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PRIME AND SEMIPRIME IDEALS IN TERNARY Γ-SO-SEMIRINGS-II

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Abstract - "The set of all partial functions over a set under a natural addition, functional composition and functional relation on the, forms a Γ-SO-semiring. The concepts of prime ideal, semi prime ideals in ternary Γ-SO-ring are introduced."

Index Terms - **"**prime ideal, semiprime ideal, Ternary Γ-SO semiring."

1. INTRODUCTION

"In 1981 the concept of Γ - semigroup as generalization of semi group introduced by Sen. H.S Vandiver develops the theory of semi ring in 1934. The notion of Γ - semirings was introduced by M.MuralikrishnaRao in 1995.

Some classical notion of ternary Γ- SO semirings are introduced in this paper. In 2019 K.Bhagyalakshmi and Dr.V.AmarendraBabu developed ideal theory in Ternary Γ-SO semirings. In this paper we familiarize the notions of irreducible, strongly irreducible bi-ideals of Ternary Γ-SO semirings and obtain characterizations of prime, semiprime, irreducible and strongly irreducible bi-ideals in regular Ternary Γ-SO semiring. Ternary Γ-SO semiring is denoted by TΓSS, Commutative Ternary Γ-SO semiring is denoted by CTΓSS, prime ideal by PI, semiprime ideal is denoted by SPI, is denoted throughout this paper, minimal prime ideal by MPI."

2. PREREQUISITES

The following are the prerequisites for this paper.

Definition2.1: "A be ***partial Γ-monoid*** is a triple (R, Γ, Σ) where R, Γ are non-empty sets and Σ is a partial addition defined on some but not necessarily all families $(a_i : i \in I)$ in R with the following laws:

1) **Unary sum axiom:** If $(a_i : i \in I)$ is a one element family in R and I = {*j*} then $\sum(a_i : i \in I)$ is defined and equal to a_j.
2) **Partition Associative axiom:** If $(a_i : i \in I)$ is a family in R and $(a_j : j \in I)$ is a partition of I, then $(a_i : i \in I)$ is sum-able if and only if $(a_i : i \in I_j)$ is sum-able for every j in J, $(\sum(a_i : i \in I_j) : j \in J)$ is sum-able and $\sum(a_i : i \in I_j) = \sum(\sum(a_i : i \in I_j) : j \in J)$."

Definition 2.2: "Let M, Γ be partial Γ-monoids then M is said to be ***partial ternary gamma semiring*** provided ∃ a mapping $M \times \Gamma \times M \times \Gamma \times M \rightarrow M$ satisfying the following conditions:

1) $x\alpha y\beta(z\delta p\gamma q) = x\alpha(y\beta z\delta p)\gamma q = (x\alpha y\beta z)\delta p\gamma q$
2) a family $(a_i : i \in I)$ is sum-able in M implies that $(x\alpha y\beta a_i : \text{ for odd } i \in I)$ is sum-able in M and $x\alpha y\beta\left[\sum(a_i : i \in I)\right] = \sum(x\alpha y\beta a_i : \text{ for odd } i \in I)$
3) family $(a_i : i \in I)$ is sum able in M implies that $(x\alpha a_i\beta y : \text{ for odd } i \in I)$ is sum able in M and $x\alpha\left[\sum(a_i : i \in I)\right]\beta y = \sum(x\alpha a_i\beta y : \text{ for odd } i \in I)$
4) family $(a_i : i \in I)$ is sum able in M implies that $(a_i\alpha x\beta y : \text{ for odd } i \in I)$ is sum able in M and $\left[\sum(a_i : i \in I)\right]\alpha x\beta y = \sum(a_i\alpha x\beta y : \text{ for odd } i \in I)$"

Definition 2.3: **"**A partial ternary Γ-semiring said to have a left (lateral, right) unity element provided there exist a family $(e_i : i \in I)$ of M and $(\alpha_i, \beta_i : i \in I)$ of Γ ∋

$\sum e_i\alpha_i e_i\beta_i a = a(\sum e_i\alpha_i a\beta_i e_i = a, \sum a\alpha_i e_i\beta_i e_i = a)$ for any $a \in$ M."

Definition 2.4: **"**The sum ordering relation $\leq$ in partially ternary Γ-monoid M is the binary relation such that $a \leq b$ iff there exist an element *c* in M such that $b = a + c \; \forall \; a, b \in$ M."

Definition 2.5: **"**A ***sum ordered partially ternary Γ-monoid (ternary Γ-so-monoid)*** in which partial sum ordering is a partial ordering."

Definition 2.6: **"**A partial ternary Γ-semiring M is said to be ***sum ordered partial ternary Γ-semiring (Ternary Γ-SO-semiring)*** if the partial Γ-monoid is SO-Γ-monoid."

Definition 2.7: **"**Let M be a partial ternary Γ-semiring. A non-empty subset of M is said to be ***left (Lateral, right) partial ternary Γ-ideal*** of M provided

(i) $(a_i : i \in I)$ is a sum able family of M and $x_i \in$ A for all $i \in$ I implies $\sum_i x_i \in A$

(ii) for all $x, y \in$ M, $z \in$ A implies that $z\alpha x\beta y \in$ A ($x\alpha z\beta y \in$ A, $x\alpha y\beta z \in$ A)

If A is left, lateral and right partial ternary Γ-ideal of M, then A is called partial ternary Γ-ideal of M."

Definition 2.8: **"**Let M be a ternary Γ-so-semiring. A non-empty subset A of M is said to be a ***left (lateral, right) ternary Γ-ideal*** of M, if it satisfies the following:

(i) A is a left (lateral, right) partial ternary Γ-ideal of M.

(ii) $x \in$ M and $y \in$ A such that $x \leq y$ then $x \in$ A.

If A is left, lateral as well as right ternary Γ-ideal of M, then A is known as ternary Γ-ideal of M."

Definition 2.9: **"**Let M be a ternary Γ-SO-semiring and A be a subset of M, then the intersection of all ternary Γ-ideals containing the set A is called ***ternary Γ-ideal generated by A*** and it is denoted by (A)."

Definition 2.10: **"**A Ternary Γ-SO-semiring M is said to be ***complete ternary Γ-SO-semiring*** if every family of elements in M is sum able."

Definition2.11: **"**A non-empty subset A of a Γ-SO-ring R is said to be Γ-sub SO-ring if

(i) A is a sub-SO monoid of R
(ii) AΓAΓA⊆ A".

3.0 Prime and semi prime ideals:

Def 3.1: **"**A proper ideal P of a TΓSS M is called as ***prime***iff for any ideals R, S, T of M, RΓSΓT ⊆ P ⟹ R ⊆ P or S ⊆ P or T ⊆ P".

Example 3.2:"Let R= [0, 1] be the unit interval of real numbers. For any family $(a_i{:}i \in I)$ in R define $\sum_i a_i = Sup\{a_i / i \in I\}$ then R is a partial ternary monoid. If we take Γ=W then R is a partial ternary Γ-monoid. Consider the mapping $(x, \alpha, y, \beta, z) \to \inf(x, \alpha, y, \beta, z)$ of R ×Γ× R×Γ× R⟶ R then R is a partial ternary Γ-semiring. Then R is a ternary Γ-SO semiring with usual $\leq$ of real numbers. Let $x \in R$. Take P= [0, x].

Let E, F, G are ideals of R such that EΓFΓG ⊆ P. Then $\exists \; y, z, w \in R$ such that E= [0, y], F=[0, z] & G=[0, w]. Now EΓFΓG=[0, y]Γ[0, z]Γ[0, w]=[0,inf{y, α, z, β, w}] for every $\alpha, \beta \in \Gamma$ and so, [0,inf{y, α, z, β, w}]⊆ P=[0, x]. Either $y \leq x$ or $z \leq x$ or $w \leq x$. Thus E=[0, y] ⊆[0, x]=P or F=[0, z] ⊆[0, x]=P or G=[0, w] ⊆[0,x]. Hence P= [0, x] is a prime ideal of R".

Definition 3.3: **"**A proper ideal P of a TΓSSM is called as ***semiprime*** if and only if for any ideals R of M, RΓRΓR ⊆ P ⟹ R ⊆ P".

Example 3.4:"Consider the TΓSSM, as define in Example 3.2. Take P= [0, x].

Let E be any ideal of M such that EΓEΓE⊆ P. Then $\exists \; y \in M$ such that E= [0, y]. Now EΓEΓE=[0, y]Γ[0, y]Γ[0, y]=[0,inf{y, α, y, β, y}] for every $\alpha, \beta \in \Gamma$ and so,

[0,inf{y, α, y, β, y}]⊆ P=[0, x]⇒ $y \leq x$. Thus E= [0, y] ⊆[0, x]=P. Hence P= [0, x] is a semiprime ideal of M".

Def3.5: A subset A ($\neq \phi$) TΓSSM, is entitled an m-system if for each $a, b, c \in A$ there exists elements w_1, w_2, w_3, w_4 of M ∋ $a\alpha w_1\beta b\gamma w_2\delta c \in A$ or $a\alpha w_1\beta w_2\gamma b\delta w_3\eta w_4\phi c \in A$ or

$a\alpha w_1\beta w_2\gamma b\delta w_3\eta c\phi x w_4 \in A$ or $w_1\alpha a\beta w_2\gamma b\delta w_3\eta w_4\phi c \in A$.

Th3.6: "A proper ideal L of a TΓSSM is prime iff its complement L^c is an m-system".

Pf: Let L be a PI of a TΓSS M. Suppose $d,e,f \notin L$. Then $d,e,f \in L^c$.

If possible, let L^c be not an m-system.

Then for all $w_1,w_2,w_3,w_4 \in M$ $d\alpha w_1\beta e\gamma w_2\delta f \notin L^c$ or $d\alpha w_1\beta w_2\gamma e\delta w_3\eta w_4\phi f \notin L^c$ or $d\alpha w_1\beta w_2\gamma e\delta w_3\eta f\phi w_4 \notin L^c$ or $w_1\alpha d\beta w_2\gamma e\delta w_3\eta w_4\phi f \notin L^c$.

$\Rightarrow d\alpha w_1\beta e\gamma w_2\delta f \in L$ or $d\alpha w_1\beta w_2\gamma e\delta w_3\eta w_4\phi f \in L$ or $d\alpha w_1\beta w_2\gamma e\delta w_3\eta f\phi w_4 \in L$ or $w_1\alpha d\beta w_2\gamma e\delta w_3\eta w_4\phi f \in L$ for all $w_1,w_2,w_3,w_4 \in M$. $\because$ L is a prime ideal of M, by known Th. we ought to $d \in L$ or $e \in L$ or $f \in L$ a contradiction. $\therefore L^c$ is an m-system.

On the contrary, let L^c be an m-system.

Then $d,e,f \in L^c$ implies that there exist $w_1,w_2,w_3,w_4 \in M \ni d\alpha w_1\beta e\gamma w_2\delta f \in L^c$ or $d\alpha w_1\beta w_2\gamma e\delta w_3\eta w_4\phi f \in L^c$ or $d\alpha w_1\beta w_2\gamma e\delta w_3\eta f\phi w_4 \in P^c$ or $w_1\alpha d\beta w_2\gamma e\delta w_3\eta w_4\phi f \in P^c$.

Thus $d,e,f \notin P$ implies that $d\Gamma M\Gamma e\Gamma M\Gamma f \not\subset L$, $d\Gamma M\Gamma M\Gamma e\Gamma M\Gamma Mf \not\subset L$ and $M\Gamma d\Gamma M\Gamma e\Gamma M\Gamma M\Gamma f \not\subset L$. Hence by known theorem L is a prime ideal of M.

Def3.7:"LetM be a TΓSS and L be an ideal of M. Then L is called a maximal ideal of M if $L \neq M$ and there does not exist any other ideal I of M such that $L \subset I \subset M$."

Th3.8:"Let A be an m-system and N an ideal of a TΓSSM such that $N \cap A = \Phi$. Then there exists a maximal ideal L of M containing N such that $L \cap A = \Phi$. Moreover, L is also a prime ideal of M".

Pf: The collection I(M) of all proper ideals of M containing N and each of which has an empty intersection with A is non-empty because N itself is a member of I(M) & I(M) is also partially ordered under the usual set inclusion relation. Now any chain of such ideals in the partially ordered set I(M) has an upper bound which is their union. Hence by Zorn's lemma

I(M) possesses a maximal element L $\ni L \cap A = \Phi$.

If possible, let L be not a PI of M. Let $x \notin L, y \notin L \,\&\, z \notin L.$ Let $I = L+<x>, J = L+<y> \,\&\, K = L+<z>$ be three ideals of M such that $I\Gamma J\Gamma K \subseteq L$ but $I \not\subset L, J \not\subset L \,\&\, K \not\subset L.$ Then we see that all the ideals I, J&K Properly contain L and hence by maximality of L, we get $I \cap A \neq \phi, J \cap A \neq \phi \,\&\, K \cap A \neq \phi.$ So $\exists\, e,f,g \in A$ such that $e \in I, f \in J \,\&\, g \in K.$ $\because$ A is an m-system, $\exists$ elements x_1,x_2,x_3,x_4 of M such that $e\alpha x_1\beta f\gamma x_2\delta g \in A$ or $e\alpha x_1\beta x_2\gamma f\delta x_3\eta x_4\phi g \in A$ or $e\alpha x_1\beta x_2\gamma f\delta x_3\eta g\phi x_4 \in A$ or $x_1\alpha e\beta x_2\gamma f\delta x_3\eta x_4\phi g \in A$.

If $e\alpha x_1\beta x_2\gamma f\delta x_3\eta x_4\phi g = (e\alpha x_1\beta x_2\gamma) f (\delta x_3\eta x_4\phi g)$

$\in I\Gamma J\Gamma K \subseteq M$

If $e\alpha x_1\beta x_2\gamma f\delta x_3\eta x_4\phi g \in A$, $e\alpha x_1\beta x_2\gamma f\delta x_3\eta x_4\phi g = (e\alpha x_1\beta x_2\gamma) f (\delta x_3\eta x_4\phi g)$

$\in I\Gamma J\Gamma K \subseteq M$

If $e\alpha x_1\beta x_2\gamma f\delta x_3\eta g\phi x_4 \in A, e\alpha x_1\beta x_2\gamma f\delta x_3\eta g\phi x_4 = (e\alpha x_1\beta x_2\gamma) f (\delta x_3\eta g\phi x_4)$

$\in I\Gamma J\Gamma K \subseteq M$

Thus in any case, we arrive at the contradiction that $L \cap A \neq \Phi$. Hence L is a PI of M.

Def3.9: A PI K of a TΓSS M is called a MPI belonging to an ideal I of M if $I \subseteq K$ &there exists no other PI K' of M $\ni I \subseteq K' \subset K.$

Th3.10: A PI"P" of a TΓSS M is a MPI belonging to an ideal I of M iff its complement P^c is a maximal m-system $\ni P^c \cap I = \Phi$.

Pf: Let P be a MPI belonging to I. By th.3.6 P^c is an m-system with $P^c \subseteq I^c$ so that $P^c \cap I \subseteq I^c \cap I = \Phi$ i.e. $P^c \cap I = \Phi.$ By applying Zorn's lemma to the class of m-systems of M, we can find a maximal m-system A such that $A \cap I = \Phi$ so that $P^c \subseteq A.$ Let L be a maximal ideal containing I $\ni L \cap A = \Phi$. Then by Th.3.8, L is also a prime ideal of M and $I \subseteq L \subseteq A^C \subseteq P.$ Since P is a MPI containing I, $M = A^C = P$ and hence $A = P^c$.

Conversely, suppose that P^c is a maximal m-system $\ni P^c \cap M = \Phi.$ Then $I \subseteq L \subseteq P$ and hence $P^c \subseteq M^c \subseteq I^c.$ Therefore, $L^c \cap I \subseteq I^c \cap I = \Phi$ ie., $L^c \cap I = \Phi.$ By virtue of maximality of P^c, we have $P^c = L^c$ &∴ P=L. Thus P is a prime ideal containing I. Now we have to S.T P is a MPIcontaining I. If possible, let Q be any PI containing I and properly contained in P. Then $P^c \cap I \subseteq Q^c \cap I \subseteq I^c \cap I = \Phi$ which is a contradiction.

∴ P is aMPI containing I.

Corollary3.11: Every prime ideal containing an ideal I of a TΓSS M possesses a MPI belonging to I.

Pf: If P is a prime ideal containing I, then there exists a maximal m-system A such that $P^c \subseteq A.$ Thus by Th: 3.10 $Q = A^c \subset P$ is a MPI containing I.

Def3.12: A proper ideal P of a TΓSS M is called a "completely prime ideal" of M if $d\alpha e\beta f\gamma \in P \Rightarrow$ implies that $d \in P$ or $e \in P$ or $f \in P$ for a few three elements d,e, f of M.

Note3.12: "Every completely prime ideal of M is surely a prime ideal of M but the converse may not be true, in general. However; for a CTΓSS both the concepts coincide."

Lemma3.13: A proper ideal Q of a TΓSS M is "completely prime"iff Q^c is a ternary subsemiring of M.

Pf: Suppose Q^c is a ternary subsemiring of M. Let $h\alpha i\beta j \in Q.$ If possible, let $h, i, j \in Q^c$.

Since Q^c is a ternary subsemiring of M, we have $h\alpha i\beta j \in Q^c$, a contradiction. So either

$h \in Q$ or $i \in Q$ or $j \in Q$. Consequently, Q is "completely prime".

By reversing the above argument the converse follows:

Theorem3.14: A proper ideal O of a TΓSS M is completely prime iff for each pair of non-negative integers m &n with even sum, $M^m\Gamma(h\alpha i\beta j)\Gamma M^n \subseteq \Omega$ implies $h \in O$ or $i \in O$ or $j \in O.$

Pf: Suppose O is a CPI of M and

$M^m\Gamma(h\alpha i\beta j)\Gamma M^n \subseteq O$,

Where m and n are non-negative integers with even sum.

Then $(h\alpha i\beta j)\Gamma(h\alpha i\beta j)\Gamma.....(h\alpha i\beta j)\Gamma(m+n+1)$ times belong to O.

$\because$ O is a CPI of M & by induction, we have $h\alpha i\beta j \subseteq O \Rightarrow h \in O$ or $i \in O$ or $i \in O$.

Conversely, suppose that $h\alpha i\beta j \in O.$ Ifm & n are non-negative integers with even sum, $M^m\Gamma(h\alpha i\beta j)\Gamma M^n \subseteq M^m\Gamma O\Gamma M^n \subseteq O$. Thus, $h \in O$ or $i \in O$ $j \in O$.

∴ O is completely prime.

Def3.15: A prime radical of a ideal P of TΓSS M is defined as the intersection of all prime ideals of M and is denoted by P(M).

Remark3.16: Since every prime ideal of aTΓSS contains 0, $P(S) \neq \phi$.

Theorem3.17: For a TΓSS M, P (M) = {m∈M: every m-system of M containing m contains zero of M}

Pf: Suppose that $P'(S)$={a∈M: every m-system of M containing 'a' containing zero of M}. Let $b \notin P$ for some PI P of M. By theorem3.14, P^c is an m-system.$\because 0 \in P, 0 \notin P^c$ Thus P^c is an m-system of M containing "b" but not containing zero of M. Consequently, $b \notin P'(M)$ and hence we find that $P'(M) \subseteq P(M)$ --- (1)

Again, let $b \notin P'(M)$.

Then ∃ an m-system A of M such that $b \in A$ but $0 \notin A$ Thus by theorem3.8, ∃ a PI"P" of M ∋ $P \cap A = \phi$.

Consequently, $b \notin P$ and hence $b \notin P(M)$.

So we find that $P(M) \subseteq P'(M)$ …..(2)

From (1) & (2), it follows that $P'(M) = P(M).$

Th3.18: If L is an ideal of TΓSS Mthen $P(L) = L \cap P(M)$, where P(L) denotes the prime radical of L considering L as TΓSS.

Pf: Let the collection Δ_1 be of all prime ideals of M & Δ_2 be the collection of all prime ideals of L. Then by known theorem $P \in \Delta_1$ implies that $P \cap L \in \Delta_2$.

So $P(L) = \bigcap\limits_{Q \in \Delta_2} Q \subseteq \bigcap\limits_{p \in \Delta_1} (L \cap P) = L \cap (\bigcap\limits_{P \in \Delta_1} P) = L \cap P(M)$ --- (1)

Again, suppose $a \notin P(L).$ Then $0 \notin A$ for some m-system A of Lcontaining 'a'. Since A is also an m-system of M, by Th: 3.16, $a \notin P(M)$.

Consequently, $P(M) \subseteq P(L)$

Now $L \cap P(M) \subseteq L \cap P(L) = P(L)$ ----- (2) from (1) & (2), we get $P(L) = L \cap P(M)$.

Def3.19: The completely prime radical of a TΓSS M is defined as the intersection of all completely prime ideals of M and is denoted by $P_c(M)$.

Note: Since every completely prime ideal of a TΓSS M is also a prime ideal of M, $P(M) \subseteq P_c(M)$.

Remark3.20: In a commutative TΓSS M, prime ideals of M coincide with its completely prime ideals. Hence for a commutative TΓSS M, we find that $P(M) = P_c(M)$.

Def3.21: ATΓSS M is called a prime TΓSS if the zero ideal is a prime ideal of M.

"Remark: It is to be noted here that each ternary ring ideal of a ternary ring T is also a ternary semiring ideal: however, the converse need not be true, in general. Thus if a ternary ring M is a prime ternary ring then the ternary semiring M is a prime ternary semiring."

Def3.21: A proper ideal Q of a TΓSS M is called a semiprime ideal of M if $J\Gamma J\Gamma J \subseteq Q$ implies $J \subseteq Q$ for any ideal J of M.

Note3.22: Every PI of a TΓSS M is also a SPI of M.

Th3.23:"A necessary and sufficient condition for an element 's' of a TΓSS N to belong to a semi prime ideal R of N is that $N\Gamma s\Gamma N \subseteq R$".

Pf: Assume R is a SPI of N.

If $s \in R$, then clearly $N\Gamma s\Gamma N \subseteq N\Gamma R\Gamma N \subseteq R$.

Conversely, let $N\Gamma s\Gamma N \subseteq R$.

Then $N\Gamma N\Gamma s\Gamma N\Gamma N \subseteq N\Gamma R\Gamma N \subseteq R.$

Now $<s>\Gamma<s>\Gamma<s> = (N\Gamma N\Gamma s + s\Gamma N\Gamma N + N\Gamma s\Gamma N + n\Gamma s)\Gamma$

$(N\Gamma N\Gamma s + s\Gamma N\Gamma N + N\Gamma s\Gamma N + N\Gamma N\Gamma s\Gamma N\Gamma N + n\Gamma s)\Gamma$

$(N\Gamma N\Gamma s + s\Gamma N\Gamma N + N\Gamma s\Gamma N + N\Gamma N\Gamma s\Gamma N\Gamma N + n\Gamma s) \in (N\Gamma s\Gamma N + N\Gamma N\Gamma s\Gamma N\Gamma N) \subseteq R \because$ R is semiprime, we have $<s> \subseteq R$ & $\therefore s \in R$.

The resultingTh. gives a descriptionof a semiprime ideal in a TΓSS N.

Th 3.24: Let M be a TΓSS then a proper ideal Q of M then the conditions are equivalent.

(i) Q is semiprime

(ii) sΓMΓsΓMΓs⊆Q, sΓMΓMΓsΓMΓMΓs⊆Q, sΓMΓMΓsΓMΓsΓM⊆Q&

MΓsΓMΓsΓMΓMΓs⊆Q ⇒ $s \in Q$.

(iii) $<s>\Gamma<s>\Gamma<s>\subseteq Q \Rightarrow \boldsymbol{s} \in \boldsymbol{Q}$.

***Pf:* (i)⇒ (ii)**Assume Q is a PI of M

& $s\Gamma M\Gamma s\Gamma M\Gamma s\subseteq Q$, $s\Gamma M\Gamma M\Gamma s\Gamma M\Gamma M\Gamma s\subseteq Q$,

$s\Gamma M\Gamma M\Gamma s\Gamma M\Gamma s\Gamma M\subseteq Q$ & $M\Gamma s\Gamma M\Gamma s\Gamma M\Gamma M\Gamma s\subseteq Q$.

Then $(M\Gamma s\Gamma M)\Gamma(M\Gamma s\Gamma M)\Gamma(M\Gamma s\Gamma M)=$

$M\Gamma(s\Gamma M\Gamma M\Gamma s\Gamma M\Gamma M\Gamma s)\Gamma M\subseteq M\Gamma Q\Gamma M\subseteq Q$

$(M\Gamma M\Gamma s\Gamma M\Gamma M)\Gamma(M\Gamma s\Gamma M)\Gamma(M\Gamma s\Gamma M)= (M\Gamma M\Gamma s)\Gamma(M\Gamma M\Gamma M)\Gamma(s\Gamma M\Gamma M\Gamma s\Gamma M)$

$\subseteq M\Gamma(M\Gamma s\Gamma M\Gamma s\Gamma M\Gamma M\Gamma s)\Gamma M\subseteq M\Gamma Q\Gamma M\subseteq Q$.

$(M\Gamma s\Gamma M)\Gamma(M\Gamma M\Gamma s\Gamma M\Gamma M)\Gamma(M\Gamma s\Gamma M)= M\Gamma s\Gamma(M\Gamma M\Gamma M)\Gamma(s\Gamma M\Gamma M\Gamma s\Gamma M)$

$\subseteq M\Gamma(M\Gamma s\Gamma M\Gamma s\Gamma M\Gamma M\Gamma s)\Gamma M\subseteq M\Gamma Q\Gamma M\subseteq Q$.

$(M\Gamma M\Gamma s\Gamma M\Gamma M)\Gamma(M\Gamma M\Gamma s\Gamma M\Gamma M)\Gamma(M\Gamma s\Gamma M)= (M\Gamma M\Gamma s)\Gamma M\Gamma(M\Gamma M\Gamma M)\Gamma s\Gamma(M\Gamma M\Gamma M)\Gamma s\Gamma M \subseteq M\Gamma M\Gamma(s\Gamma M\Gamma M\Gamma s\Gamma M\Gamma s\Gamma M)$

$\subseteq M\Gamma M\Gamma Q\subseteq Q$.

$(M\Gamma s\Gamma M)\Gamma(M\Gamma s\Gamma M)\Gamma(M\Gamma M\Gamma s\Gamma M\Gamma M)= M\Gamma(s\Gamma M\Gamma M)\Gamma s\Gamma(M\Gamma M\Gamma M)\Gamma(s\Gamma M\Gamma M)$

$\subseteq M\Gamma(s\Gamma M\Gamma M\Gamma s\Gamma M\Gamma s\Gamma M)\Gamma M \subseteq M\Gamma M\Gamma Q\subseteq Q$.

$(M\Gamma M\Gamma s\Gamma M\Gamma M)\Gamma(M\Gamma s\Gamma M)\Gamma(M\Gamma M\Gamma s\Gamma M\Gamma M)= (M\Gamma M\Gamma s)\Gamma(M\Gamma M\Gamma M)\Gamma s\Gamma(M\Gamma M\Gamma M)\Gamma(s\Gamma M\Gamma M) \subseteq M\Gamma M\Gamma(s\Gamma M\Gamma s\Gamma M\Gamma s)\Gamma M\Gamma M$

$\subseteq M\Gamma(M\Gamma Q\Gamma M)\Gamma M\subseteq M\Gamma Q\Gamma M\subseteq Q$.

$(M\Gamma s\Gamma M)\Gamma(M\Gamma M\Gamma s\Gamma M\Gamma M)\Gamma(M\Gamma M\Gamma s\Gamma M\Gamma M)= M\Gamma s\Gamma(M\Gamma M\Gamma M)\Gamma s\Gamma M\Gamma(M\Gamma M\Gamma M)\Gamma(s\Gamma M\Gamma M)$

$\subseteq(M\Gamma s\Gamma M\Gamma s\Gamma M\Gamma M\Gamma s)\Gamma M\Gamma M\subseteq Q\Gamma M\Gamma M\subseteq Q$.

$(M\Gamma M\Gamma s\Gamma M\Gamma M)\Gamma(M\Gamma M\Gamma s\Gamma M\Gamma M)\Gamma(M\Gamma M\Gamma s\Gamma M\Gamma M)= (M\Gamma M\Gamma s)\Gamma M\Gamma(M\Gamma M\Gamma M)\Gamma s\Gamma M\Gamma(M\Gamma M\Gamma M)\Gamma(s\Gamma M\Gamma M)$

$\subseteq M\Gamma M\Gamma(s\Gamma M\Gamma M\Gamma s\Gamma M\Gamma M\Gamma s)\Gamma M\Gamma M\subseteq M\Gamma(M\Gamma Q\Gamma M)\Gamma M\subseteq M\Gamma Q\Gamma M\subseteq Q$.

Sums of consecutive pairs are taken, we get

$(M\Gamma s\Gamma M+ M\Gamma M\Gamma s\Gamma M\Gamma M)\Gamma(M\Gamma s\Gamma M)\Gamma(M\Gamma s\Gamma M)$

$\subseteq Q+Q\subseteq Q$

$(M\Gamma s\Gamma M+ M\Gamma M\Gamma s\Gamma M\Gamma M)\Gamma(M\Gamma M\Gamma s\Gamma M\Gamma M)\Gamma(M\Gamma s\Gamma M) \subseteq Q+Q\subseteq Q$

$(M\Gamma s\Gamma M+M\Gamma M\Gamma s\Gamma M\Gamma M)\Gamma(M\Gamma s\Gamma M)\Gamma(M\Gamma M\Gamma s\Gamma M\Gamma M) \subseteq Q+Q\subseteq Q$

$(M\Gamma s\Gamma M+M\Gamma M\Gamma s\Gamma M\Gamma M)\Gamma(M\Gamma M\Gamma s\Gamma M\Gamma M)\Gamma(M\Gamma M\Gamma s\Gamma M\Gamma M) \subseteq Q+Q\subseteq Q$

for the above four relations repeat the same procedure, further we have

$(M\Gamma s\Gamma M+M\Gamma M\Gamma s\Gamma M\Gamma M)\Gamma(M\Gamma s\Gamma M+ M\Gamma M\Gamma s\Gamma M\Gamma M)\ \Gamma(M\Gamma s\Gamma M)$

$\subseteq Q+Q\subseteq Q$

$(M\Gamma s\Gamma M+M\Gamma M\Gamma s\Gamma M\Gamma M)\Gamma(M\Gamma s\Gamma M+ M\Gamma M\Gamma s\Gamma M\Gamma M)\ \Gamma(M\Gamma M\Gamma s\Gamma M\Gamma M) \subseteq Q$

$(M\Gamma s\Gamma M+M\Gamma M\Gamma s\Gamma M\Gamma M)\Gamma(M\Gamma s\Gamma M+ M\Gamma M\Gamma s\Gamma M\Gamma M)\ \Gamma(M\Gamma s\Gamma M+M\Gamma M\Gamma s\Gamma M\Gamma M$

$\subseteq Q+Q\subseteq Q$.

The above each component are ideals of M, by primeness of Q, it shows that

$(M\Gamma s\Gamma M+M\Gamma M\Gamma s\Gamma M\Gamma M) \subseteq Q$ or $(M\Gamma s\Gamma M+M\Gamma M\Gamma s\Gamma M\Gamma M)\ ) \subseteq Q$ or

$(M\Gamma s\Gamma M+M\Gamma M\Gamma s\Gamma M\Gamma M) \subseteq Q$.

Without loss of generality, suppose that $(M\Gamma s\Gamma M+M\Gamma M\Gamma s\Gamma M\Gamma M) \subseteq Q$ then

$<s>\Gamma<s>\Gamma<s>\subseteq Q$.

Q Q is a semiprime ideals of M, $<s>\subseteq Q$&thus$s \in Q$,

In the same manner if $(M\Gamma s\Gamma M+M\Gamma M\Gamma s\Gamma M\Gamma M) \subseteq Q$ then $s \in Q$,

& if $(M\Gamma s\Gamma M+M\Gamma M\Gamma s\Gamma M\Gamma M) \subseteq Q$ then $s \in Q$.

(ii)⇒ (iii)

Assume (ii)$<s>\Gamma<s>\Gamma<s>\subseteq Q$ for some $s\in M$.

$s\Gamma M\Gamma s\Gamma M\Gamma s=s\Gamma(M\Gamma s\Gamma M)\Gamma s\subseteq<s>\Gamma<s>\Gamma<s>\subseteq Q$

$s\Gamma M\Gamma M\Gamma s\Gamma M\Gamma M\Gamma s=s\Gamma(M\Gamma M\Gamma s\Gamma M\Gamma M)\Gamma s$

$\subseteq<s>\Gamma<s>\Gamma<s>\subseteq Q$

$s\Gamma M\Gamma M\Gamma s\Gamma M\Gamma s\Gamma M=s\Gamma(M\Gamma M\Gamma s)\Gamma(M\Gamma s\Gamma M) \subseteq$

$<s>\Gamma<s>\Gamma<s>\subseteq Q$

$M\Gamma s\Gamma M\Gamma s\Gamma M\Gamma M\Gamma s=(M\Gamma s\Gamma M)\Gamma s\Gamma(M\Gamma M\Gamma s) \subseteq$

$<s>\Gamma<s>\Gamma<s>\subseteq Q$

By (ii)$s \in Q$.

(iii)⇒ (i)

Suppose the condition (iii) holds and $J\Gamma J\Gamma J \subseteq Q$ for an ideal J of M.

If possible $J \not\subset Q$. Then ∃ an element $s \in J \ni s \notin J$.

Now $<s>\Gamma<s>\Gamma<s>\subseteq J\Gamma J\Gamma J \subseteq Q.$ This implies that $s \in Q$, a contradiction.

Thus $J \subseteq Q$ & hence Q is a SPI of M.

Corollary3.25: A proper ideal Q of a CTΓSS M is semiprimeiff $s\alpha s\beta s \in Q$ implies that $s \in Q$ for any element s of S.

Def3.26: A subset $D(\neq \phi)$ of a TΓSS M is so-called a p-system if for each $d \in D\,\exists$ elements s_1,s_2,s_3,s_4 of M & $\alpha,\beta,\gamma,\delta,\varepsilon \in \Gamma$ such that $d\alpha s_1\beta s_2\gamma d \in D$ or $d\alpha s_1\beta s_2\gamma d\delta s_3\eta s_4\varepsilon d \in D$ or $d\alpha s_1\beta s_2\gamma d\delta s_3\eta d\varepsilon s_4 \in D$ or $s_1\alpha d\beta s_2\gamma d\delta s_3\eta s_4\varepsilon d \in D$.

Def3.27: A proper ideal Q of a TΓSS M is called a completely semiprime ideal of M if $s\alpha s\alpha s \in Q$ implies that $s \in Q$.

Th3.28: A proper ideal Q of a TΓSS M is semiprimeiff Q^c (complement of Q) is a p-system.

Th3.29: Let D be a p-system and J be an ideal of a TΓSS M $\ni D\cap J=\phi$. Then ∃ a maximal ideal N of M containing J $\ni D\cap J=\phi$. Moreover, N is also a semiprime ideal of M.

Th: 3.30 A proper ideal Q of a TΓSS M is a minimal semiprime ideal belonging to the ideal J iff its complement Q^c is a maximal p-system such that $Q^c\cap J=\phi$.

Corollary3.31: There exist a unique minimal semiprime ideal belonging to an ideal J of M, namely, the intersection of all semiprime ideals containing J.

Corollary3.32: There exists a unique maximal p-system which does not intersect any ideal J of a TΓSS M.

Theorem3.33: A proper ideal Q of a TΓSS M is completely semiprime iff for each pair with even sum of non-negative integers m & n, $S^m\Gamma(s\alpha s\alpha s)\Gamma S^n \subseteq Q$ implies $s \in Q.$

Remark: Every m-system is a p-system and it is clear that the union of p-systems is again a p-system but not conversely.

Th3.34: A subset $D(\neq \phi)$ of a TΓSS M is p-system iff it is the union of m-systems.

Pf: Suppose D is a union of m-systems. Then D is a p-system, since any m-system is also a p-system & the union of p-systems is a p-system, by above Remark.

Conversely, Let D be a p-system and $s_0 \in D$. Now we set $C_0=\{s_0\}$ $C_{k+1}=\{s:s=a\alpha t_1\beta b\delta t_2\varepsilon c$ or $s=a\alpha t_1\beta t_2\gamma t_3\delta t_4\varepsilon c$ or $s=a\alpha t_1\beta t_2\gamma t_3\delta c\varepsilon t_4$ or $s=t_1\alpha a\beta t_2\gamma t_3\delta t_4\varepsilon c$

For some $a,b,c \in \bigcup_{j=0}^{k} C_j; t_1,t_2,t_3,t_4 \in M, \alpha,\beta,\gamma,\delta,\varepsilon \in \Gamma\}$.

Next we show that $C=\bigcup_{k=0}^{\infty}C_k$ is a subset of D.

To show this we shall have to show that if $s\in C$, then $s=s_0$ or $s=s_0\alpha r_1\beta s_0\gamma r_2\varepsilon s_0$ or $s=s_0\alpha r_1\beta r_2\gamma s_0\delta r_3\varepsilon r_4\eta s_0$ or $s=s_0\alpha r_1\beta r_2\gamma s_0\delta r_3\varepsilon s_0\eta r_4$ or $s=r_1\alpha s_0\beta r_2\gamma s_0\delta r_3\varepsilon r_4\eta s_0$ for some $r_1,r_2,r_3,r_4\in M,\alpha,\beta,\gamma,\delta,\varepsilon,\eta\in\Gamma$.

We shall show this by using induction on k. If $s\in C_0$, then clearly; $s=s_0\in D$.

Suppose the result is true for all j≤k. Now if $s\in C_{k+1}$, then $\exists\, a,b,c\in\bigcup_{j=0}^{k}C_j\ni s=a\alpha t_1\beta b\gamma t_2\delta c$ or $s=a\alpha t_1\beta t_2\gamma b\delta t_3\varepsilon t_4\eta c$ or $s=a\alpha t_1\beta t_2\gamma b\delta t_3\varepsilon c\eta t_4$ or $s=t_1\alpha a\beta t_2\gamma b\delta t_3\varepsilon t_4\eta c$ for some $t_1,t_2,t_3,t_4\in M,\alpha,\beta,\gamma,\delta,\varepsilon,\eta\in\Gamma$.

Now by hypothesis of induction, we have $a=s_0\alpha u_1\beta s_0\gamma u_2\delta s_0$ or $a=s_0\alpha u_1\beta u_2\gamma s_0\delta u_3\varepsilon u_4\eta s_0$ or $a=s_0\alpha u_1\beta u_2\gamma s_0\delta u_3\varepsilon s_0\eta u_4$ or $a=u_1\alpha s_0\beta u_2\gamma s_0\delta u_3\varepsilon u_4\eta s_0$

for some $u_1,u_2,u_3,u_4\in M,\ \alpha,\beta,\gamma,\delta,\varepsilon,\eta\in\Gamma$

$b=s_0\alpha v_1\beta s_0\gamma v_2\delta s_0$ or $b=s_0\alpha v_1\beta v_2\gamma s_0\delta v_3\varepsilon v_4\eta s_0$ or $b=s_0\alpha v_1\beta v_2\gamma s_0\delta v_3\varepsilon s_0\eta v_4$ or $b=v_1\alpha s_0\beta v_2\gamma s_0\delta v_3\varepsilon v_4\eta s_0$ for some $v_1,v_2,v_3,v_4\in M,\ \alpha,\beta,\gamma,\delta,\varepsilon,\eta\in\Gamma$ $c=s_0\alpha w_1\beta s_0\gamma w_2\delta s_0$ or $c=s_0\alpha w_1\beta w_2\gamma s_0\delta w_3\varepsilon w_4\eta s_0$ or $c=s_0\alpha w_1\beta w_2\gamma s_0\delta w_3\varepsilon s_0\eta w_4$ or $c=w_1\alpha s_0\beta w_2\gamma s_0\delta w_3\varepsilon w_4\eta s_0$ for some $w_1,w_2,w_3,w_4\in M,\ \alpha,\beta,\gamma,\delta,\varepsilon,\eta\in\Gamma$.

Thus by considering each of the 4^4 cases the result follows because if we consider $s=a\alpha t_1\beta b\delta t_2\varepsilon c$, $a=s_0\alpha u_1\beta u_2\gamma s_0\delta u_3\varepsilon u_4\eta s_0$, $b=v_1\alpha s_0\beta v_2\gamma s_0\delta v_3\varepsilon v_4\eta s_0$ $c=s_0\alpha w_1\beta w_2\gamma s_0\delta w_3\varepsilon s_0\eta w_4$ then

$s=(s_0\alpha u_1\beta u_2\gamma s_0\delta u_3\varepsilon u_4\eta s_0)\alpha t_1\beta(v_1\alpha s_0\beta v_2\gamma s_0\delta v_3\varepsilon v_4\eta s_0)\delta t_2\varepsilon(s_0\alpha w_1\beta w_2\gamma s_0\delta w_3\varepsilon s_0\eta w_4)$

$=s_0\alpha(u_1\beta u_2\gamma s_0\delta u_3\varepsilon u_4\eta s_0\alpha t_1\beta v_1\alpha s_0)\beta(v_2\gamma s_0\delta v_3\varepsilon v_4\eta s_0\delta t_2\varepsilon s_0\alpha w_1\beta w_2)\gamma s_0\delta w_3\varepsilon s_0\eta w_4$

$=s_0\alpha r_1\beta r_2\gamma s_0\delta r_3\varepsilon s_0\eta r_4$

Where $r_1=u_1\beta u_2\gamma s_0\delta u_3\varepsilon u_4\eta s_0\alpha t_1\beta v_1\alpha s_0$, $r_2=v_2\gamma s_0\delta v_3\varepsilon v_4\eta s_0\delta t_2\varepsilon s_0\alpha w_1\beta w_2$, $r_3=w_3,\ \&r_4=w_4$

Similarly, for the other cases we find that

$s=s_0\alpha r_1\beta s_0\gamma r_2\varepsilon s_0$ or $s=s_0\alpha r_1\beta r_2\gamma s_0\delta r_3\varepsilon r_4\eta s_0$ or $s=s_0\alpha r_1\beta r_2\gamma s_0\delta r_3\varepsilon s_0\eta r_4$ or $s=r_1\alpha s_0\beta r_2\gamma s_0\delta r_3\varepsilon r_4\eta s_0$ for some $r_1,r_2,r_3,r_4\in M,\ \alpha,\beta,\gamma,\delta,\varepsilon,\eta\in\Gamma$ and

Hence $C=\bigcup_{k=0}^{\infty}C_k$ is a subset of D.

Now it remains show that $C=\bigcup_{k=0}^{\infty}C_k$ is an m-system.

To show this let a, b, c are belongs to C.

Then $a,b,c\in C_k$ for some natural number k.

Consequently, $s=a\alpha t_1\beta b\gamma t_2\delta c\in C_{k+1}\subseteq C$ or $s=a\alpha t_1\beta t_2\gamma b\delta t_3\varepsilon t_4\eta c\in C_{k+1}\subseteq C$ or $s=a\alpha t_1\beta t_2\gamma b\delta t_3\varepsilon c\eta t_4\in C_{k+1}\subseteq C$ or $s=t_1\alpha a\beta t_2\gamma b\delta t_3\varepsilon t_4\eta c$ for some $t_1,t_2,t_3,t_4\in M\ \ \alpha,\beta,\gamma,\delta,\varepsilon,\eta\in\Gamma$.

Thus C is an “m-system”.

Theorem3.35: A proper ideal Q of a TΓSS M is semiprimeiff if the complement Q^c is the union of m-system of M.

Def3.36: The prime radical P(J) of the ideal J in a TΓSS M is the intersection of all prime ideals in M which contain J.

We have the following characterization of prime radical of an ideal in a TΓSS M.

Th3.37: For a proper ideal J of TΓSS M,

P(J)={r∈M: every m-system in M which contains r has a non-empty intersection with J}.

Pf: Suppose $P^c(J)$ ={r∈M: every m-system in M which contains r has a non-empty intersection with J}.

Let $a \notin P^c$.

∃an m-system A in M ∋ $a \in A \& A \cap J = \phi$.

Then by Th.3.8, ∃ a PI P of M containing J & $A \cap P = \phi$. This implies that $a \notin P$ & hence $a \notin P(J)$ consequently, $P(J) \subseteq P^c(J)$.

Again, let $b \notin P(J)$.

Then ∃ a prime ideal P of M ∋ $b \notin P$ i.e. $b \in P^c$, the complement of P.

Now by Th:3.6 , P^c is an m-system of M.

since P contains J, $P^c \cap J = \phi$.

Thus ∃ an m-system P^c in M which contains b but has an empty intersection with J.

So, $b \notin P^c(J)$ & hence $P^c(J) \subseteq P(J)$.

Thus it follows that $P^c(J) = P(J)$ and this completes the proof of the theorem.

Th3.38: An ideal J of a TΓSS M is semiprime iff P(J)=J.

Pf: Let J be a semiprime ideal of a TΓSS M. Then by Th.3.8 the complement J^c is a p-system.

Hence by Th.3.34 $I^c = \bigcup_{i \in \Delta} B_i$, where each B_i is an m-system contained in I^c.

$\because I \cap B_i = \phi$ for each i∈Δ, it follows from Th.,3.8 that ∃ a maximal ideal P_i containing J such that $B_i \cap P_i = \phi$ for each i∈Δ, and each P_i is a prime ideal of M.

Thus $J \subseteq \bigcap_{i \in \Delta'} P_i \subseteq \bigcap_{i \in \Delta} P_i \subseteq \bigcap_{i \in \Delta} B^c_i = (\bigcup_{i \in \Delta} B_i)^c = (J^c)^c = J$.

Consequently, $J = \bigcap_{i \in \Delta'} P_i = P(J)$, where Δ' is the set of all prime ideals of M containing J.

Conversely, suppose that J=P(J). Then $J = \cap \{P : P$ is a prime ideal of M containing J}.

$\Rightarrow J^c = \cup \{P^c : P$ is a prime ideal of M containing J}.

Since P is a prime ideal of M, by Th: 3.8, it follows that P^c is an m-system and hence by using Th: 3.6we get J^c is a p-system. Consequently, J is a SPI of M, by using Th: 3.8

Corollary: If J is an ideal of a TΓSS M, then P(J) is the smallest semiprime ideal of M contains J.

Def3.39: A proper ideal J of TΓSS M is known to be weakly irreducible if for ideals K, L, N of M, $K \cap L \cap N = J \Rightarrow$ J=K or J=L or J=N.

The termwe simply use irreducible to mean weakly irreducible.

Def3.40:A proper ideal J of TΓSS M is said to be weakly irreducible if for ideals K, L, N of M, $K \cap L \cap N \subseteq J \Rightarrow K \subseteq J$ or $L \subseteq J$ or $N \subseteq J$.

Note3.41:“It is be noted here that a strongly irreducible ideal of a TΓSS M, is an irreducible ideal of M”.

Def3.42: A non-empty subset A of a TΓSS M, is named an “i-system” if $l, m, n \in A$ implies $< l > \cap < m > \cap < n > \cap A \neq \phi$.

Th3.43: The conditions followed in a TΓSS M, are equivalent:

(i) J is a strongly irreducible ideal of M

(ii) If for $l,m,n \in M$; $<l> \cap <m> \cap <n> \subseteq J$ then $l \in J$ or $m \in J$ or $n \in J$

(iii) The complement of J ie., J^c is an i-system.

Pf: $(i) \Rightarrow (ii)$ This is a consequence of the Def. 2.2.24

$(ii) \Rightarrow (iii)$ If possible let $l,m,n \in J^c$ & $<l> \cap <m> \cap <n> \cap J^c = \phi.$

Then $<l> \cap <m> \cap <n> \cap J^c = \phi$ implies that $<l> \cap <m> \cap <n> \subseteq J$ and hence by using (ii), we have $l \in J$ or $m \in J$ or $n \in J$, which is a contradiction. Consequently $<l> \cap <m> \cap <n> \cap J^c \neq \phi$ & hence J^c is an i-system.

$(iii) \Rightarrow (i)$ Let E, F, G be three ideals of M $\ni E \not\subset J, F \not\subset J$ & $G \not\subset J$.

Then $\exists\, l \in E - J, m \in F - J$ & $n \in G - J.$

Now from (iii), it follows that $<l> \cap <m> \cap <n> \cap J^c \neq \phi$

ie., there exists an element $o \in (<l> \cap <m> \cap <n>) - J.$

$\Rightarrow o \in E \cap F \cap G$ & $o \notin J.$

$\therefore E \cap F \cap G \not\subset J$. $\Rightarrow$ J is strongly irreducible.

Lemma3.45: Let "l" be a non-zero element of a TΓSS M and let J be a proper ideal of M not containing 'l'. Then there exists an irreducible H of M containing J and not containing "l".

Th3.46: Any proper ideal J of a TΓSS M is the intersection of all irreducible ideals containing it.

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