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Second Order Qualitative Inspection of Difference Equation

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Abstract - We have established some additional oscillatory conditions of difference equation in the form, by generating nonoscillatory solutions with monotonous features. Our oscillatory findings are simply a continuation of those that came before them. We derived this using summation averaging technique and comparison principle. .

Index Terms - Oscillation; Nonlinear; Neutral Difference Equation. AMS subject classification: 39A10

INTRODUCTION

In this study, we have found certain conditions for second order difference equation

$$\Delta(a_e \Delta x_e) - p_e x_{e+l} = 0, e \geq e_0 \qquad (1)$$

One or more of the following assumptions will be applied throughout the rest of our investigations:

(H_1) $\{a_e\}, \{p_e\}$ are positive real sequences, where $N = \{e, e_0, e_1, e_2, e_3, e_4 ....\}$

(H_2) $\sum_{s=e_0}^{\infty} \frac{1}{a_s} < \infty$

(H_3) $D_e = \sum_{n=e}^{\infty} a_n$

If the solution of (1) is neither eventually positive nor eventually negative, it is said to be oscillatory; otherwise, it is nonoscillatory.

In [1], the authors considered the oscillation of advanced functional difference equation in the form $\Delta(a_n \Delta(y_n)) + q_n y_{\sigma(n)} = 0$ are established in canonical form, the Riccati equation is used to construct multiple oscillation criteria.

In [2], the authors found some adequate criteria for all the solutions of difference equation in the form $\Delta(a_n \Delta(y_n)) + p_n y_n + q_n y_{\sigma(n)} = 0$ are established in canonical form,

In [3], the authors studied the criteria for second order difference equation of the form $\Delta(a_n \Delta(y_n + p_n y_{n+k})) + q_n y_{n+l} + v_n y^{\alpha}{}_{n+m} = 0$ are obtained by the condition of oscillation of solutions in canonical form.

In [7], the authors considered the neutral difference equations in the form $\Delta(a_n \Delta(x_n + p_n x_{n-l})) - q_n f(x_{n-m}) = 0$ are obtained by the oscillatory and non oscillatory behavior.

In [8], the authors studied the oscillation of second order nonlinear difference equation with mixed neutral terms.

In [9], Mohanapriya and Mehar Banu concerned the nonlinear oscillation criteria for the advanced type of difference equations in the form $\Delta(a(n)\Delta x(n)) + q(n) x(\sigma^{\alpha}(n)) = 0$ are established.

In [10], the authors studied the noncanonical difference equation in the form $\Delta(r(v)\Delta x(v)) + q(v) x(v+\sigma) = 0$ are obtained by the nonoscillatory solution of monotonical property.

As observed in the previous study of literature, all the oscillatory results established for advanced type difference equations are linear and in canonical form. As a result, we derived the oscillation requirements for a second-order advanced difference equation in noncanonical form. The goal of this study is to find some new adequate condition of difference equation for the oscillation of (1). The findings in this research were influenced by those found in [1, 3, 9].

OSCILLATORY RESULTS

In this part, we have established some adequate conditions for the oscillation of (1).

Lemma 1

Assume $\sum_{e=e_0}^{\infty} P_e = \infty \quad for \quad e \geq e_1 \geq e_0$ (2)

and $\{x_e\}$ of (1) has the positive solution. Then $x_e > 0,\ \Delta x_e < 0, \Delta(a_e \Delta x_e) \leq 0$ (3)

and $\left\{\frac{x_e}{D_e}\right\}$ is increasing for $e \geq e_1$

Proof:

Let $\{x_e\}$ be the positive solution of (1)

$\Delta(a_e \Delta x_e) = P_e x_{e+l} < 0$

To prove $\Delta x_e < 0$

Define $Z_e = \frac{a_e(\Delta x_e)}{x_{e+l}} < 0$

$$\Delta Z_e = \frac{\Delta(a_e \Delta x_e)}{x_{e+l}} - \frac{a_{e+1}(\Delta x_{e+1})}{x_{e+l}\ x_{e+l+1}} \Delta x_{e+1}$$

$$\Delta Z_e = P_e \tag{4}$$

Take Summation from e_2 to $e-1$

$$\sum_{n=e_n}^{e-1} \Delta Z_n \leq \sum_{n=e_n}^{e-1} P_n$$

From (3), we get $\Delta x_e < 0$

Monotonicity of $a_e \Delta x_e$

$$x_e \geq a_e \Delta x_e \sum_{n=e}^{s} a_n$$

When $S \to \infty$, we get

$$x_e \geq a_e \Delta x_e D_e \tag{5}$$

$$\Delta\left(\frac{x_e}{D_e}\right) = \frac{a_e D_e \Delta x_e}{a_e D_e D_{e+1}} \geq 0 \tag{6}$$

Hence $\left\{\frac{x_e}{D_e}\right\}$ is increasing and hence the proof.

Theorem 1

Assume $\sum_{e=e_0}^{\infty} \left(\frac{1}{a_e} \sum_{n=e_0}^{e-1} P_n\right) = \infty$ (7)

and if $\{x_e\}$ of (1) has the positive solution for any $e \geq e_1 \geq e_0$ then $\{x_e\}$ satisfy (3) and $\lim_{e\to\infty} x_e = 0.$

Proof:

Let $\{x_e\}$ be the positive solution of (1)

The unboundedness of $\sum_{n=e_n}^{e-1} P_e$

is clear from (7) and (4) indicate that (2) holds

$\{x_e\}$ Satisfy (3) for any $e \geq e_1$ by lemma 1

Since $\{x_e\}$ is positive, there exist a constant $R > 0$,

$$\lim_{e \to \infty} x_e = R$$

Assume that $R > 0$, then there exist a integer $e_2 \geq e_1$, Such that

$$\Delta(a_e \Delta x_e) = P_e R, \quad e \geq e_2$$

Take Summation from e_2 to $e - 1$, in the above inequality,

$$\Delta x_e \geq R\left(\frac{1}{a_e} \sum_{n=e_2}^{e-1} P_n\right) \tag{8}$$

In the above inequality take summation from e_2 to $e - 1$, we get

$$x_e \geq x_{e_2} + R \sum_{n=e_2}^{e-1} \left(\frac{1}{a_e} \sum_{s=e_2}^{e-1} P_s\right)$$

Using (7) in the above inequality that is $x_e \to \infty$ as $e \to \infty$ and $R = 0$

Hence the proof.

Theorem 2

Let $\{x_e\}$ be the non oscillatory solution of (1) then $\sum_{e=e_0}^{\infty} \left(\frac{1}{a_e} \sum_{n=e_0}^{e-1} D_{n+l} P_n\right) = \infty$ (9)

Proof:

Let $\{x_e\}$ be the non oscillatory solution $\Delta(a_e \Delta x_e) - P_e x_{e+l} = 0$

Since $x_e > 0, x_{e+l} > 0$ for any $e \geq e_1 \geq e_0$

It is worth noting that (2) is required for (9) to hold.

The unboundedness of $\sum_{n=e_n}^{e-1} D_{n+l} P_n$ and (H_3) indicate that (2) holds due to (3)

$\{x_e\}$ Satisfy (4) for any $e \geq e_1$

Since $\left\{\frac{x_e}{D_e}\right\}$ is increasing then there exist the constant $R > 0$, Such that $e_2 \geq e_1$

$$x_e \geq R D_e \text{ for any } \quad e \geq e_2$$

Substitute above inequality in (1), we get

$$\Delta(a_e \Delta x_e) \geq R P_e D_{e+l} \tag{10}$$

Take Summation from e_2 to $e - 1$, we get

$$a_e \Delta x_e \geq R \sum_{n=e_2}^{e-1} P_n D_{n+l}$$

Summing from e_2 to $e - 1$

$$x_{e_2} \geq x_e + \sum_{n=e_2}^{e-1} \frac{R}{a_n} \left(\sum_{s=e_2}^{e-1} P_s D_{s+l}\right) \to \infty \quad \text{as} \quad e \to \infty$$

Hence the proof.

Theorem 3

Let $\{x_e\}$ be the non oscillatory solution of (1) then for any $e_1 \geq e_0$ of

$$\lim_{e \to \infty} Sup\, D_{e+l} \sum_{n=e_1}^{e-1} P_n < 1 \tag{11}$$

Proof:

Assume $\{x_e\}$ be the non oscillatory solution of (1), Such that $x_e > 0, x_{e+l} > 0$ for any $e \geq e_1 \geq e_0$

The inequality (11) and (3) implies (2)

$\{x_e\}$ satisfy (3) for any $e \geq e_1$ by lemma 1

$$\Delta(a_e \Delta x_e) - P_e x_{e+l} = 0$$

Take summation from e_1 to $e-1$

$$\sum_{n=e_1}^{e-1} a_e \Delta x_e = \sum_{n=e_1}^{e-1} P_e x_{e+l}$$

$$a_e \Delta x_e = x_{e+l} \sum_{n=e_1}^{e-1} P_e$$

Since x_e is increasing

Substitute (5) in above inequality

$$a_e \Delta x_e \geq a_{e+l}(\Delta x_{e+l}) D_{e+l} \sum_{n=e_1}^{e-1} P_e$$

$$\lim_{e\to\infty} Sup\, D_{e+l} \sum_{n=e_1}^{e-1} \leq 1$$

Hence the proof.

Theorem 4

If $\{a_e \Delta x_e\}$ is eventually negative then $\{x_e\}$ be the non oscillatory solution of (1)

Proof:

Assume the contrary that $x_e > 0$ for $e \geq N$

$$\Delta(a_e \Delta x_e) - P_e x_{e+l} = 0 \text{ for } e \geq N \tag{12}$$

To prove $x_e < 0$

Take $\Delta x_{e_1} \geq 0$ for $e_1 > N$

From (12), we get $\Delta(a_e \Delta x_e) < 0$

$$\Delta(a_e (x_{e+1} - x_e)) < 0$$

$$a_e x_e < a_e x_{e+1} - a_{e+1} \Delta x_{e+1}$$

This is the contradiction to the fact that $x_e < 0$.

Hence our assumption is wrong and the proof completed.

Theorem 5

Assume that x_e be the eventually positive solution of (1) then $x_e > 0, \Delta x_e > 0, \Delta(a_e \Delta x_e) > 0$ for large e.

Proof:

Let x_e be the positive solution of (1)

$$\Delta(a_e \Delta x_e) - P_e x_{e+l} = 0$$

$$\Delta(a_e \Delta x_e) \leq 0$$

There exist $e > e_1$, Such $x_{e+l} > 0$

We claim $\Delta(a_e \Delta x_e) > 0$ for any $e \geq e_2$

Suppose that $\Delta(a_e \Delta x_e) \leq 0$

$$a_e \Delta x_e \leq a_{e_2} \Delta x_{e_2} \leq 0$$

Taking Summation e_2 to e, we get

$$\sum_{e_2}^{e}(x_{e+1}-x_e) \le a_{e_2}\Delta x_{e_2}\frac{1}{a_e}$$

Then $x_e \to -\infty$ as $e \to \infty$

Which is a contradiction.

Hence $\Delta(a_e \Delta x_e) > 0$ for any e.

CONCLUSION

Using summation average techniques and the comparison principle, several new oscillatory criteria for neutral delay difference equation are generated. The purpose of this study is to develop some criteria for solving the second order oscillatory neutral delay difference equation. So that we can fall back on them if the other conditions fail. In the future, we hope to apply these findings to higher order neutral delay difference equations. As a result, the findings in this research relate to and improved on previous findings in the literature

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