ISSN: 0974-5823 Vol. 7 No. 2 February, 2022
International Journal of Mechanical Engineering

Infra Soft α – Open (Closed) sets on Infra Soft Topological Space

M John Peter[1,2,a] and R Manoharan[3,b]

[1]Assistant Professor, Panimalar Institute of Technology, Chennai-600123, India.

[2]Research Scholar, Sathyabama Institute of Science and Technology, Chennai-600119, India.

[3]Associate Professor, Sathyabama Institute of Science and Technology, Chennai-600119, India.

Abstract: In this research paper, we elucidate a new soft open (closed) set called infra soft α-open (closed) set & establish some fundamental features of this soft open (closed) set. The connection between infra soft α-open (closed) set and other soft topological open (closed) sets are investigated. The findings mentioned in this study are preliminary and serve as an introduction to more advanced research in theoretical and practical areas.

AMS Subject Classifications: 54A40, 54C08, 54C10

Keywords: Soft Open (Closed) Set, Soft Topology, Infra Soft Topology, Infra soft α - open (closed) set.

1. Introduction

Molodtsov [1] presented soft sets, a novel mathematical method for dealing with ambiguity, in 1999. He explained how it relates to fuzzy sets and demonstrated how it can be used in various fields. Many scholars and researchers have since researched soft set applications in a variety of areas, such as computer science, decision-making problems and medicine. Maji et al. [6] started researching the fundamental principles & conceptions of soft set theory in 2003. They looked at the union and intersection operators, as well as the difference and complement of two soft sets. However, significant flaws in their definitions were discovered by Ali et al., leading to the reformulation of the majority of these definitions and the presentation of new types of them. Ali et al. [17] developed new operators & procedures to conserve many features and conclusions in soft set theory.

Soft open sets were used to define soft topological spaces (STS) in 2011 by Cagman et al. [11] and Shabir et al. [5] and they described main concepts on STS. Many investigations on topological notions in soft topologies, such as soft connectedness, soft compactness, soft basis, soft separation axioms, soft bioperators and generalized soft open sets(SOS), have been conducted since then. [7-10] Many SOS (Soft closed i.e SCS) were described over some specified universe U with parameter set E, then they were extended to related soft topologies and studied their properties. Soft topology has been extended to a number of structures with weaker or stronger topological properties, one of which is the infra soft topological space (ISTS) [12], in which a soft space is closed under finite soft intersection and a soft null set is a soft space member. Many topological features are retained in ISTSs, additionally the easy building of instances that explain the connections between the topological concepts, which is one of the grounds for continuing to investigate infra soft topological (ISTS) structure. The notions of infra soft connectedness, infra soft local connectedness and infra soft compactness were researched in this area in [12-18].

In this article, we elucidate infra soft α-interior (ISα-Int), infra soft α-closure (ISα-Cl), infra soft α-open sets (ISα-OS), infra soft α-closed sets (ISα-CS) and establish someof their basic characterizes. This means that the ISTSs are flexible areato discuss the topological ideas and investigate the relationshipsbetween them. On ISTSs, many aspects of soft topological spaces are still valid, and initiating cases that prove specific links between certain topological notions is easier.

2. Preliminaries

In this chapter, we are going to recollect few fundamental definitions and propositions which are essential to the development of this research article.

Firstly, we look back on fundamental definitions and basic operations on soft topology which are described over some standard universe and fixed parameter set

Definition 2.1:[1] Let D be a group of parameter and U be a given universe. Let C be a nonempty subset of D, and P(U) be the collection of sub sets of U. A soft set over U with D is a pair (Γ, D), where Γ is a function provided by $\Gamma: C \rightarrow P(U)$. In different way, a soft set over U is a parameterized system of soft subsets. (p) can be thought of as the set of p-approximate element of soft set.

Definition 2.2:[5] i) A "Null soft set" is known as soft set (Γ, C) or Γ_C over universe U with parameter set D, expressed by Φ_C; if, $e \in C$, $\Gamma_C(e) = \emptyset$. ii) An "Absolute soft set", represented by (Γ, C) or Γ_C over universe U with parameter set D, if $p \in C$, $\Gamma_C(e) = U$. iii) Let us define a soft set η_O as the "union of two soft sub sets" ξ_M and χ_N on the universal set U , where M, N and O are subsets of the parameter set D, $M \cup N=O$ and for all $p \in O$. The components of the soft set η_O is described as follows, $\eta(p) = \xi(p)$ if $p \in M-N$, $\eta(p) =\chi(p)$ if $p \in N - M$ and $\eta(p) = \xi(p)\sqcup\chi(p)$ if $p \in M \cap N$, we can express $\eta_O = \xi_M \sqcup \chi_N$. iv) Let us define a soft set η_O as the "intersection of two soft sub sets" of ξ_M and χ_N on the universal set U with parameter set D, where $M \cap N= O$ and for all $p \in O$. The members of the soft set η_O is noted as follows, $\eta(p) = \xi(p) \sqcap \chi(p)$ if $O= N\cap M$. We can express $\eta_O= \xi_M \sqcap \chi_N$.

Definition 2.3 [5]: Let χ_D be a soft set over universe U with parameter set D. $x_e \in U_D$, we called that, (a) $x_e \in \chi_D$ (element x_e is completely member of χ_E) if $x_e \in \chi(p)$ for every $p \in D$

(b) $x_e \notin \chi_D$ (element x_e doesn't moderately member of χ_D) if $x_e \notin \chi(p)$ for some $p \in D$

(c) $x_e \Subset \chi_D$ (element x_e partly member of χ_E) if $x_e \in \chi(p)$ for some $p \in D$

(d) $x_e \not\Subset \chi_D$ (element x_e doesn't completely member of χ_D) if $x_e \notin \chi(p)$ for every $p \in D$.

After established soft sets definitions and operations, then recollect definition of soft topology on SOS which is defined over some universe U and parameter set E.

Definition 2.4: [5] Let τ be the system of soft open sets(SOS) over universe U with parameter E, then τ is referred as "soft topology" on $SS(U_E)$, if it meets the proceeding axioms, (1) Φ_E and U_E member of τ, where $\Phi(p) = \Phi_E$, and $U(p) = U_E$, $\forall p \in E$ (2) If $\Gamma_{E1},\Gamma_{E2}, \Gamma_{E3,\ldots} \Gamma_{En} \in \tau$, then $\Gamma_{E1}\sqcup\Gamma_{E2}\sqcup\Gamma_{E3,\ldots}\sqcup\Gamma_{En} \in \tau$, (3) If $\Gamma_{E1}, \Gamma_{E2,}, \Gamma_{E3,\ldots} \Gamma_{En} \in \tau$, then $\Gamma_{E1}\sqcap\Gamma_{E2}\sqcap\Gamma_{E3,\ldots}\sqcap\Gamma_{En} \in \tau$

The system (U, E, τ) is referred as STS then the elements of τ will be SOS in $SS(U_E)$. The respective 'complement of a soft set (Γ, C)' is noted by $(\Gamma, C)^C$ & is described by $(\Gamma,C)^C = (\Gamma^C, C)$, here $\Gamma^C: C \rightarrow P(U)$ is a function referred by $\Gamma^C(e) = U-\Gamma(p)$ for all $p \in C$. A soft set Γ_E on U is called SCS in U_E if its complement Γ_E^C is soft open subset in τ^c.

Soft topology has been extended to a variety of structures with weaker or stronger topological properties, one of which is ISTS. Many soft topological properties are valid for ISTS also.

Definition 2.5: [12] The family ϑ of soft sets over universe U with parameter D is said to be an ISTSon U if it is closed under finite soft intersection and the null soft set is a member of ϑ.

The triplet (U, ϑ, D) is called an ISTS. Each member of ϑ is called an infra soft open set (ISOS), and its corresponding complement is said to be infra soft closed set (ISCS).

Remark 2.6: [12] It will be scrutinized that the intersection of any collection of ISTSs is ISTS. However, the union of two ISTSs is not required to be an ISTS

Definition 2.7: [8] A soft set (λ, D) on STS (U, τ, D) is said to be soft α-open set if $(\lambda, D) \sqsubseteq int(cl(int(\lambda, D)))$. The relative complement of soft α-open set (Sα-OS) is called soft α-closed set (Sα-CS).

Definition 2.8 [8] Let I be a non-null family of soft sets over a universal set U with parameter D, then $I \sqsubseteq SS(U_D)$ is said to be a soft ideal if

(1) $(\lambda, D) \in I$ & $(\chi, D) \in I \Rightarrow (\lambda, D) \sqcup (\chi, D) \in I$,

(2) $(\lambda, D) \in I$ & $(\chi, D) \sqsubseteq (\lambda, D) \Rightarrow (\chi, D) \in I$,

i.e. I is closed under finite soft unions of soft subsets.

Definition 2.9: [8] Let (U, τ, D) be a STS and I be a soft ideal over universe U with the parameter set D. Then $(\xi ,D)^* (I , \tau)$ or $(\xi_D) = \sqcup \{x_e \in SS(U_D) : O_{xe}\sqcap (\xi, D) \notin I$ for all $O_{xe} \in \tau\}$ is said to be soft local function of (ξ, D) with respect to I and τ, where O_{xe} is a τ -soft open set containing x_e.

Theorem 2.10 [8] Let (U, τ, D) be a STS and I be a soft ideal over U with parameters D. Then the soft closure operator cl*: $SS(U_D) \rightarrow SS(U_D)$ described by: $Cl^*(\xi, D) = (\xi, D) \sqcup (\xi, D)^*$ convinces Kuratwski's axioms.

Definition 2.11: [18] A soft set $(\xi, C) \in SS(U_D)$ is said to be:

a) Soft Semi*open or infra soft semi open set if $(\xi, C) \sqsubseteq Cl^*(Int(\xi, C))$.

b) Soft Semi*closed or infra soft semi closed set if $Int^*(Cl(\xi, C)) \sqsubseteq (\xi, C)$.

c) Soft Pre* open (supra soft -preopen) if $(\xi, C) \sqsubseteq Int^*(Cl(\xi, C))$ and Soft Pre* closed (supra soft-preclosed) if $Cl^*(Int(\xi, C))) \sqsubseteq (\xi, C)$.

d) Soft α*-open (supra soft-α-open) if $(\xi, C) \sqsubseteq Int^*(Cl(Int^*(\xi, C)))$ and soft α*- closed (supra soft-α-closed) if $Cl^*(Int(Cl^*(\xi, C))) \sqsubseteq (\xi, C)$

The collection of all infra soft semi open set, infra soft semi closed set, supra soft -preopen, supra soft-preclosed, supra soft-α-open and supra soft-α-closed sets in U_D shall be noted as ISSOS(U_D), ISSCS(U_D), SSPOS(U_D), SSPCS(U_D), SSα -OS(U_D) and SSα − CS(U_D), respectively.

Definition 2.12: [16] Let (λ, D) any soft set. Then,

(i) $Cl^*(\lambda, D) = \sqcap \{(\lambda, M): (\lambda, M) \sqsupset (\lambda, D); (\lambda, M)$ is a generalized SCS of $U_D\}$ is said to be closure*.

(ii) $Int^*(\lambda, D) = \sqcup \{(\lambda, C) : (\lambda, C) \sqsubseteq (\lambda, D), (\lambda, C)$ is a generalized SOS of $U_D\}$ is said to be Interior*.

Lemma 2.13: [16] Let (λ, C) be a soft set. Then, i) $(\lambda, C) \sqsubseteq Cl^*(\lambda, C) \sqsubseteq Cl(\lambda, C)$.

ii) $Int(\lambda, C) \sqsubseteq Int^*(\lambda, C) \sqsubseteq (\lambda, C)$.

3. Some Properties on Infra Soft α -Open (Closed) Sets

In this chapter, we are going to define infra soft α –interior (ISα-Int), infra soft α-closure (ISα-Cl) with some examples and also investigate many of their basic properties. We instigate the idea of ISα-OSs which constitutes a class of generalizations of ISOSs. We give some characterizations of ISα-OSs and ISα-CSs and initiate some properties. We further show that this soft class is closed under arbitrary unions and identify the conditions under which it is closed under finite intersection.

Definition 3.1: A soft subset (λ, M) of universal set U with parameter M is said to be ISα-OS [ISα-CS] set if $(\lambda, M) \sqsubseteq Int(Cl^*(Int(\lambda, M)))$ $[Cl(Int^*(Cl(\lambda, M))) \sqsubseteq (\lambda, M)]$. The class of all IS$\alpha$-OS (IS$\alpha$-CS) sets in U_M will be denoted as ISα-OS(U_M) [ISα- CS(U_M)].

Definition 3.2: Let us consider some soft set (λ, B) & (λ, M), then we may define some soft closure and soft interior over universal set U with parameter D in the following way

- ISα-Cl(λ, M) = ⊓{(λ, B) : (λ,B) ⊒ (λ, M), (λ, B) is an ISα-CS of U is called an ISα-Cl}.
- ISα-Int(λ, M) = ⊔{(λ, B) : (λ, B) ⊑ (λ, M), (λ, B) is an ISα-OS in U iscalled an ISα-Int}
- ISs-Cl(λ, M) = ⊓{(λ, B) : (λ, B) ⊒ (λ, M), (λ, B) is an IS-SCS of U iscalled an infra soft-semi-closure}
- ISs-Int(λ, M) = ⊔{(λ, B) : (λ, B) ⊑ (λ, M), (λ, B) is an IS-SOS in U iscalled an infra soft-semi-interior}.

Theorem 3.3: A soft set (ξ, C) ∈ ISα-OS(U_D) iff (if and only if) ∃ a SOS (ξ, C) such that $(\xi, D) \sqsubseteq (\xi, C) \sqsubseteq Int(Cl^*(\xi, D))$.

Proof: If (ξ, C) ∈ ISα-OS(U_D), then $(\xi, C) \sqsubseteq Int(Cl^*(Int(\xi, C)))$, Put (ξ, D) = Int(ξ, C), then (ξ ,D) is SOS and$(\xi, D) \sqsubseteq (\xi, C) \sqsubseteq Int(Cl^*(\xi, D))$. In other way, Let (ξ, D) be an SOS such that $(\xi, D) \sqsubseteq (\xi, C) \sqsubseteq Int(Cl^*(\xi, D))$, ⇒ $Int(Cl^*(\xi, D)) \sqsubseteq Int(Cl^*(Int(\xi, C)))$, then $(\xi, C) \sqsubseteq Int(Cl^*(Int(\xi, C)))$.

Theorem 3.4: A soft set (ξ, D) ∈ ISα-CS(U_D) iff ∃ a SCS (ξ, C) such that $Cl(Int^*(\xi, C)) \sqsubseteq (\xi, D) \sqsubseteq (\xi, C)$.

Proof: If (ξ, D) ∈ ISα-CS(U_D), then $Cl(Int^*(Cl(\xi ,D))) \sqsubseteq (\xi, D)$. Put (ξ, A) = Cl(ξ, D), then(ξ, C) is a SCS and $Cl(Int^*(\xi, C)) \sqsubseteq (\xi, D) \sqsubseteq (\xi, C)$.

In other way, Let (ξ, C) be a SCS such that $Cl(Int^*(\xi, C)) \sqsubseteq (\xi, D) \sqsubseteq (\xi, C)$, this indicates that $Cl(Int^*(Cl(\xi, D))) \sqsubseteq Cl(Int^*(\xi ,C))$, then $Cl(Int^*(\xi, D)) \sqsubseteq Cl(\xi, D)$.

Theorem 3.5: Let (ξ, M) be a soft set over universe U and parameter set M. Then, the proceeding properties are true: i) ISs-Int(ξ, M) = (ξ, M) ⊓ Cl*(Int(ξ, M)) ii) ISs-Cl(ξ, M) = (ξ, M)⊔ Int*(Cl(ξ, M)).

Proof: (i) Using definition that ISs-Int is infra soft-semiopen (IS-SOS),

then ISs-Int(ξ, M) ⊑ Cl*(Int(ISs-Int(ξ, M))) ⊑ Cl*(Int(ξ, M)).

So, ISs-Int(ξ, M) ⊑ (ξ, M) ⊑ Cl*(Int(ξ, M)) → (a)

We have Int(ξ, M) ⊑ (ξ, M) ⊓ Cl*(Int(ξ, M)) ⊑ Cl*(Int(ξ, M)). by definition 3.2 (ξ, M) ⊓

Cl*(Int(ξ, M)) is an IS-SOS and (ξ, M) ⊓ Cl*(Int(ξ, M) ⊑ (ξ, M), then (ξ, M) ⊓ Cl*(Int(ξ, M)) ⊑ ISs-Int(ξ, M) → (b)

From (a) and (b), we get ISs-Int(ξ, M) = (ξ ,M) ⊓ Cl*(Int(ξ, M)).

ii) Similar to the above proof (i)

Corollary 3.6: Let (ξ, M) be a soft set over universe U with parameter set M. Then, the proceeding assertions are true:

(a) If (ξ, M) is a generalized SCS, then ISs-Int(ξ, M) = Cl*(Int(ξ, M)).

(b) If (ξ, M) is a generalized SOS, then ISs-Cl(ξ, M) = Int*(Cl(ξ, M)).

Proof: a) By using definition, Int*$(\xi, M) \sqsubseteq$ Int*(Cl(ξ, M)) but Int*$(\xi, M) = (\xi, M)$, this implies that $(\xi, M) \sqsubseteq$ Int*(Cl(ξ, M)), then ISs-Cl(ξ, M) = Int*(Cl(ξ, M)).

b) Similar to the proof a)

Theorem 3.7: For any soft subset (ξ, D) of a soft space SS(U_D), the proceeding implication

(a) $\Rightarrow$(b)$\Rightarrow$(c)$\Rightarrow$(d) hold:

(a) $(\xi, D) \in$ ISα-CS(U_D),

(b) Cl*(Int(ξ, C) $\sqsubseteq (\xi, D)$; for soft closed set (ξ, C),

(c) ISs-Int(ξ, C) $\sqsubseteq (\xi, D) \sqsubseteq (\xi, C)$; for soft closed set (ξ, C),

(d) ISs-Int(cl(ξ, D)) $\sqsubseteq (\xi, D)$.

Proof: We may prove this theorem by using Lemma 2.13, Definition 3.1, Theorem3.5 and Corollary3.6

Theorem 3.8: Let us consider the soft subset (ξ, D) of a soft space SS(U_D), the proceeding statements are hold:

(i) If $(\xi, D) \sqsubseteq (\xi, C) \sqsubseteq$ Int(Cl*(ξ, D)) and $(\xi, D) \in$ ISα-OS(U_D), then$(\xi, C) \in$ ISα-OS(U_D).

(ii) If Cl(Int*(ξ, D)) $\sqsubseteq (\xi, C) \sqsubseteq (\xi, D)$ and $(\xi, D) \in$ ISα-CS(U_D), then $(\xi, C) \in$ ISα-CS(U_D).

Proof: (i) Let $(\xi, D) \in$ ISα-OS(U_D), then $\exists$ (ξ, B) an SOS such that $(\xi, B) \sqsubseteq (\xi, D) \sqsubseteq$ Int(Cl*(ξ, B)), this implies that $(\xi, B) \sqsubseteq (\xi, C)$ and $(\xi, D) \sqsubseteq$ Int(Cl*(ξ, B)). Therefore, Int(Cl*(ξ, D)) $\sqsubseteq$ Int(Cl*(ξ, B)) and $(\xi, B) \sqsubseteq (\xi, C) \sqsubseteq$ Int(Cl*(ξ, B)), then $(\xi, C) \in$ ISα-OS(U_D)

(ii) Same as the proof (i)

Proposition 3.9: Let (ξ, D) & (ξ, C) be two soft sets in U_E & $(\xi, D) \sqsubseteq (\xi, C)$. Then, the proceeding statements are true

1) ISα-Int(ξ, D) is the largest ISα-OS contained in (ξ, D).

2) ISα-Int$(\xi, D) \sqsubseteq (\xi, D)$.

3) ISα-Int$(\xi, D) \sqsubseteq$ ISα-Int(ξ, C).

4) ISα-Int(ISα-Int(ξ, D)) = ISα-Int(ξ, D)

5) $(\xi, D) \in$ ISα-OS(U_D) $\Leftrightarrow$ ISα-Int$(\xi, D) = (\xi ,D)$.

Proposition 3.10 Let (ξ, D) & (ξ, C) be two soft sets in SS(U_D) and $(\xi, D) \sqsubseteq (\xi, C)$. Then the proceeding statements are true:

1) ISα-Cl(ξ, D) is the smallest ISα-CS containing (ξ, D).

2) $(\xi, D) \sqsubseteq$ ISα-Cl(ξ, D) .

3) ISα-Cl$(\xi, D) \sqsubseteq$ ISα-Cl(ξ, C).

4) ISα-Cl(ISα-Cl(ξ, D)) = ISα-Cl(ξ, D).

5) $(\xi, D) \in$ ISα-Cl(U_D) $\Leftrightarrow$ ISα-Cl$(\xi, D) = (\xi, D)$.

Theorem 3.11: Let (ξ, D) be a soft set of SS(U_D). Then, the proceeding assertions are true:

i) (ISα-Int(ξ, D))^C = ISα-Cl(ξ, D).

ii) (ISα- Cl(ξ, D))^C = ISα-Int(ξ, D)

iii) ISα-Int(ξ, D) $\sqsubseteq$ $(\xi, D) \sqcap$ Int(Cl*(Int(ξ, D))).

iv) ISα- Cl(ξ, D) $\sqsupseteq$ $(\xi, D) \sqcup$ Cl(Int*(Cl(ξ, D)))

Proof: (i) (ISα-Int(ξ, D))^C = $\sqcup$ $\{(\xi , C) : (\xi, C) \subseteq (\xi, D), (\xi, C)$ is an ISα-OS of $U_D\}^C$

= ISα-Cl(ξ, D)

(ii) Proof is trivial.

(iii) Proof is trivial.

(iv) Since $(\xi, D) \sqsubseteq$ ISα-Cl(ξ, D) and ISα-Cl(ξ, D) is an ISα-CS.

Hence, Cl(Int*(Cl(ISα-Cl(ξ, D)))) $\sqsubseteq$ ISα-Cl(ξ, D). Then, ISα-Cl(ξ, D) $\sqsupseteq$ $(\xi, D) \sqcup$ Cl(Int*(Cl(ξ, D)))

Corollary 3.12: Let (ξ, D) be a soft set of SS(U_D) . Then, the proceeding assertions are true:

a) If (ξ, D) is a SOS, then ISα-Int$(\xi, D) \sqsubseteq$ Int(Cl*(Int(ξ, D))).

b) If (ξ, D) is a SCS, then ISα-Cl(ξ, D) $\sqsupseteq$ Cl(Int*(Cl(ξ, D))).

Theorem 3.13: (a) The arbitrary union of an ISα-OS is an ISα-OS. (b) The arbitrary intersection of an ISα-CS is an ISα-CS.

Proof: (a) Let $\{(\xi, D)_i\}$ be family of an ISα-OS . Then, for every i, $(\xi, D)_i \sqsubseteq Int(Cl^*(Int(\xi, D)_i))$ and $\sqcup(\xi, D)_i \sqsubseteq \sqcup (Int(Cl^*(Int(\xi, D)_i))) \sqsubseteq Int(Cl^*(Int(\sqcup(\xi, D)_i)))$. Hence $(\xi, D)_i$ is an ISα-OS.

(b) It is trivial.

Theorem 3.14: Let (λ, M) be a soft set over the universes U parameter M. Then, $Int^*(\lambda, M) \sqsubseteq$ ISα-Int$(\lambda, M) \sqsubseteq (\lambda, M) \sqsubseteq$ ISα-Cl$(\lambda, M) \sqsubseteq Cl^*(\lambda, M)$.

Proof: We know that $Int^*(\lambda, M) \sqsubseteq (\lambda, M)$, this continues that ISα-Int$(Int^*(\lambda, M)) \sqsubseteq$ ISα-Int(λ, M). Then, ISα-Int$(Int^*(\lambda, M)) = Int^*(\lambda, M)$ and so, $Int^*(\lambda, M) \sqsubseteq$ ISα-Int(λ, M) $\rightarrow$ (a)

Also, using known result $(\lambda, M) \sqsubseteq Cl^*(\lambda, M)$, this implies that ISα-Cl$(\lambda, M) \sqsubseteq$ ISα-Cl$(Cl^*(\lambda, M))$. Then, ISα-Cl$(Cl^*(\lambda, M) = Cl^*(\lambda, M)$ and so, ISα-Cl$(\lambda, M) \sqsubseteq Cl^*(\lambda, M)$ $\rightarrow$ (b)

Combine the results (a) and (b), we will get $Int^*(\lambda, M) \sqsubseteq$ ISα-Int$(\lambda, M) \sqsubseteq (\lambda,M) \sqsubseteq$ ISα-Cl$(\lambda, M) \sqsubseteq Cl^*(\lambda, M)$.

Theorem 3.15: Let (ξ, D) be a soft set on ISTS (U, ϑ, D).Then the proceeding assertions hold: (a) If (ξ, D) is an ISα-OS (ISα-CS), then (ξ, D) is a Sα-OS(Sα-CS).

(b) If (ξ, D) is an ISα-OS (ISα-CS), then (ξ, D) is a soft α*- open (supra soft α- open) (soft α*-closed (supra α-closed)) set.

(c) If (ξ, D) is an ISα-OS (ISα-CS) set, then (ξ, D) is a soft pre* open (supra soft -preopen) (soft pre*closed (supra soft-preclosed)) set.

(d) If (ξ, D) is an ISα-OS (ISα-CS) set, then (ξ, D) is a soft semi open (ISSOS) (soft semi* closed (ISSCS)) set.

(e) If (ξ, D) is a SOS (SCS), then (ξ, D) is an ISα-OS (ISα-CS).

Proof: It is clear that from the definitions 2.1, 2.2 & 3.1 and basic relationships with other soft sets, we can prove above results.

Remark 3.16: Any SOS imply supra soft α- open set. Every ISα-OS imply Sα-OS. Any Sα-OS imply supra soft α- open set. Every Sα-OS imply SSOS and SPOS. Every ISα-OS imply ISSOS and Supra soft- preopen set. Every SPOS imply supra soft preopen set. Any ISSOS imply SSOS. There is no connections between ISSOS and supra soft preopen set.

The proceeding examples will show that the converses of above relationships need not to be true in general.

Example 3.17: Let us consider the soft space (U, M, τ) where, $U = \{1, 2, 3, 4\}$, $M= \{a, b\}$. Let $\Gamma_{M1}, \Gamma_{M2}, \Gamma_{M3}, \Gamma_{M4}, \Gamma_{M5}, \Gamma_{M6}$ and Γ_{M7} be soft sets of U_M defined as: $\Gamma_{M1} = \{(a, \{1\})\}$, $\Gamma_{M2} = \{(a, \{2\})\}$, $\Gamma_{M3} = \{(b, \{1, 2\})\}$, $\Gamma_{M4} = \{(a, \{1, 2, 3\})\}$, $\Gamma_{M5} = \{(a, \{1,4\})\}$, $\Gamma_{M6} = \{(b, \{2, 4\})\}$, $\Gamma_{M7} = \{(b,\{1, 2, 4\})\}$. Let $\tau = \{\varphi_A, \Gamma_{M1}, \Gamma_{M2}, \Gamma_{M3}, \Gamma_{M4}, U_M\}$. We may identify that

- Γ_{M5} is an ISSOS but it is not a supra soft-preopen set.
- Γ_{M6} is an ISSOS but it is not a SOS.
- Γ_{M4} is a supra soft-preopen set but it is not an ISSOS.
- Γ_{M7} is a supra soft-preopen set but it is not a SOS.
- Γ_{M7} is a supra soft-preopen set but it is not an ISα-OS.
- Γ_{M6} is an ISSOS but it is not an ISα-OS.

Example 3.18: Let us consider the soft space (U, M, τ) where, $U = \{1, 2, 3\}$ & $M = \{a, b\}$. Let Γ_{M1} and Γ_{M2} be soft sets of U_M defined as: $\Gamma_{M1}= \{(a, \{2\})\}$, $\Gamma_{M2} = \{(b, \{2, 3\})\}$, Let $\tau = \{\varphi_A, \Gamma_{M1}, U_M\}$. We identify that

- Γ_{M2} is a supra soft α-open set but it is not an ISα-OS.
- Γ_{M2} is a supra soft α-open set but it is not soft open set.
- Γ_{M2} is a Sα-OS but it is not an ISα-OS.
- Γ_{M2} is a SSOS but it is not an ISα-OS.
- Γ_{M2} is a SPOS but it is not an ISα-OS.

4. Conclusion

In this article, we define and study ISα-Int (ISα-Cl) & ISα-OS (ISα-CS) as a new structure on some soft topology and established their basic properties. Moreover, we studied their relationship with other soft open (closed) sets in STS. We think that the discoveries in this study are just the inception of a new structure that will not only serve as a theoretical foundation for future applications of soft topology and also for the enlargement of information systems.

References

[1] D. Molodtsov, "Soft set theory-first results," *Computers andMathematics with Applications*, vol. 37, no. 4-5, pp. 19–31, 1999.

[2] D. Molodtsov, V. Y. Leonov, and D. V. Kovkov, "Soft sets technique and its application," *Nechetkie Sistemyi Myagkie Vychisleniya*, vol. 1, no. 1, pp. 8–39, 2006.

[3] P. K. Maji, R. Biswas, and A. R. Roy, "Fuzzy soft sets," *Journal ofFuzzy Mathematics*, vol. 9, no. 3, pp. 589–602, 2001.

[4] I. Zorlutuna, M. Akdag, W. K. Min, and S. Atmaca, "Remarkson soft topological spaces," *Annals of Fuzzy Mathematics and Informatics*, vol. 3, no. 2, pp. 171–185, 2011.

[5] M. Shabir and M. Naz, "On soft topological spaces," *Computers and Mathematics with Applications*, vol. 61, no. 7, pp. 1786–1799,2011.

[6] P. K. Maji, R. Biswas, and A. R. Roy, "Soft set theory," *Computers and Mathematics with Applications*, vol. 45, no. 4-5, pp. 555–562,2003.

[7] D. Andrijevic, "Semi-preopen sets," *Mathematics Vesnik*, vol. 38,no. 1, pp. 24–32, 1986.

[8] M. Akdag and A. Ozkan, "Soft α-open sets and soft α- continuous functions," *Abstract and Applied Analysis*, vol. 2014, Article ID 891341, 7 pages, 2014.

[9] M. Akdag and A. Ozkan, "Soft β -open sets and soft β -continuous functions," Hindawi Publishing corporations, The Scient. World Journal, vol. 2014, Article ID 843456, 6 pages, 2014

[10] B. Chen, "Soft Semi-open sets and related properties in soft topological spaces," *Applied Mathematics and Information Sciences*, vol. 7, no. 1, pp. 287–294, 2013.

[11] N. Çağman, S. Karataş, and S. Enginoğlu, "Soft topology," *Computer and Mathematics with Applications*, vol. 62, no. 1, pp. 351–358, 2011.

[12] T. M. Al-shami, "New soft structure: infra soft topological spaces," *Mathematical Problems in Engineering*, vol. 2021, Article ID 3361604, 12 pages, 2021

[13] T. M. Al-shami, "Infra soft compact spaces and application tofixed point theorem," *Journal of Function Spaces*, vol. 2021, Article ID 3417096, 9 pages, 2021

[14] T. M. Al-shami and E. A. Abo-Tabl, "Connectedness and localconnectedness on infra soft topological spaces," *Mathematics*, vol. 9, no. 15, 2021

[15] H. A. Othman, *On Fuzzy supra-preopen sets*, Ann. Fuzzy Math. Inform., **12,** No. 3 (2016) 361 - 371.

[16] H. A. Othman and Md. Hanif. Page, *On an Infra-α-Open Sets*, Global Journal of Mathematical Analysis, **4,** No. 3 (2016) 12-16.

[17] M. I. Ali, F.Feng, X.Liu, W.K.Min and M.Shabir " On some new operations in soft set theory", *Computers and Mathematics with Applications*, Vol5,no.9, pp 1547-1553,2009

[18] Tareq M. Al-shami and A.A.Azzam, " Infra soft semiopen sets and Infra soft semicontinuity" ,*Journal of Function Spaces*, vol. 2021, Article ID 5716876, 11 pages,