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Effect of Surface Tension on Gravity Driven Convection in a Rotating Ferrofluid Fluid Layer Subject to Robin Thermal Boundary Condition

Mahesh Kumar R and C E Nanjundappa

MES Pre-University College of Arts, Commerce and Science, Bengaluru- 560003, India

Dr. Ambedkar Institute of Technology, Bengaluru-56, India

Abstract - Combined effect of buoyancy and surface tension forces in a rotating ferrofluid layer heated from below is studied using linear stability analysis of the Navier-Stokes equations supplemented by Maxwell's equations and the appropriate magnetic force. The lower boundary is considered to be rigid at either conducting or insulating to temperature perturbations, while upper boundary free open to the atmosphere is flat and subject to a Robin-type of thermal boundary condition. The weighted residual Galerkin technique is employed to extract the critical stability parameters numerically. It is shown that convection sets in oscillatory motions provided that the Prandtl number ( ) is less than unity. A mechanism for suppressing or augmenting Bénard–Marangoni ferroconvection by Coriolis force ( ), Biot number ( ), magnetic Rayleigh number ( ) and nonlinearity of fluid magnetization ( ) is discussed in detail. It is found that the onset of Bénard–Marangoni ferroconvection is delayed with an increase in , but opposite is the case with an increase in , . A few results are known as recovered to special cases.

Index Terms - ferrofluid, rotation, surface-tension, thermal boundary condition, Prandtl number, Biot number, Galerkin technique.

INTRODUCTION

There has been a significant attention in the study of ferrofluids (FFS) or magnetic fluids (MFs) [1-3]. The heat transfer processes in FFs were first studied by Neuringer and Rosenswieg [4]. The thermomechnical interactions taking place in FFs may give rise to convection imposed by externally magnetic field and temperature gradients. This study is of great interest because it influences upon the function efficiency of many practical devices employing FFs. The convection in FF (ferroconvection; FC) layer heated from below in the presence of a vertical magnetic field has been studied by Finlayson [5]. FC Ferroconvection can also be induced by surface tension forces provided it is a function of temperature [6-18]. The effect of viscosity variations on Bénard–Marangoni-ferroconvection was investigated by Nanjundappa et al. [19]. Sekhar et al. [20] have studied the effect of variable viscosity on thermal convection in Newtonian ferromagnetic liquid by different forms of boundary conditions.

Marangoni convection arises when the surface tension of fluid interface depends on the temperature. Schwab [20] experimentally examined the stability of flat FF layer when a vertical-temperature gradient and - magnetic field were applied. Based on the energy method, a nonlinear stability has been developed by Qin & Kaloni [21] to discuss the impact of gravity and surface tension on the motion in a FF layer. Venkatasubramanian & Kaloni [22] analyzed the influence of rotation on thermo-convective instability in FF layer. A linear stability analysis in FF layer with deformable free surface and placed in a magnetic field has been discussed by Weilepp & Brand [23]. Shivakumara and Nanjundappa [24] have studied the effects of Coriolis force and different basic temperature gradients on Marangoni ferroconvection. Shivakumara and Nanjundappa [25] have investigated the effect of rotation on the onset of coupled Benard-Marangoni ferroconvection in a horizontal ferrofluid layer. Nanjundappa et al. [26] have investigated the combined effect of rotation and MFD viscosity on Bénard-Marangoni ferroconvection. Several investigators have studied both types of instabilities in isolation or together in a horizontal ferrofluid layer.

Motivated by the fact that Coriolis force gives rise to interesting situations in practical and also the importance of buoyancy forces, however small it may be, even under reduced gravity environment, the objective of the present work is to study a general problem of coupled BBM ferroconvection in a rotating FF layer. In the present investigation, the lower surface rigid with isothermal is considered, whereas the non-deformable upper free surface and subjected to surface tension is a function of temperature. The problem of eigenvalue is applied numerically by using a GT with Tchebychev polynomials of second kind as basis function.

THE PROBLEM FORMULATION

Consider layer of FF of constant depth d in the occurrence of perpendicular magnetic field H_0. The surfaces are maintained the constant temperatures at $T_0 + \Delta T/2 \quad (z=0)$ and $T_0 - \Delta T/2 \quad (z=d)$. The angular velocity, $\vec{\Omega} = \Omega\,\hat{k}$, is rotating uniformly about the vertical axis and bounded above by a non–deformable free–insulating surface. The gravity, $\vec{g} = -g\,\hat{k}$, acting downward direction.

The stream of Bénard-Marangoni convection for thermocapillary forces (surface tension force), buoyancy forces and viscous forces is due to the linearly temperature dependent surface tension (σ) and viscosity (η), respectively. The following relations are considered:

$$\sigma = \sigma_0 \left\{1 - \sigma_T \left(T - T_0\right)\right\} \tag{1}$$

$$\mu = \mu_0 \left\{1 - \eta\left(T - T_0\right)\right\} \tag{2}$$

where σ_T , σ_0 , μ_0 and η are positive constants.

The Maxwell's equations for the magnetic field are implemented:

$$\nabla \times \vec{H} = 0 \quad \text{or} \quad \vec{H} = \nabla \varphi \tag{3}$$

$$\nabla \cdot \vec{B} = 0 \tag{4}$$

where,

$$\vec{B} = \mu_0\left(\vec{M} + \vec{H}\right) \text{with } \vec{M} = \frac{M\left(H,T\right)}{H} \vec{H} \tag{5}$$

$$M = M_0 + \chi(H - H_0) - K(T - T_a) \tag{6}$$

The equation of momentum for an incompressible FF with rotating frame of reference is

$$\rho_0 \left\{\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q}\right\} = -\nabla p + \rho \vec{g} + \mu_0 (\vec{M} \cdot \nabla) \vec{H} + \mu \nabla^2 \vec{q} + 2\rho_0 \vec{q} \times \vec{\Omega} + \frac{\rho_0}{2} \nabla(|\vec{\Omega} \times \vec{r}|^2) \tag{7}$$

The heat equation for an incompressible FF by ignoring the viscous dissipation is

$$\left[\rho_0\, C_{V,H} - \mu_0 \vec{H} \cdot \left(\frac{\partial \vec{M}}{\partial T}\right)_{V,H}\right] \frac{DT}{Dt} + \mu_0 T \left(\frac{\partial \vec{M}}{\partial T}\right)_{V,H} \cdot \frac{D\vec{H}}{Dt} = k_t \nabla^2 T \tag{8}$$

The mass conservation equation is

$$\nabla \cdot \vec{q} = 0. \tag{9}$$

The state equation is

$$\rho = \rho_0 \left[1 - \alpha_t (T - T_0)\right] \tag{10}$$

The state of undisturbed quiescent is

$$\vec{q} = 0, \quad p = p_b(z), \quad \rho = \rho_b(z), \quad T_b = T_0 - \beta z \left(\beta = \frac{\Delta T}{d}\right), \tag{11}$$

$$\vec{H}_b = \left\{H_0 - \frac{K \beta z}{1+\chi}\right\} \hat{k}, \; \vec{M}_b = \left\{M_0 + \frac{K \beta z}{1+\chi}\right\} \hat{k} \tag{12}$$

From (4) and (5), the standard linear stability analysis procedure), we obtain

$$H_i + M_i = \left\{1 + \frac{M_0}{H_0} M_0 / H_0\right\} H_i, \quad i = 1,\ 2 \tag{13}$$

$$M_3 + H_3 = -K\, T + (1+\chi)\, H_3 \tag{14}$$

where, H_i and M_i are the components of perturbed magnetic field and magnetization, respectively.

Taking the curl of (7), linearizing and the resulting equation in z-component is

$$\rho_0 \frac{\partial \xi}{\partial t} = 2\rho_0 \Omega \frac{\partial w}{\partial z} + \mu \nabla^2 \xi \tag{15}$$

is called the equation of vorticity transport with $\xi = \partial v / \partial x - \partial u / \partial y$. By taking curl double of (7), linearizing and applying (13) and (14) with (3), the resulting equation in the z-component (after by ignoring primes)

$$\left(\rho_0 \frac{\partial}{\partial t} - \mu\nabla^2 \right)\nabla^2 w = \rho_0 \alpha g \nabla_1^2 T - 2\rho_0 \Omega \frac{\partial \xi}{\partial z} - \mu_0 K \beta \frac{\partial}{\partial z}\left(\nabla_1^2 \phi \right) + \frac{\mu_0 K^2 \beta}{1+\chi} \nabla_1^2 T \tag{16}$$

As before, using (11) and (12) in (8), and linearizing yields

$$\rho_0 C_0 \frac{\partial T}{\partial t} = \mu_0 K T_0 \frac{\partial}{\partial t}\left(\frac{\partial \phi}{\partial z} \right) + \left[\rho_0 C_0 - \frac{\mu_0 K^2 T_0}{(1+\chi)} \right] w \beta + k_t \nabla^2 T \tag{17}$$

On using (13) and (14) with (3), yields

$$\left[\frac{1 + M_0 / H_0}{1+\chi} \right] \nabla_1^2 \varphi + \frac{\partial^2 \varphi}{\partial z^2} - \frac{K}{(1+\chi)} \frac{\partial T}{\partial z} = 0 . \tag{18}$$

The expanded form of each variable in the normal mode analysis

$$F(t,\ x,\ y,\ z) = F(t,\ z) e^{i\{l x + m y\}} \tag{19}$$

Using (19), (15) – (18) yields

$$\left[\rho_0 \frac{\partial}{\partial t} - \mu \left(\frac{\partial^2}{\partial z^2} - a^2 \right) \right] \left(\frac{\partial^2}{\partial z^2} - a^2 \right) \mathrm{w} = -a^2 \alpha g \theta + a^2 \mu_0 K \beta \frac{\partial \phi}{\partial z} - \frac{a^2 \mu_0 K^2 \beta}{1+\chi} \theta - 2\rho_0 \Omega \frac{\partial \xi}{\partial z} \tag{20}$$

$$\frac{\partial \theta}{\partial t} - \kappa \left(\frac{\partial^2}{\partial z^2} - a^2 \right) \theta - \frac{\mu_0 K T_0}{\rho_0 C_0} \frac{\partial}{\partial t}\left(\frac{\partial \phi}{\partial z} \right) = \left(1 - \frac{\mu_0 K^2 T_0}{(1+\chi) \rho_0 C_0} \right) w \beta \tag{21}$$

$$\frac{\partial^2 \phi}{\partial z^2} - \frac{(1 + M_0 / H_0)}{1+\chi} a^2 \phi - \frac{K}{1+\chi} \frac{\partial \theta}{\partial z} = 0 \tag{22}$$

$$\rho_0 \frac{\partial \xi}{\partial t} = \mu \left(\frac{\partial^2}{\partial z^2} - a^2 \right) \xi + 2\rho_0 \Omega \frac{\partial w}{\partial z} . \tag{23}$$

Thus, (20) – (23) are the governing linearized perturbation equations and they are non-dimensionalized quantities by applying the following quantities:

$$z^* = \frac{z}{d},\ \ w^* = \frac{d}{\nu} w,\ \ a^* = a\mathrm{d},\ \ t^* = \frac{\nu}{d^2} t,\ \ \xi^* = \frac{d^2}{\nu} \xi,\ \theta^* = \frac{\kappa}{\beta\ \nu\ d} \theta, \varphi^* = \frac{(1+\chi)\ \kappa}{K\ \beta\ \nu\ d^2} \varphi . \tag{24}$$

After using (24) in (20) – (23), we obtain (after neglecting the asterisks)

$$\left[\frac{\partial^2}{\partial z^2} - a^2 - \frac{\partial}{\partial t} \right] \left[\frac{\partial^2}{\partial z^2} - a^2 \right] \mathrm{w} = \sqrt{Ta} \frac{\partial \xi}{\partial z} + a^2 R_t\ \theta + a^2 R_m \theta - a^2 R_m \frac{\partial \phi}{\partial z} \tag{25}$$

$$\left[\frac{\partial^2}{\partial z^2} - a^2 - Pr \frac{\partial}{\partial t} \right] \theta + Pr\, M_2 \frac{\partial}{\partial t}\left(\frac{\partial \varphi}{\partial z} \right) = (M_2 - 1)\mathrm{w} \tag{26}$$

$$\left(\frac{\partial^2}{\partial z^2} - a^2 M_3 \right) \varphi - \frac{\partial \theta}{\partial z} = 0 \tag{27}$$

$$\left(\frac{\partial^2}{\partial z^2} - a^2 - \frac{\partial}{\partial t} \right) \xi = -Ta^{1/2} Dw . \tag{28}$$

We look for the solutions to (25)-(28) of normal modes kind is

$$\{w,\ T,\ \varphi,\ \xi\}(z,\ t)=\{W,\ \Theta,\ \Phi,\ \xi\}(z)\exp[\omega t] \tag{29}$$

where, ω denoted as complex frequency, substituting into (25)-(28), we obtain

$$\left[D^2-a^2-\omega\right]\left[D^2-a^2\right]\mathrm{W}=\sqrt{Ta}\ D\xi+a^2R_t\ \theta\ +a^2R_m\theta-a^2R_m D\Phi \tag{30}$$

$$\left(\mathrm{D}^2-a^2-\ \mathrm{Pr}\,\omega\right)\Theta=-\mathrm{W} \tag{31}$$

$$\left(\mathrm{D}^2-a^2M_3\right)\Phi-D\Theta=0 \tag{32}$$

$$\left(D^2-a^2-\omega\right)\xi=-Ta^{1/2}DW\ . \tag{33}$$

The boundary conditions are

$$W(0)=DW(0)=\Phi(0)=\xi(0)=0;\quad \Theta(0)=0\ \text{ or }\ D\Theta(0)=0 \tag{34}$$

$$W(1)=D^2W(1)+Ma\ a^2\Theta(1)=D\Phi(0)=D\xi(0)=0;\quad D\Theta(1)+Bi\,\Theta(1)=0. \tag{35}$$

METHOD OF SOLUTION

The GT is applied to obtain the problem of eigenvalue is to study the linear system of Eqs.30–33 with 34 and 35. The unknown factors W,Θ and Φ can be expanded upon the complete set:

$$W=\sum_{i=1}^{n}A_i\,W_i(z),\ \ \Theta(z)=\sum_{i=1}^{n}C_i\,\Theta_i(z)$$
$$\Phi(z)=\sum_{i=1}^{n}D_i\,\Phi_i(z),\ \ \xi=\sum_{i=1}^{n}E_i\,\xi_i(z) \tag{36}$$

Substitute in (30)-(33), multiplying the resulting equations respectively by $W_i(z)$, $\Theta_i(z)$, $\Phi_i(z)$ and $\xi_i(z)$ and carrying out the integration by parts from z = 0 to z = 1 and using (30) and (33), we obtain

$$\begin{bmatrix} C_{ij} & D_{ij} & E_{ij} & F_{ij} \\ G_{ij} & H_{ij} & 0 & 0 \\ 0 & I_{ij} & J_{ij} & 0 \\ K_{ij} & 0 & 0 & L_{ij} \end{bmatrix}\begin{bmatrix} A_i \\ C_i \\ D_i \\ E_i \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{37}$$

where

$$C_{ji}=\int_0^1\left\{D^2W_jD^2W_i+(2a^2+\omega)DW_jDW_i+a^2(a^2+\omega)W_jW\right\}dz$$

$$D_{ji}=-a^2\left(R_t+R_m\right)\int_0^1\Theta_jW_i\,dz+a^2Ma\,DW_j(1)\Theta_i(1)$$

$$E_{ji}=a^2R_m\int_0^1W_jD\Phi_i\,dz$$

$$F_{ji}=-\sqrt{Ta}\int_0^1W_jD\zeta_i\,dz$$

$$G_{ji}=-\int_0^1\Theta_jW_i\,dz$$

$$H_{ji} = \int_0^1 \left\{ D\Theta_j D\Theta_i + (a^2 + \omega \Pr) \Theta_j \Theta_i \right\} dz + Bi \; \Theta_j(1)\Theta_i(1)$$

$$I_{ji} = \int_0^1 \Phi_j D\Theta_i \, dz$$

$$J_{ji} = \int_0^1 \left\{ D\Phi_j D\Phi_i + a^2 M_3 \Phi_j \Phi_i \right\} dz$$

$$K_{ji} = -\sqrt{Ta} \int_0^1 \zeta_j DW_i \, dz$$

$$L_{ji} = \int_0^1 \left\{ D\zeta_j D\zeta_i + (a^2 + \omega) \zeta_j \zeta_i \right\} dz$$

Equation (37) will have a non-trivial solution, if

$$\begin{vmatrix} C_{ij} & D_{ij} & E_{ij} & F_{ij} \\ G_{ij} & H_{ij} & 0 & 0 \\ 0 & I_{ij} & J_{ij} & 0 \\ K_{ij} & 0 & 0 & L_{ij} \end{vmatrix} = 0 \tag{38}$$

The eigenvalue is extracted from (38). A trivial function W_i, Θ_i, Φ_i and ζ_i can be considered to satisfy the boundary conditions (34) and (35), such as

$$W_i = z^2(1-z)T^*_{i-1}, \quad \Phi_i = \xi_i = z^2\left(1 - \frac{2}{3}z\right)T^*_{i-1},$$

For lower insulating case: $\Theta_i = z^{i-1}T^*_{i-1}$, and For lower conducting case: $\Theta_i = z\left(1-\frac{z}{2}\right)T^*_{i-1}$ (39)

At this juncture to look at $i = j = 1$ and (38) yields

$$Ma = -\frac{(\eta_1 + 2\omega \Pr)}{1575\, a^2 < W\Theta >}\left[\frac{147\,Ta}{(\eta_2 + 13\omega)} + 2(\eta_3 + \eta_4 \omega)\right] - \frac{63\, N < W\, D\Phi >}{2\eta_5} - 2(N+R) < W\Theta > \tag{40}$$

where $\eta_1 = 2a^2 + 5 + Bi/4$, $\eta_2 = 42 + 13a^2$, $\eta_3 = a^4 + 28\,a^2 + 420$, $\eta_4 = 14 + a^2$ and $\eta_5 = 42 + 13\, M_3\, a^2$.

To study the stability of the system, we consider $\omega = i\,\omega$ in (40) yields

$$Ma = -\frac{1}{1575 a^2 < W\Theta >}\left[\frac{147\, Ta\,(\eta_1\eta_2 + 26\omega^2 \Pr)}{(\eta_2^2 + 169\omega^2)} + 2(\eta_1\eta_3 - 2\omega^2\, \eta_4 \Pr)\right] - 2(N+R) < W\Theta > - \frac{63\, N < W\, D\Phi >}{2\eta_5} + i\,\omega\Delta \tag{41}$$

where

$$\Delta = -\frac{1}{1575 a^2 < W\Theta >}\left[\frac{147\, Ta\,(2\eta_2 \Pr - 13\eta_1)}{(\eta_2^2 + 169\omega^2)} + 2\left(2\eta_3 \Pr + + \eta_1\eta_4\right)\right]. \tag{42}$$

The steady onset is governed by $\omega = 0$ and it occurs at $Ma = Ma^S$,

where $$Ma^S = -\frac{\eta_1}{1575\, a^2 < W\Theta >}\left(\frac{147\, Ta}{\eta_2} + 2\eta_3\right) - 2(N+R) < W\Theta > - \frac{63\, N < W\, D\Phi >}{2\eta_5} \tag{43}$$

The oscillatory convection (i.e.,Hopf bifurcation) occurs at $Ma = Ma^0$, where

$$Ma^o = -\frac{2(a_1 a_4^2 + a_2 a_4 + a_3)}{1575 a_4 a^2 < W\Theta >} - 2(N+R) < W\Theta > - \frac{63 N < W D\Phi >}{2\eta_5} . \quad (44)$$

Here, $a_1 = \eta_1\eta_2 - \frac{26}{169}\Pr\eta_2^2$, $a_2 = \eta_1\eta_3 + \frac{2}{169}\Pr\eta_4\eta_2^2 + \frac{147}{13}Ta\Pr$, $a_3 = -\frac{147}{169}Ta\Pr\eta_4$ and $a_4 = \frac{\eta_1\eta_4 + 2\Pr\eta_3}{13\eta_1 - 26\Pr\eta_2}$.

The resultant frequency of oscillations is given by

$$\omega^2 = -\frac{\eta_2^2}{169} + \frac{147 Ta}{26\eta_4}\left[\frac{1-2\beta_1\Pr}{1+2\beta_2\Pr}\right] \quad (45)$$

where $\beta_1 = \frac{42+13a^2}{65+26a^2}$ and $\beta_2 = \frac{a^4+28a^2+420}{2a^4+33a^2+70}$.

For the oscillatory occurrence on the onset ω^2 should be positive, the necessary conditions are

$$\Pr < \frac{(a^2+2.5)}{(a^2+3.23)} \quad \text{and} \quad Ta > \frac{26}{24843}\eta_2^2\eta_4\left[\frac{1+2\beta_2\Pr}{1-2\beta_1\Pr}\right]. \quad (46)$$

From (46), it is evident that oscillatory FTC occurs $\Pr < 1$ and Ta exceeds a threshold. This behaviour is reminiscent of that observed in classical viscous case. However, for most of the commercially available FFs,

whether the based on organic liquid case or water, $\Pr > 1$, hence the oscillatory FTC is ruled out for the instability mode [see Venkatasubramanian and Kaloni [22], and Aurenhammer and Brand [27]. Substituting Eq.39 into 38 and leads to

$f(R_t, R_m, Ma, Ta, M_1, M_3, a) = 0.$

NUMERICAL RESULTS AND DISCUSSION

To solve the eigenvalue problem from (30)–(33) by employing the Galerkin-type of WRM. In order to confirm the numerical technique is applied, the values (Ma_c, a_c) are very close to the existing values of Vidal and Acrivos [28] and Davis [29] for $Bi = R_t =$ under the limiting condition in Table 1 and 2, respectively.

Comparison of calculated present results agree well with results of previous numerical investigations are given in Tables 1 and 2.

Table 1. Comparison of (Ma_c, a_c) for $Bi = R_t = R_m = 0$

Ta	**Vidal and Acrivos [28]**		**Present study**	
	Ma_c	a_c	Ma_c	a_c
0	80	2.0	79.61	1.99
10^2	92	2.2	91.31	2.17
10^3	164	3.0	163.11	2.97
10^4	457	5.0	456.23	4.99
10^5	1400	8.6	398.36	8.86

Table 2. Comparison of (Ma_c, a_c) for R_t and Bi with $R_m = Ta = 0$

Bi	R	**Davis [29]** Ma_c	**Present study** Ma_c
0	**0**	79.61	79.608
	100	68.43	68.484

	200	57.12	57.116
	300	45.49	45.491
	400	33.59	33.589
	500	21.39	21.387
	600	8.857	8.857
	669	0.000	0,000
10	**0**	413.4	413.444
	100	378.7	378.741
	300	305.0	304.980
	500	225.1	225.116
	700	138.6	138.634
	900	44.73	44.730
	989.49	0.000	0.000

The various levels of approximation to Ma_c and the corresponding a_c are also obtain for variation of Ta when classical Marangoni convection and results are shown graphically in Fig. 2. It is seen that with an increase in Galerkin approximations, Ma_c goes on increasing and finally $i=j=8$ the present results converge compare well with results of previous study by Pradhan [30] and these results obtained by Fourier series method.

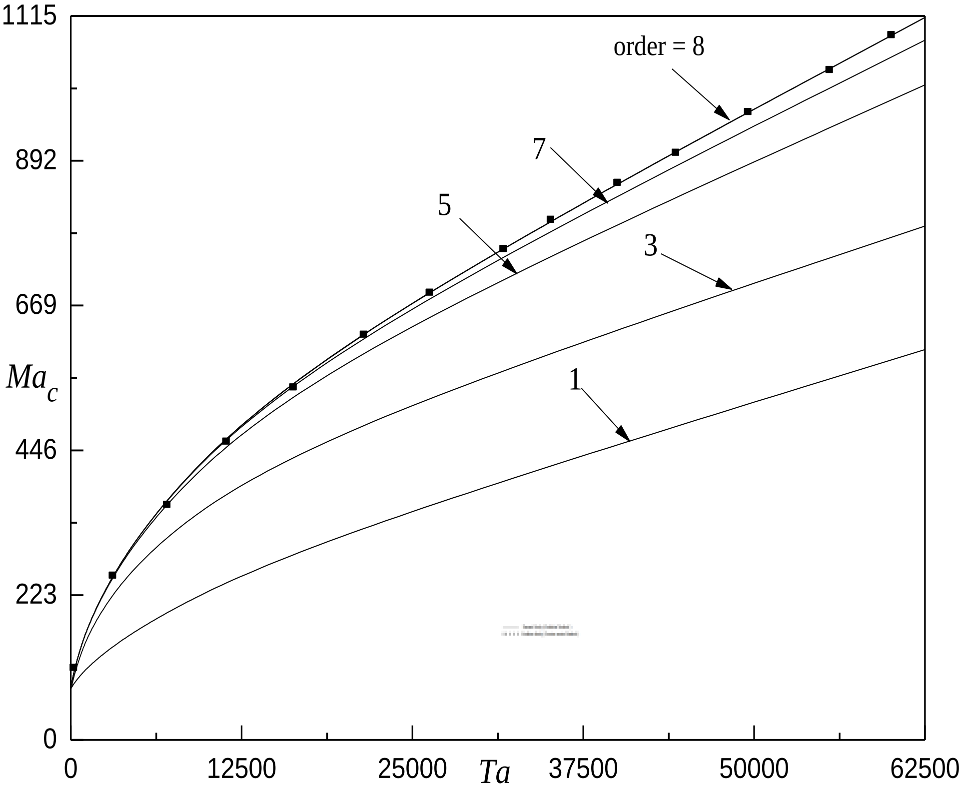

Fig. 2 . Variation of Ma_c verses Ta for different orders by Galerkin method (present study) and for Fourier series method (previous study) when $M_3 = 1$, $R_m = 0$ and $Bi = 0$

Figures 3-7 illustrates the neutral stability curves corresponding for different Ta, Bi, R_t, R_m and M_3 as well as different bounding surfaces (lower conducting and lower insulating). The neutral stability curves are concave upward for each of these boundaries and the curves of lower conducting case lie above lower insulating surfaces. The neutral stability

curves move upward with increasing Ta (Fig.3), Bi (Fig.4) indicating that their effect is to increase the stability region. Besides, decrease the stability of the region by increasing R_t (Fig.5), R_m (Fig.6) and M_3 (Fig.3).

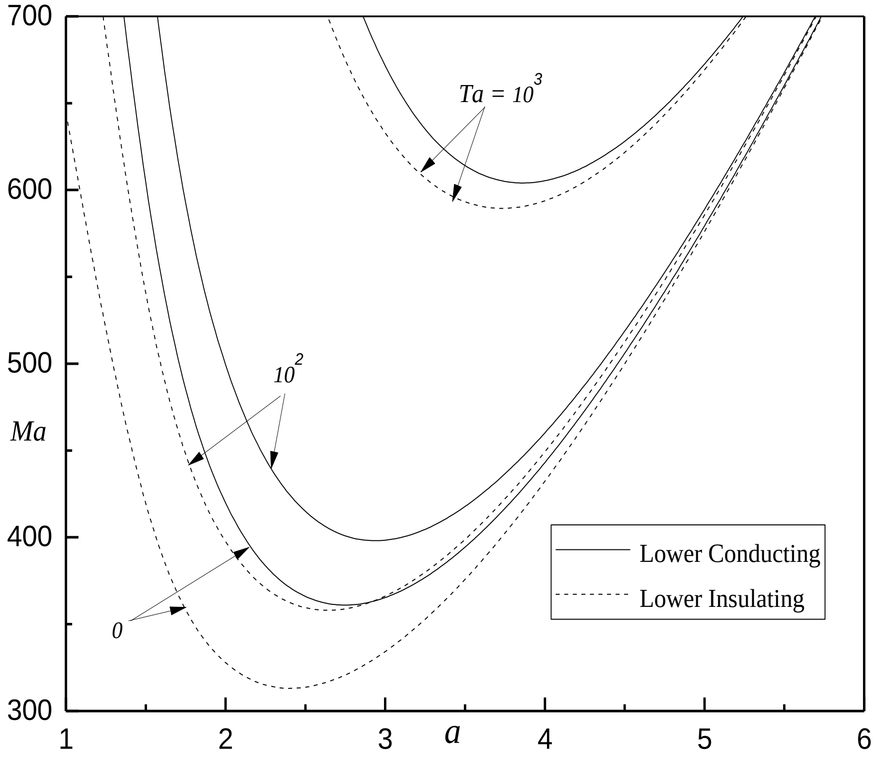

Fig. 3. Ma against a for $Bi = 10$, $R_t = R_m = 100$ and $M_3 = 1$

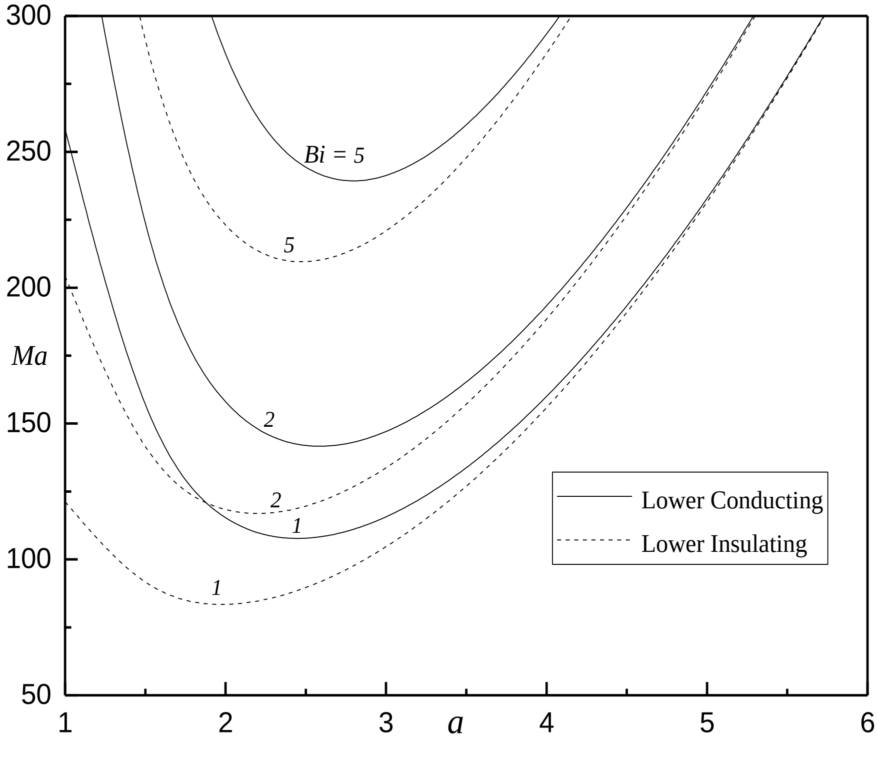

Fig. 4. Ma against a for $R_t = R_m = Ta = 100$ and $M_3 = 1$

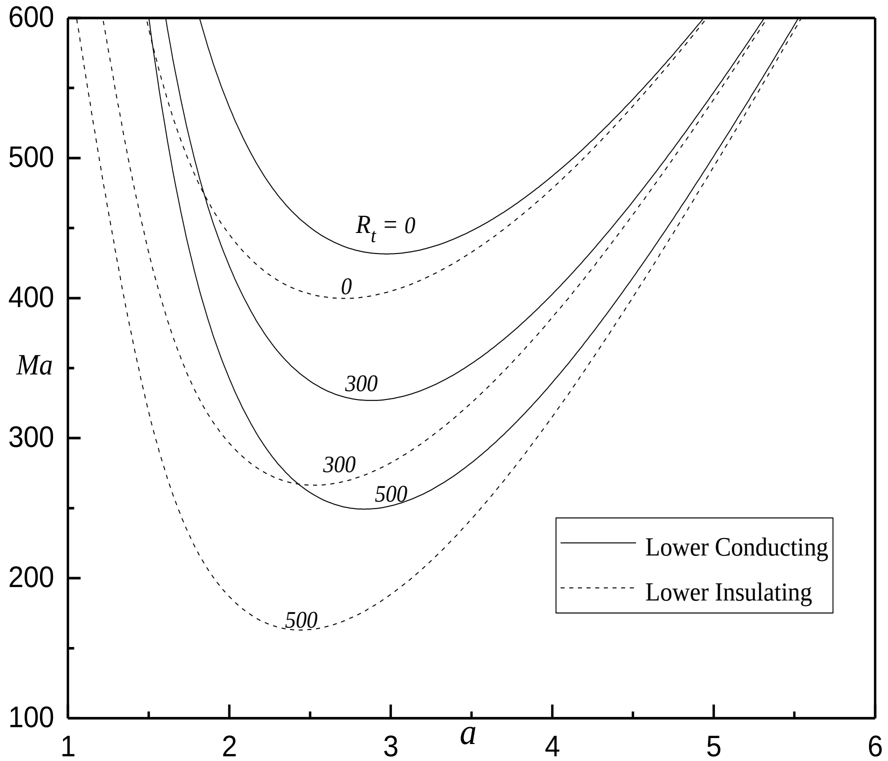

Fig. 5. Ma against a for $Bi = 10$, $R_m = Ta = 100$ and $M_3 = 1$

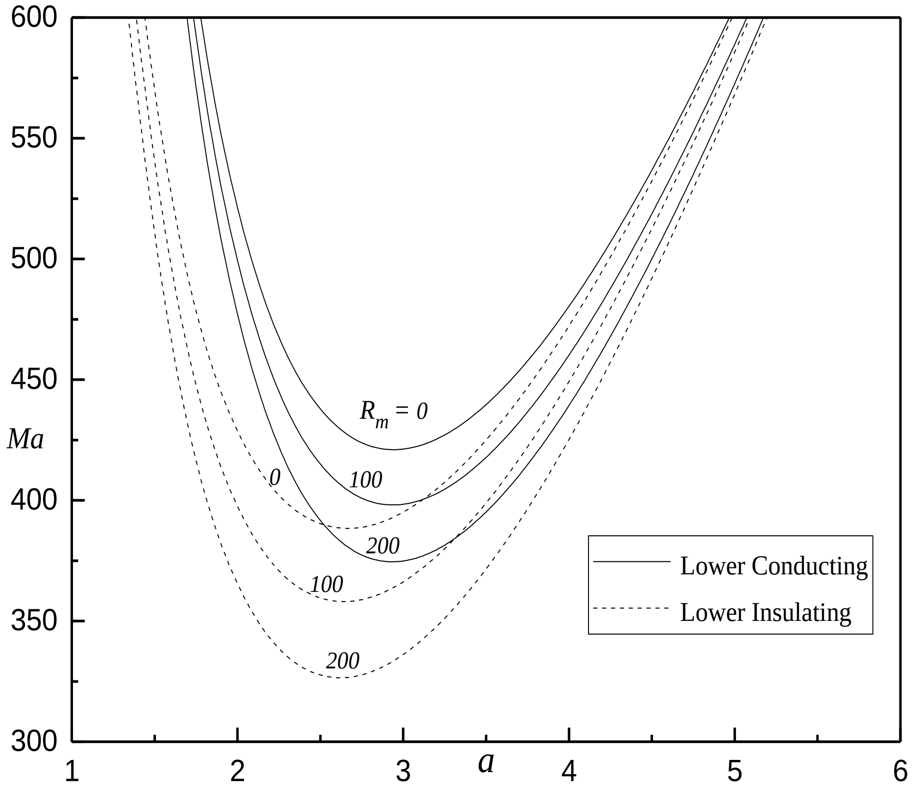

Fig. 6. Ma against a for $Bi = 10$, $R_t = Ta = 100$ and $M_3 = 1$

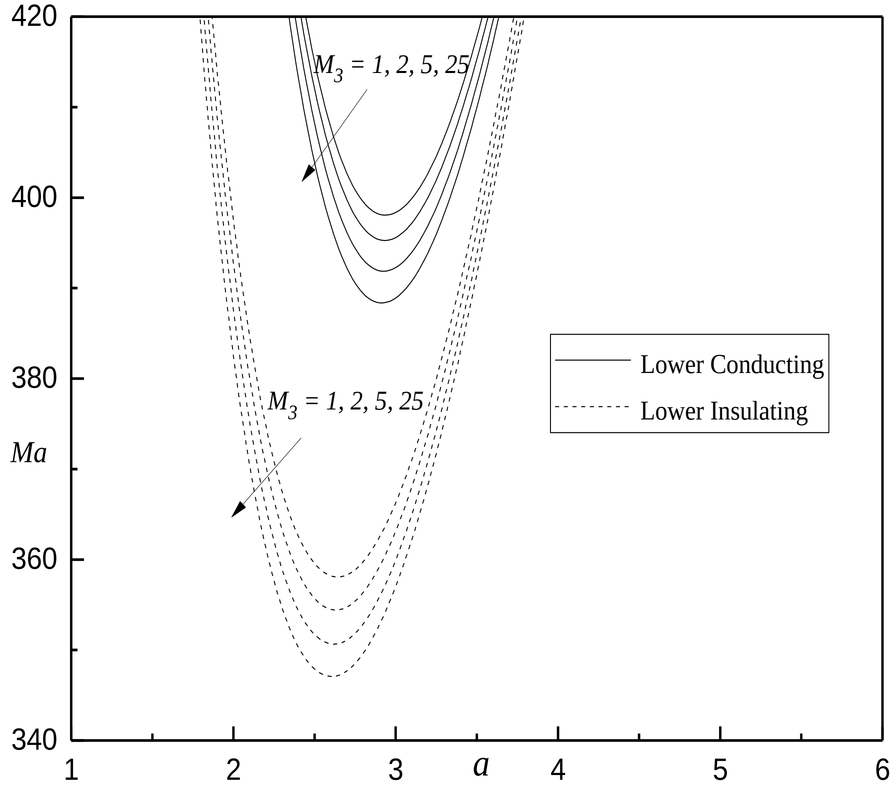

Fig. 7. Ma against a for $Bi = 10$ and $R_t = R_m = Ta = 100$

In Figs.8-11 analogous to solid curves are corresponding to lower conducting and dotted curves corresponding to lower insulating. The locus plot of R_{tc} against Ma_c for various Ta for $Bi = 10$, $R_m = 100$ and $M_3 = 1$ (see Fig.8). It shows that they are bridging the space between lower conducting and lower insulating by increasing in Ta. Clearly, the results of BMC advances the FTC compared to lower conducting and lower insulating. Figure 8 reveals that the linear stability analysis can be expressed in terms of R_{tc} and Ma_c, the system with R_{tc} eigenvalue is unstable compared to Ma_c eigenvalue, it is noted that $Ma_c < R_{tc}$. Besides, it can be observed that an increasing Ta, the critical stability parameters (R_{tc} and Ma_c) increases, thus it has a stabilizing effect on the system.

From Fig.9, it is evident that the deviation of Bi from 0 to 2 significantly increase in Ma_c and R_{tc} in both the cases of temperature boundary conditions considered; the least being for $Bi = 0$ and the maximum correspond to $Bi = 2$. Thus the system is found to be more unstable for upper insulating case as compared to upper isothermal condition. This behavior is not surprising as the nature of upper surface changes drastically from an insulated surface to a conductive boundary with an increase in Bi. It is evident that with an increase in Bi the temperature perturbations will not grow so easily and therefore higher R_{tc} and Ma_c are needed for the onset.

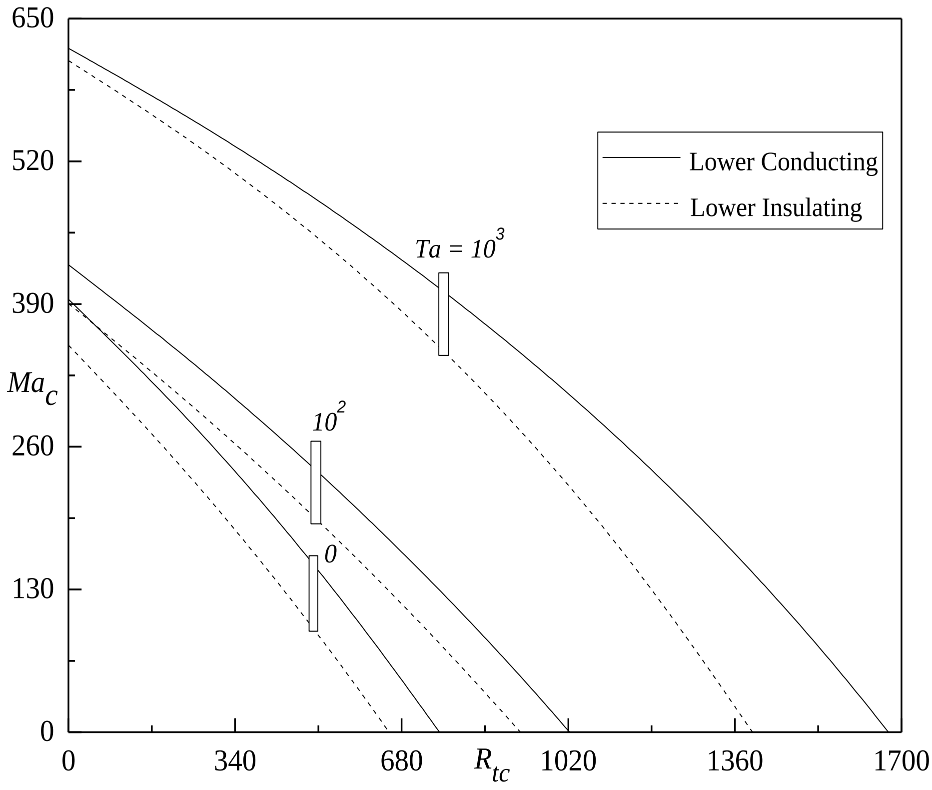

Fig. 8. Ma_c against R_{tc} for different Ta when $Bi = 10$, $R_m = 100$ and $M_3 = 1$

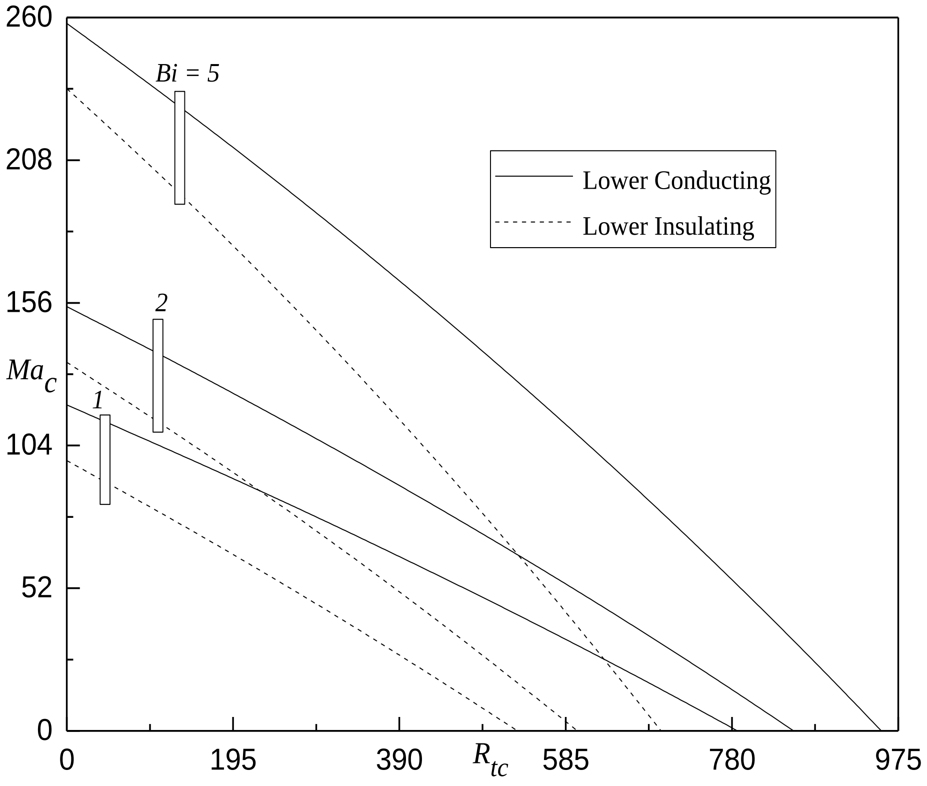

Fig. 9. Ma_c against R_{tc} for different Bi when $Ta = R_m = 100$ and $M_3 = 1$

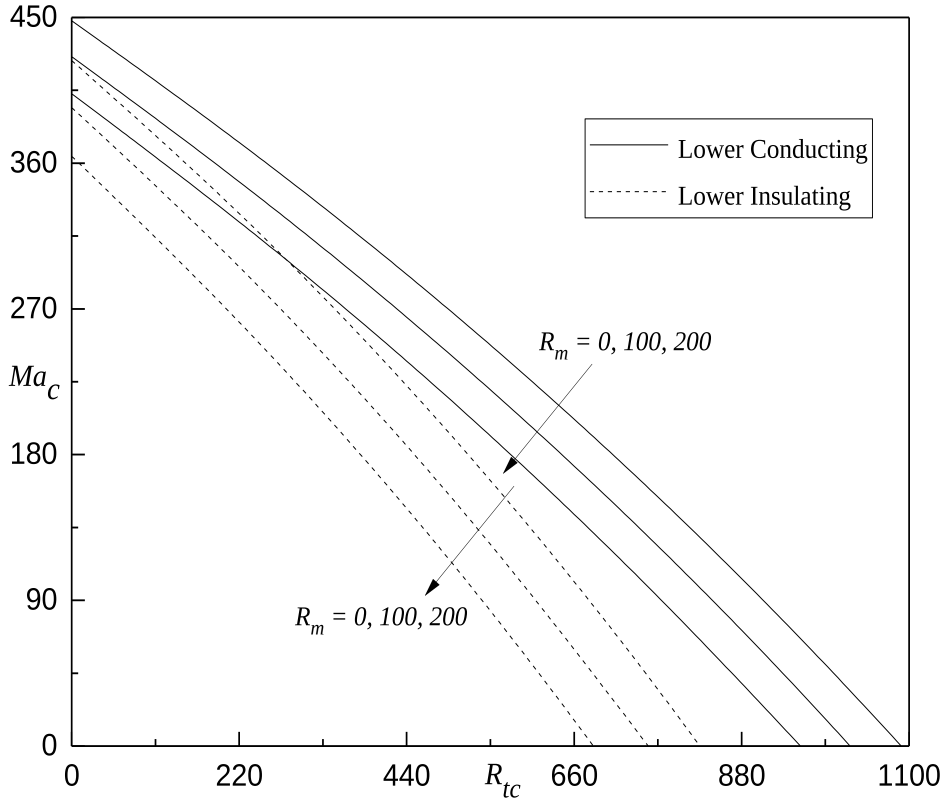

Fig. 10. Ma_c against R_{tc} for different R_m when $Bi = 10$, $Ta = 100$ and $M_3 = 1$

The variations of Ma_c against R_{tc} is shown in Fig.10 for two types of temperature boundary conditions when $Bi = 10$, $Ta = 100$ and $M_3 = 1$. For $R_m = 0$, the case corresponds to only the gravitational force are in effect. The amount of $R_m \neq 0$ is associated to the importance of magnetic force. It is observed that an increase in R_m leads to decrease Ma_c and R_{tc} signifying that the FFs carry more heat efficiency than the ordinary viscous fluid case. This is due to an increase the destabilizing magnetic force with increasing R_m which the fluid to flow more easily.

The result of increase in nonlinearity of magnetization (i.e.M_3) is shown in Fig. 11 for $Bi = 10$ and $Ta = R_m = 100$. It is noticed that, increase in M_3 is to decrease R_{tc} and Ma_c, thus mechanism of R_{tc} has destabilizing effect on the system but this effect is very marginal.

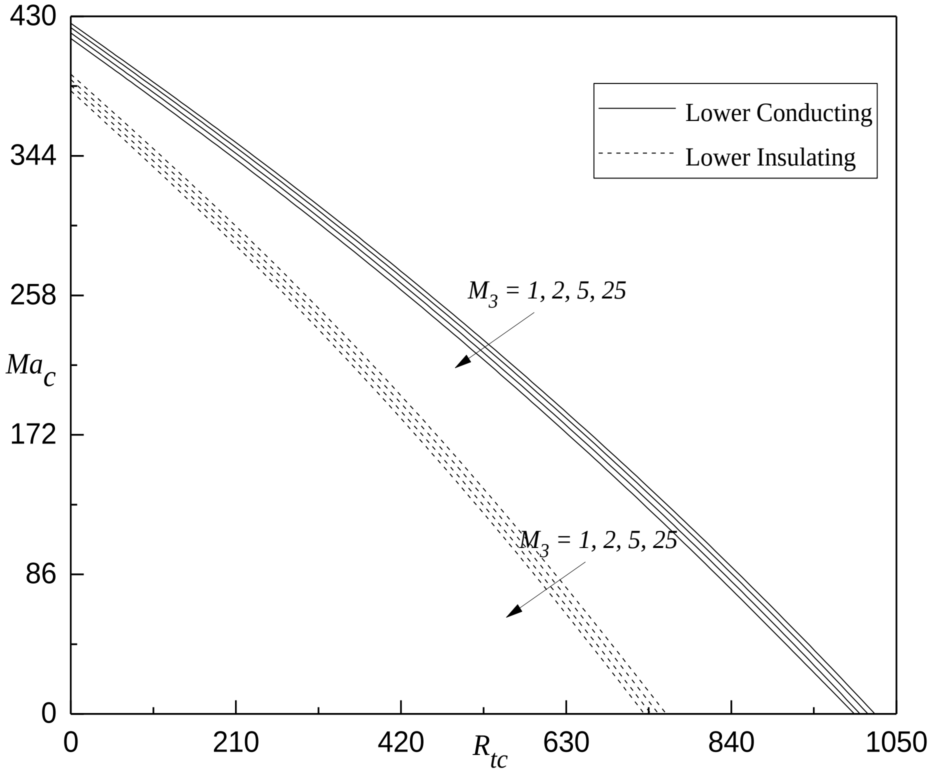

Fig. 11. Ma_c against R_{tc} for different M_3 when $Bi = 10$ and $Ta = R_m = 100$

CONCLUSIONS

The effect of Coriolis force is to suppress the FTC and hence rotation plays a stabilizing role on the system. Variations in R_{tc} and Ma_c are significant for large *Ta* values and found to be obscure for small *Ta*. The increase in magnetic force and buoyancy/surface tension force is to destabilize the system. Their effects are complementary in the sense that the R_{tc} and Ma_c decrease with an increase in R_m . The increase in Bi and decrease in R_m and M_3 are having stabilizing effect on the system. The Taylor number and the Biot number significantly influence the dimensions of the convective cells..

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