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SOME APPLICATIONS OF DISCRETE LAPLACE TRANSFORMS BY NABLA OPERATOR

[1,*]Shiny N.S & [2] Dominic Babu.G

[1]*Research Scholar* & [2] *Associate professor*

[1,2]*P.G and Research Department of Mathematics, Annai Velankanni College, Tholayavattam,*

Kanyakumari District, 629157, ***Affiliated to Manonmaniam Sundaranar University****,*

Abishekapatti, Tirunelveli-627012, Tamilnadu, India;

Abstract

In this paper, we define the generalized positive polynomial factorial and the generalized difference operator Δ_ℓ .A level of quality approach of numerical integration of differential equations is to replace it by suitable difference equation whose solution can be acquired in a suitable difference equation in a stable manner and without trouble from round –off errors. A definition for the Laplace transform corresponding to the nabla difference operator is given.

Keywords: Inverse Difference Operator,Generalized Laplace Transform, Nabla Operator, Exponential, Hyperbolic and Trigonometric Functions.

1. Introduction

The Knowledge of Laplace transforms become an essential part of the study of engineers and scientists. This provides easy and effective solutions for many problems arising in engineering [9]. This subject originated from the operational methods by the English engineer Oliver Heaviside (1850-1925) to applied linear transform in problems of electrical engineering [9]. Then it has been developed by Bromwich and Carson during 1916-17. The method of Laplace transforms has the advantage of directly giving the solution of differential equations with given boundary values. The Laplace transform of f(t) is defined by L(f(t) $=\int_0^\infty e^{-st} f(t)dt$ provided that the integral exists, s is a parameter which may be a real or complex number.

Definition 1.1

If n and ℓ are any two positive integers, then the generalized positive polynomial factorial is defined as

$$k_\ell^{(n)} = k(K-\ell)(k-2\ell)\ldots(k-(n-1)\ell) \text{ and } k_\ell^{(0)}=1,\ k_\ell^{(1)}=k \qquad (1)$$

Definition 1.2

If u(k) is a sequence of numbers and ℓ is any positive integer, then we define the generalized difference operator Δ_ℓ as

$$\Delta_\ell u(k) = u(k+1) - u(k) \qquad (2)$$

Theorem 1.3

If n is a positive integer and ℓ >0 then

$$\Delta_\ell k_\ell^{(n)} = n\,\ell\, k_\ell^{(n-1)} \qquad (3)$$

Definition 1.4 [5]

Let >0 and u(k) ,w(k) are real valued bounded functions. Then

$$\Delta_\ell^{-1} u(k)\, w(k) = u(k)\, \Delta_\ell^{-1} w(k) - \Delta_\ell^{-1} (\Delta_\ell^{-1} w(k+\ell)\Delta_\ell u(k)) \qquad (4)$$

Definition 1.5

Let ℓ >0 and $a^{s\ell}$-1 $\neq$ 0, then $\Delta_\ell^{-1} e^{sk} = \frac{e^{sk}}{e^{s\ell}-1}$ (5)

Definition 1.6

Let ℓ >0 and $a^{-s\ell}$-1 $\neq$ 0, then $\Delta_\ell^{-1} e^{-sk} = \frac{e^{-sk}}{e^{-s\ell}-1}$ (6)

Definition 1.7

For a given function u(k) ,the generalized Laplace transform is defined as

$L_\ell\, u(k) = \ell\, e^{sk} \Big|_0^\infty$

2.GENERALIZED LAPLACE TRANSFORM OF EXPONENTIAL FUNCTIONS

Lemma 2.1

Assume that $s \neq 0$, and ∇_ℓ^{-1} be the inverse difference operator, then

$$\Delta_\ell^{-1}\, \ell\, \frac{k_\ell^{(n)}}{n!}\, e^{-sk} \Big|_0^\infty = \frac{1}{s^{n+1}}$$

Proof

From the definition of generalized Laplace transform, we have

When n=1, $\Delta_\ell^{-1}\, \ell\, \frac{k_\ell^{(1)}}{1!}\, e^{-sk} \Big|_0^\infty = \ell\, \Delta_\ell^{-1} [k_\ell^{(1)}\, e^{-sk}] \Big|_0^\infty = \ell\, [k_\ell^{(1)} \frac{e^{-sk}}{e^{-s\ell}-1} - \frac{e^{-s(k+\ell)}}{(e^{-s\ell}-1)^2}\, \ell\,] \Big|_0^\infty$

$$\Rightarrow \Delta_\ell^{-1}\, \ell\, \frac{k_\ell^{(1)}}{1!}\, e^{-sk} \Big|_0^\infty = \frac{1}{s^2} \text{ as } \ell \to 0$$

When n=2, $\Delta_\ell^{-1}\, \ell\, \frac{k_\ell^{(2)}}{2!}\, e^{-sk} \Big|_0^\infty = \frac{\ell}{2} \Delta_\ell^{-1} [k_\ell^{(2)}\, e^{-sk}] \Big|_0^\infty = \frac{\ell}{2} [k_\ell^{(2)} \frac{e^{-sk}}{e^{-s\ell}-1} - 2\ell \Delta_\ell^{-1}(\frac{e^{-s(k+\ell)}}{e^{-s\ell}-1}\, k_\ell^{(1)})] \Big|_0^\infty$

$$= \frac{\ell}{2}\; [\, k_\ell^{(2)} \frac{e^{-sk}}{e^{-s\ell}-1} - 2\ell\; (k_\ell^{(1)} \frac{e^{-s(k+\ell)}}{(e^{-s\ell}-1)^2} - \frac{e^{-s(k+2\ell)}}{(e^{-s\ell}-1)^3}\, \ell)\,] \Big|_0^\infty$$

$$\Rightarrow \Delta_\ell^{-1}\, \ell\, \frac{k_\ell^{(2)}}{2!}\, e^{-sk} \Big|_0^\infty = \frac{1}{s^3} \text{ as } \ell \to 0$$

When n=3, $\Delta_\ell^{-1}\, \ell\, \frac{k_\ell^{(3)}}{3!}\, e^{-sk} \Big|_0^\infty = \frac{\ell}{6} \Delta_\ell^{-1} [k_\ell^{(3)}\, e^{-sk}] \Big|_0^\infty = \frac{\ell}{6} [\, k_\ell^{(3)} \frac{e^{-sk}}{e^{-s\ell}-1} - 3\ell\, \Delta_\ell^{-1}(\frac{e^{-s(k+\ell)}}{e^{-s\ell}-1}\, k_\ell^{(2}\,)] \Big|_0^\infty$

$$= \frac{\ell}{6}\; [\, k_\ell^{(3)} \frac{e^{-sk}}{e^{-s\ell}-1} - 3\ell\, (\, k_\ell^{(2)} \frac{e^{-s(k+\ell)}}{(e^{-s\ell}-1)^2} - 2\ell\, \Delta_\ell^{-1}\, (k_\ell^{(1)} \frac{e^{-s(k+2\ell)}}{(e^{-s\ell}-1)^2}\,))] \Big|_0^\infty$$

$$= \frac{\ell}{6}\; [\, k_\ell^{(3)} \frac{e^{-sk}}{e^{-s\ell}-1} - 3\ell\, (\, k_\ell^{(2)} \frac{e^{-s(k+\ell)}}{(e^{-s\ell}-1)^2} - 2\ell\, (k_\ell^{(1)} \frac{e^{-s(k+2\ell)}}{(e^{-s\ell}-1)^3} - \frac{e^{-s(k+3\ell)}}{(e^{-s\ell}-1)^4}\, \ell))] \Big|_0^\infty \quad \text{(from (4)\&(5))}$$

$$\Rightarrow \Delta_\ell^{-1}\, \ell\, \frac{k_\ell^{(3)}}{3!}\, e^{-sk} \Big|_0^\infty = \frac{1}{s^4} \text{ as } \ell \to 0$$

When n=4, $\Delta_\ell^{-1}\, \ell\, \frac{k_\ell^{(4)}}{4!}\, e^{-sk} \Big|_0^\infty = \frac{\ell}{24} \Delta_\ell^{-1} [k_\ell^{(4)}\, e^{-sk}\,] \Big|_0^\infty$

$$= \frac{\ell}{24}\; [\, k_\ell^{(4)} \frac{e^{-sk}}{e^{-s\ell}-1} - 4\ell\, (\, k_\ell^{(3)} \frac{e^{-s(k+\ell)}}{(e^{-s\ell}-1)^2} - 3\ell\, \Delta_\ell^{-1}\, (k_\ell^{(2)} \frac{e^{-s(k+2\ell)}}{(e^{-s\ell}-1)^2}\,))] \Big|_0^\infty$$

$$= \frac{\ell}{24}\; [\, k_\ell^{(4)} \frac{e^{-sk}}{e^{-s\ell}-1} - 4\ell\, (\, k_\ell^{(3)} \frac{e^{-s(k+\ell)}}{(e^{-s\ell}-1)^2} - 3\ell\; (k_\ell^{(2)} \frac{e^{-s(k+2\ell)}}{(e^{-s\ell}-1)^3} - 2\ell\, \Delta_\ell^{-1}\, (k_\ell^{(1)} \frac{e^{-s(k+2\ell)}}{(e^{-s\ell}-1)^3}))))] \Big|_0^\infty$$

$$= \frac{\ell}{24}\; [\, k_\ell^{(4)} \frac{e^{-sk}}{e^{-s\ell}-1} - 4\ell\, (\, k_\ell^{(3)} \frac{e^{-s(k+\ell)}}{(e^{-s\ell}-1)^2} - 3\ell\; (k_\ell^{(2)} \frac{e^{-s(k+2\ell)}}{(e^{-s\ell}-1)^3} - 2\ell\; (k_\ell^{(1)} \frac{e^{-s(k+3\ell)}}{(e^{-s\ell}-1)^4} - \frac{e^{-s(k+4\ell)}}{(e^{-s\ell}-1)^5}\, \ell))))] \Big|_0^\infty$$

$$\Rightarrow \Delta_\ell^{-1}\, \ell\, \frac{k_\ell^{(4)}}{4!}\, e^{-sk} \Big|_0^\infty = \frac{1}{s^5} \text{ as } \ell \to 0$$

In general, $\Delta_\ell^{-1}\, \ell\, \frac{k_\ell^{(n)}}{n!}\, e^{-sk} \Big|_0^\infty = \frac{1}{s^{n+1}}$

Theorem 2.2

Let $K \epsilon\, (0,\infty)$ and $\ell > 0$, then $\Delta_\ell^{-1}\, \ell\, e^{k_\ell^{(1)}} e^{-sk} \Big|_0^\infty = \frac{1}{s-1}$

Proof

We have, $e^{k_\ell^{(1)}} = 1 + \frac{k_\ell^{(1)}}{1!} + \frac{k_\ell^{(2)}}{2!} + \frac{k_\ell^{(3)}}{3!} + \frac{k_\ell^{(4)}}{4!} + \frac{k_\ell^{(5)}}{5!} + \ldots$

$\Delta_\ell^{-1}\, \ell\, e^{k_\ell^{(1)}} e^{-sk} \Big|_0^\infty = \Delta_\ell^{-1}\, \ell (\, 1 + \frac{k_\ell^{(1)}}{1!} + \frac{k_\ell^{(2)}}{2!} + \frac{k_\ell^{(3)}}{3!} + \frac{k_\ell^{(4)}}{4!} + \frac{k_\ell^{(5)}}{5!} + \ldots\,)\, e^{-sk} \Big|_0^\infty$ (from Lemma (1.1))

$$= \frac{1}{s} + \frac{1}{s^2} + \frac{1}{s^3} + \frac{1}{s^4} + \frac{1}{s^5} + \frac{1}{s^6} + \dots$$

This completes the proof

Corollary 2.3

Let Kϵ (0,∞) and $\ell > 0$, then $\Delta_\ell^{-1}\, \ell\, e^{-k_\ell^{(1)}} e^{-sk} \Big|_0^\infty = \frac{1}{s+1}$

3. Generalized Laplace Transform of Exponential & Trigonometric Functions

3.1 Generalized Laplace Transform of Sine Function

Lemma 3.1.1

If for any positive integer n, the polynomial factorial k(K- ℓ)(k-2 ℓ)…(k-(n-1) ℓ) ,then

$$\Delta_\ell^{-1}\ell\, \sin a k_l^{(1)}\, e^{-sk} \Big|_0^\infty = \frac{a}{s^2+a^2}$$

Proof

$$\Delta_\ell^{-1}\ell\, \sin a k_l^{(1)}\, e^{-sk} \Big|_0^\infty = \ell\, [\Delta_\ell^{-1}(a k_\ell^{(1)} - -\frac{a^3 k_\ell^{(3)}}{3!} + \frac{a^5 k_\ell^{(5)}}{5!} - \cdots) e^{-sk}] \Big|_0^\infty \quad (7)$$

Now,

$$\ell\, [\Delta_\ell^{-1}(a k_\ell^{(1)}\, e^{-sk})] \Big|_0^\infty = \ell a\, [\, k_\ell^{(1)} \frac{e^{-sk}}{e^{-s\ell}-1} - \Delta_\ell^{-1}(\frac{e^{-s(k+\ell)}}{e^{-s\ell}-1})] \Big|_0^\infty \text{ (from (4)\&(5))}$$

$$= \ell\, a\, [\, k_\ell^{(1)} \frac{e^{-sk}}{e^{-s\ell}-1} - \frac{e^{-s(k+\ell)}}{(e^{-s\ell}-1)^2}\ell] \Big|_0^\infty$$

$$\ell\, [\Delta_\ell^{-1}(a k_\ell^{(1)}\, e^{-sk})] \Big|_0^\infty = \frac{a}{s^2} \quad \text{as } \ell \longrightarrow 0 \quad (8)$$

Also,

$$\ell\, [\Delta_\ell^{-1}(\frac{a^3 k_\ell^{(3)}}{3!} e^{-sk})] \Big|_0^\infty = \frac{\ell a^3}{6} [k_\ell^{(3)} \frac{e^{-sk}}{e^{-s\ell}-1} - \Delta_\ell^{-1} (\frac{e^{-s(k+\ell)}}{e^{-s\ell}-1}\, 3\, \ell\, k_\ell^{(2)})] \Big|_0^\infty$$

$$= \frac{\ell a^3}{6} [k_\ell^{(3)} \frac{e^{-sk}}{e^{-s\ell}-1} - 3\,\ell\, (k_\ell^{(2)} \frac{e^{-s(k+\ell)}}{(e^{-s\ell}-1)^2} - \Delta_\ell^{-1} (\frac{e^{-s(k+2\ell)}}{(e^{-s\ell}-1)^2}\, 2\, \ell (k_\ell^{(1)})))] \Big|_0^\infty \text{(from (3) \&(4))}$$

$$= \frac{\ell a^3}{6} [k_\ell^{(3)} \frac{e^{-sk}}{e^{-s\ell}-1} - 3\,\ell\, (k_\ell^{(2)} \frac{e^{-s(k+\ell)}}{(e^{-s\ell}-1)^2} - 2\,\ell\, (k_\ell^{(1)} \frac{e^{-s(k+2\ell)}}{(e^{-s\ell}-1)^3} - - \Delta_\ell^{-1}(\frac{e^{-s(k+3\ell)}}{(e^{-s\ell}-1)^3}\, \ell))) \Big|_0^\infty$$

$$\ell\, [\Delta_\ell^{-1}(\frac{a^3 k_\ell^{(3)}}{3!} e^{-sk})] \Big|_0^\infty = \frac{a^3}{s^4} \quad \text{as } \ell \longrightarrow 0 \quad (9)$$

Continuing this process, we get

$$\ell\, [\Delta_\ell^{-1}(\frac{a^5 k_\ell^{(5)}}{5!} e^{-sk})] \Big|_0^\infty = \frac{a^5}{s^6} \quad \text{as } \ell \longrightarrow 0 \quad (10)$$

Substituting (8),(9),(10) in (7) ,we get

$$\nabla_\ell^{-1}\ell \sin a k_l^{(1)}\, e^{-sk} \Big|_0^\infty = \frac{a}{s^2} - \frac{a^3}{s^4} + \frac{a^5}{s^6} - \dots$$

This gives the proof

Corollary 3.1.2

Let k ϵ (0,∞) and ℓ >0 , then we have $\nabla_\ell^{-1}\ell\, \sin k_l^{(1)}\, e^{-sk} \Big|_0^\infty = \frac{1}{s^2+1^2}$

3.2 Generalized Laplace transform of cosine Function

Lemma 3.2.1

Assume that s$\neq$ 0, and ∇_ℓ^{-1} be the inverse difference operator, then

$$\Delta_\ell^{-1}\ell\, \cos a k_l^{(1)}\, e^{-sk} \Big|_0^\infty = \frac{s}{s^2+a^2}$$

Proof

$$\Delta_\ell^{-1}\ell\, \cos a k_l^{(1)}\, e^{-sk} \Big|_0^\infty = \ell\, [\Delta_\ell^{-1}(1 - -\frac{a^2 k_\ell^{(2)}}{2!} + \frac{a^4 k_\ell^{(4)}}{4!} - \cdots) e^{-sk}] \Big|_0^\infty \quad (11)$$

Now,

$$\ell\ [\Delta_{\ell}^{-1}(\ e^{-sk})]\ \Big|_0^{\infty}\ = [\ \frac{e^{-sk}}{1-e^{s\ell}}\ell]\ \ \Big|_0^{\infty}\ = \frac{1}{s}\quad \text{as}\ \ \ell\longrightarrow 0 \qquad (12)$$

$$\ell\ [\Delta_{\ell}^{-1}(\frac{a^2 k_{\ell}^{(2)}}{2!}e^{-sk})]\ \Big|_0^{\infty}\ = \frac{\ell a^2}{2}[k_{\ell}^{(2)}\frac{e^{-sk}}{1-e^{s\ell}}\ -\ \Delta_{\ell}^{-1}(\ \frac{e^{-s(k-\ell)}}{1-e^{s\ell}}\ 2\ \ell\ (k-\ell)_{\ell}^{(1)})]\ \ \Big|_0^{\infty}$$

$$= \frac{\ell a^2}{2}[k_{\ell}^{(2)}\frac{e^{-sk}}{1-e^{s\ell}}\ - 2\ \ell\ (\ k_{\ell}^{(1)}\frac{e^{-s(k-\ell)}}{(1-e^{s\ell})^2} -$$

$$\Delta_{\ell}^{-1}\ (\frac{e^{-s(k-2\ell)}}{(e^{s\ell})^2}\ \ \ell))]\ \Big|_0^{\infty}(\text{from}(4)\ \&\ (5))$$

$$\ell\ [\Delta_{\ell}^{-1}(\frac{a^2 k_{\ell}^{(2)}}{2!}e^{-sk})]\ \Big|_0^{\infty}\ = \frac{a^2}{s^3}\quad \text{as}\ \ \ell\longrightarrow 0 \qquad (13)$$

Continuing like this process, we get

$$\ell\ [\Delta_{\ell}^{-1}(\frac{a^4 k_{\ell}^{(4)}}{4!}e^{-sk})]\ \Big|_0^{\infty} = \frac{a^4}{s^5}\quad \text{as}\ \ \ell\longrightarrow 0 \qquad (14)$$

Substituting (12),(13),(14) in (11) ,we get

$$\Delta_{\ell}^{-1}\ell\ \cos a k_l^{(1)}\ e^{-sk}\ \ \Big|_0^{\infty}\ = \ \frac{1}{s}\ -\ \frac{a^2}{s^3}\ +\ \frac{a^4}{s^5}\ \ -\ \ldots$$

This yields the proof.

Corollary 3.2.2

Assume that s$\neq$ 0, and ∇_{ℓ}^{-1} be the inverse difference operator, then

$$\nabla_{\ell}^{-1}\ell\ \cos k_l^{(1)}\ e^{-sk}\ \ \Big|_0^{\infty}\ = \frac{s}{s^2+1^2}$$

4. Generalized Laplace Transform of Hyperbolic Functions

4.1Geeralized Laplace Transform of Hyperbolic Sine Function

Definition 4.1.1

Let ℓ >0 and a is a parameter, then sinh$k_{\ell}^{(1)}$ $=\frac{e^{k_{\ell}^{(1)}}-e^{-k_{\ell}^{(1)}}}{2}$

Definition 4.1.2

Let ℓ >0 and a is a parameter, then sinh$ak_{\ell}^{(1)}$ $=\frac{e^{ak_{\ell}^{(1)}}-e^{-ak_{\ell}^{(1)}}}{2}$

Lemma 4.1.3

$$\Delta_{\ell}^{-1}\ \ell e^{ak_{\ell}^{(1)}}\ sinhbk_{\ell}^{(1)}\ e^{-sk_{\ell}^{(1)}}\quad \Big|_0^{\infty} = \frac{b}{(s-a)^2-b^2}$$

Proof

$$\Delta_{\ell}^{-1}\ \ell e^{ak_{\ell}^{(1)}}\ sinhbk_{\ell}^{(1)}e^{-sk_{\ell}^{(1)}}\quad \Big|_0^{\infty} = \ell\ \ \Delta_{\ell}^{-1}\ (\frac{e^{bk_{\ell}^{(1)}}-e^{-bk_{\ell}^{(1)}}}{2}\ )e^{-(s-a)k_{\ell}^{(1)}}\quad \Big|_0^{\infty}$$

$$= \frac{\ell}{2}(\ \Delta_{\ell}^{-1}\ e^{-(s-a-b)k}\ \ -\ \ \Delta_{\ell}^{-1}(e^{-\{s-a+b)k})\quad \Big|_0^{\infty}$$

$$= \frac{1}{2}\ (\ \frac{1}{s-a-b}\ -\frac{1}{s+a+b}\ )$$

This completes the proof

Corollary 4.1.4

Assume that s$\neq$ 0, and ∇_{ℓ}^{-1} be the inverse difference operator, then

$$\Delta_{\ell}^{-1}\ \ell e^{-ak_{\ell}^{(1)}}\ sinhbk_{\ell}^{(1)}\ e^{-sk_{\ell}^{(1)}}\quad \Big|_0^{\infty} = \frac{b}{(s+a)^2-b^2}$$

Lemma 4.1.5

Assume that s$\neq$ 0, and ∇_{ℓ}^{-1} be the inverse difference operator, then

$$\Delta_{\ell}^{-1}\ \ell k_{\ell}^{(1)}\ sinhak_{\ell}^{(1)}\ e^{-sk_{\ell}^{(1)}}\quad \Big|_0^{\infty} = \frac{2s}{(s^2-a^2)^2}$$

Proof

$$\Delta_{\ell}^{-1}\ \ell k_{\ell}^{(1)}\ sinhak_{\ell}^{(1)} e^{-sk_{\ell}^{(1)}}\quad \Big|_0^{\infty} = \ell\quad \Delta_{\ell}^{-1} k_{\ell}^{(1)}\ \Big(\frac{e^{ak_{\ell}^{(1)}}-e^{-ak_{\ell}^{(1)}}}{2}\Big) e^{-sk_{\ell}^{(1)}}\quad \Big|_0^{\infty}$$

$$= \frac{\ell}{2}\big(\ \Delta_{\ell}^{-1} k_{\ell}^{(1)}\ e^{-(s-a)k}\ \ -\ \ \Delta_{\ell}^{-1} k_{\ell}^{(1)} e^{-\{s+a)k}\big)\ \Big|_0^{\infty} \qquad (15)$$

$$\frac{\ell}{2}\big(\ \Delta_{\ell}^{-1} k_{\ell}^{(1)}\ e^{-(s-a)k}\ \big)\ \ \Big|_0^{\infty}\ \ = \frac{\ell}{2}\Big(k_{\ell}^{(1)}\ \frac{e^{-(s-a)k}}{e^{-(s-a)\ell}-1}\ -\ \frac{e^{-(s-a)(k+\ell)}}{(e^{-(s-a)\ell}-1)^{\ 2}}\ \Big|_0^{\infty}\ = \frac{1}{2(s-a)^2} \qquad (16)$$

Similarly we can prove,

$$\frac{\ell}{2}\big(\ \Delta_{\ell}^{-1} k_{\ell}^{(1)}\ e^{-(s+a)k}\ \big)\ \ \Big|_0^{\infty}\ \ = \frac{1}{2(s+a)^2} \qquad (17)$$

Substituting (16) & (17) in (15)we get

$$\Delta_{\ell}^{-1}\ \ell k_{\ell}^{(1)}\ sinhk_{\ell}^{(1)} e^{-sk_{\ell}^{(1)}}\quad \Big|_0^{\infty} = \quad \frac{1}{2(s-a)^2}\ -\ \frac{1}{2(s+a)^2}$$

This completes the proof

4.2 Generalized Laplace Transform of Hyperbolic Cosine Functions

Definition 4.2.1

Let ℓ >0 and a is a parameter, then cosh$k_{\ell}^{(1)}$ =$\frac{e^{k_{\ell}^{(1)}}-e^{-k_{\ell}^{(1)}}}{2}$

Definition 4.2.2

Let ℓ >0 and a is a parameter, then cosh$ak_{\ell}^{(1)}$ =$\frac{e^{ak_{\ell}^{(1)}}+e^{-ak_{\ell}^{(1)}}}{2}$

Lemma 4.2.3

Assume that s$\neq$ 0, and ∇_{ℓ}^{-1} be the inverse difference operator, then

$$\Delta_{\ell}^{-1}\ \ell e^{ak_{\ell}^{(1)}}\ coshk_{\ell}^{(1)}\ e^{-sk_{\ell}^{(1)}}\qquad \Big|_0^{\infty} = \frac{s-a}{(s-a)^2-1}$$

Proof

$$\Delta_{\ell}^{-1}\ \ell e^{ak_{\ell}^{(1)}}\ coshk_{\ell}^{(1)} e^{-sk_{\ell}^{(1)}}\qquad \Big|_0^{\infty} = \ell\quad \Delta_{\ell}^{-1}\ \Big(\frac{e^{k_{\ell}^{(1)}}+e^{-k_{\ell}^{(1)}}}{2}\Big) e^{-(s-a)k_{\ell}^{(1)}}\qquad \Big|_0^{\infty}$$

$$= \frac{\ell}{2}\big(\ \Delta_{\ell}^{-1}\ e^{-(s-a-1)k}\ \ +\ \Delta_{\ell}^{-1}(e^{-\{s-a+1)k}\big)\quad \Big|_0^{\infty}$$

$$= \frac{1}{2}\ \Big(\ \frac{1}{s-a-1}\ + \frac{1}{s+a+1}\ \Big)$$

This completes the proof

Corollary 4.2.4

Assume that s$\neq$ 0, and ∇_{ℓ}^{-1} be the inverse difference operator, then

$$\Delta_{\ell}^{-1}\ \ell e^{-ak_{\ell}^{(1)}}\ coshk_{\ell}^{(1)}\ e^{-sk_{\ell}^{(1)}}\quad \Big|_0^{\infty} = \frac{s+a}{(s+a)^2-1}$$

Lemma 4.2.5

Assume that s$\neq$ 0, and ∇_{ℓ}^{-1} be the inverse difference operator, then

$$\Delta_{\ell}^{-1}\ \ell e^{k_{\ell}^{(1)}}\ coshak_{\ell}^{(1)}\ e^{-sk_{\ell}^{(1)}}\qquad \Big|_0^{\infty} = \frac{s-1}{(s-1)^2-a^2}$$

Proof

$$\Delta_{\ell}^{-1}\ \ell e^{k_{\ell}^{(1)}}\ coshak_{\ell}^{(1)} e^{-sk_{\ell}^{(1)}} \Big|_0^{\infty} = \ell\ \ \Delta_{\ell}^{-1}\ (\frac{e^{ak_{\ell}^{(1)}}+e^{-ak_{\ell}^{(1)}}}{2})e^{-(s-1)k_{\ell}^{(1)}} \Big|_0^{\infty}$$

$$= \frac{\ell}{2}(\ \Delta_{\ell}^{-1}\ e^{-(s-1-a)k}\ +\ \Delta_{\ell}^{-1}(e^{-(s-1+a)k})\ \Big|_0^{\infty}$$

$$= \frac{1}{2}\ (\frac{1}{s-1-a} + \frac{1}{s-1+a}\ )$$

This completes the proof

Corollary 4.2.6

Assume that s≠ 0, and ∇_{ℓ}^{-1} be the inverse difference operator, then

$$\Delta_{\ell}^{-1}\ \ell e^{-k_{\ell}^{(1)}}\ coshak_{\ell}^{(1)}\ e^{-sk_{\ell}^{(1)}} \Big|_0^{\infty} = \frac{s+1}{(s+1)^2-a^2}$$

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