0.7641	0.8761	0.112
R_5	0.9	0.8252	0.8761	0.0509
R_3	0.7	0.8275	0.8761	0.0486
R_4	0.6	0.8408	0.8761	0.0353

Through the above table ,it is not possible to rely on this measure when the values of the units are different , but it is possible to reach correct results and adopt it after making the following improvement

$$I^B(R_7/t) = \frac{0.3241}{0.8} = 0.405125 , I^B(R_2/t) = \frac{0.1557}{0.8} = 0.194625 , I^B(R_6/t) = \frac{0.112}{0.6} = 0.1866$$

$$I^B(R_1/t) = \frac{0.1457}{0.9} = 0.16188 , I^B(R_3/t) = \frac{0.0486}{0.7} = 0.0694 , I^B(R_4/t) = \frac{0.0353}{0.6} = 0.0588$$

$$I^B(R_5/t) = \frac{0.0509}{0.9} = 0.05655$$

5. Conclusions

We used the methods of measuring importance in reliability and showed defects (Improvement Potential (1)) and (Improvement Potential (2)) in the series system or the parallel system . Then we worked on improving these systems and showed this improvement on a complex system with reliability values equal once and different times. This improvement gave accurate results that can be adopted in measuring the importance of the reliability of the units

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