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Improvement Maximum Flow and Minimum Cut

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Abstract - We will randomly take a map of one of cities and study the problem of the maximum flow, as well as the problem of the minimum cut, where there are cities and roads leading to those cities on the map. These paths look at the edge that has the highest amplitude and then make the rest of the edges of that path the same amplitude in order to accommodate the largest possible amount of flow and thus improve the maximum flow. As for improving the problem of minimum cut, we try to cut the edge that has the highest amplitude in that path after that we used to cut the edge of the lowest capacity in order to be at the lowest cost. To ensure that the process is more efficient, we cut the edge that has the highest capacity in that path, even if the cost increases by a small amount and does not have a significant impact on the general resource of the project and in order to give better results and so the results are equal to the maximum flow and minimum cutoff.

Keywords - Maximum Flow, Minimum Cut, Lowest Cost.

INTRODUCTION

The maximum flow problems and the minimum cut problems are so important in linear programming field. The minimum cost flow problem is the fundamental network flow problem, where, determining max flow at min cost from the source to the sink. The shortest path problem is the specializations of this problem [1]. In this paper we discuss the concept of maximum flow and its algorithm, as well as the minimum cut and its algorithm, and then we suggest improving for these two algorithms. The authors introduced many papers in varied fields of science such as operation research [2- 15], and optimization [16- 36].

1.1. Maximum Flow Problem

In order to explain this problem in simplified way, we will express the point in the drawing as a city and about the edges as the road that accommodates a certain number of cars that want to pass from the starting point to the target point so that we can take the largest number possible cars that can pass on one road [37].

1.1.1. Steps of Maximum Flow Problem

Step 1: Given a directed graph represents roads, weight on edge represents maximum of cars to pass on this car.

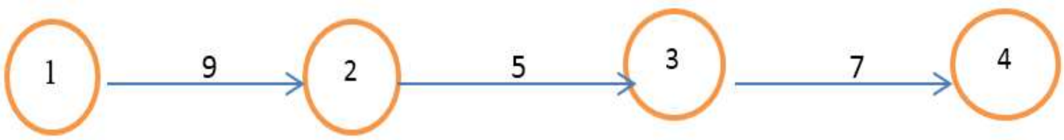
Step 2: Send maximum number of cars from first to second.

Step 3: Find set of paths to send cars through them from start to end.

Step 4: Call first node source (S) and call second node sink (T).

Step 5: Call answer S-T flow, flow means flow over edges [38].

For example, to find the maximum flow for the following graph:



We can send on 1-2 = 9 cars

We can send on 2-3 = 5 cars

We can send on 3-4 = 7 cars

But edge 2-3 is bottleneck as if we sent from 1-2 = 9 cars we can’t pass more than 5 so max flow of path is the minimum value on it.

1.2. Minimum Cut Problem

We will explain this problem by giving an example from the real life and linking it to the topic directly. For example, in a war, there is a network of roads and the enemy delivers weapons through these roads to other areas in order to address this problem needs to destroy these roads and it's better to destroy the road that has the least capacity for cars to pass because we are looking for the lowest cost.

Cutting one of these roads prevents weapons from reaching to other areas. We can destroy all the roads or more of them, but it requires more cost, so we resort to less destruction to number of ways until the enemy is unable to deliver weapons to other Ares [39].

1.2.1. Steps of Minimum Cut Problem

Step 1: Given a weighted directed graph that each node represents a city, we are in a war we would like to destroy the road to prevent city (A) to send weapons to city (B)

Step 2: Edges costs is cost of destroying a road.

Step 3: Any set of edges to disconnect (A) from (B) is called cut.

Step 4: The minimum edges to achieve the task are Min cut edges.

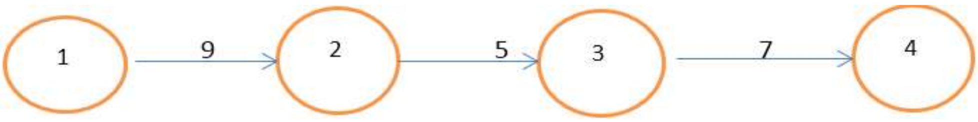
For example, to find the minimum cut of the following graph:

We can destroy all edges but this is very costive, as well as we have just 1 path make sense to destroy 1 edge, this edge is the minimum edge = 5.

IMPROVEMENT MAXIMUM FLOW

We have previously mentioned the maximum flow and explained that the points in the drawing represent the cities and the edges represent the streets leading to the cities, we suggest that we improve the maximum flow by choosing the road that accommodates greater number of flow and making the rest of the roads with the same capacity for that larger road so that it can pass the maximum number of flutes in each path and the same amplitude, as in the following examples.

Example 1: Find the maximum flow of the following graph.



We can send on (1-2) = 9 cars

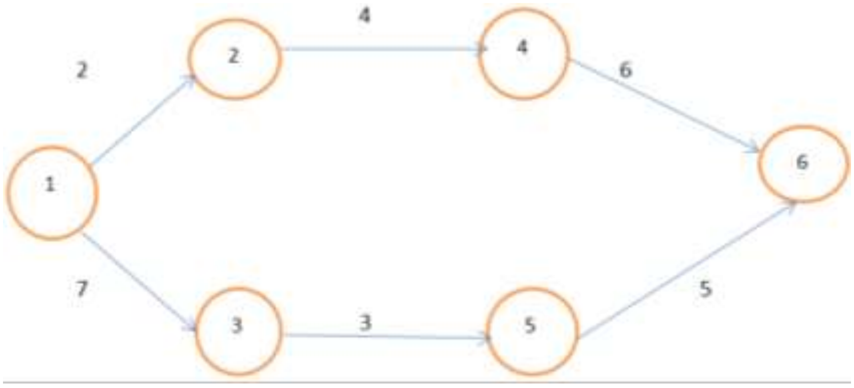
We can send on (2-3) =5 cars

We can send on (3-4) =7 cars

But edge (1-2) is the largest value of the path, we must make all the paths the same value for the largest flow:

(2-3) = 5 cars, 5+4 = 9 cars, (4-3) = 7 cars, 7+2 = 9 cars, the maximum flow is 9.

Example 2: Find the maximum flow for the following graph:



Path 1: 1- 2- 4- 6

Path 2: 1- 3- 5- 6

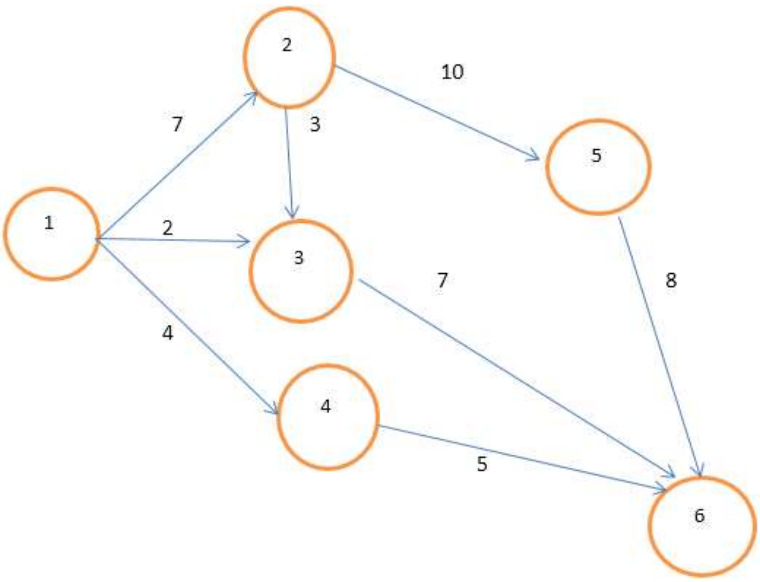
In path 1 the edge (4- 6) = 6 is the largest value, we must make all the path the same value for the largest flow:

(1- 2) = 2+4 = 6, (2- 4) = 4+2 = 6

In path 2 the edge (1- 3) =7 is the largest value, we must make all the path the same value for the largest flow

(3- 5) = 3+4 = 7, (5- 6) = 5+2 = 7, The Maximum flow is 7+6 = 13

Example 3: Find the maximum flow of the following graph:



Path 1: 1- 2- 5- 6

The edge (2- 5) is the largest value, we must make all the edges of the path same value for the largest flow.

(1- 2) = 7+3 = 10, (5- 6) = 8+2 = 10, Path 2: 1- 2- 3- 6

The edges (1- 2) and (3- 6) are the largest values of this path, we must make edge (2- 3) same value for largest flow:

(2- 3) = 3+4 = 7

Path 3: 1- 3- 6

The edge (3- 6) is the largest value of the path, we must make edge (1- 3) same value for largest flow:

(1- 3) = 2+5 = 7

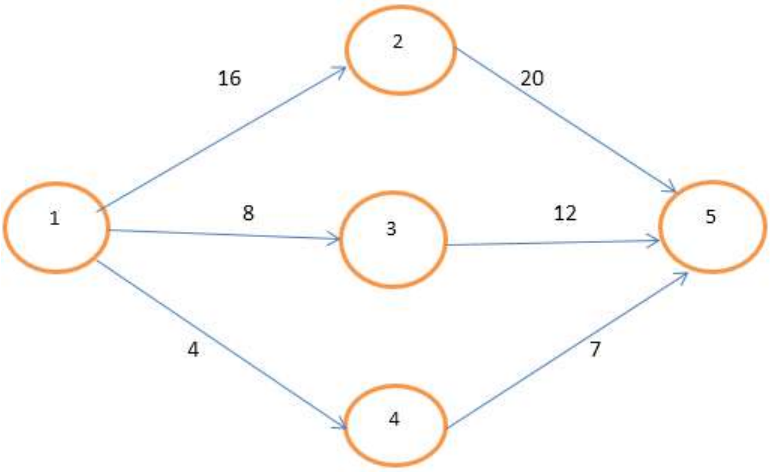
Path 4: 1- 4- 6

The edge (4- 6) is the largest value of the path:

(1- 4) = 4+1= 5

The Maximum flow is 10 +7+7+5 = 29.

Example 4: Find the maximum flow of the following graph:



Path 1: 1- 2- 5

The edge (2- 5) is the largest value of the path then the edge (1- 2) = 16+4 = 20

Path 2: 1- 3- 5

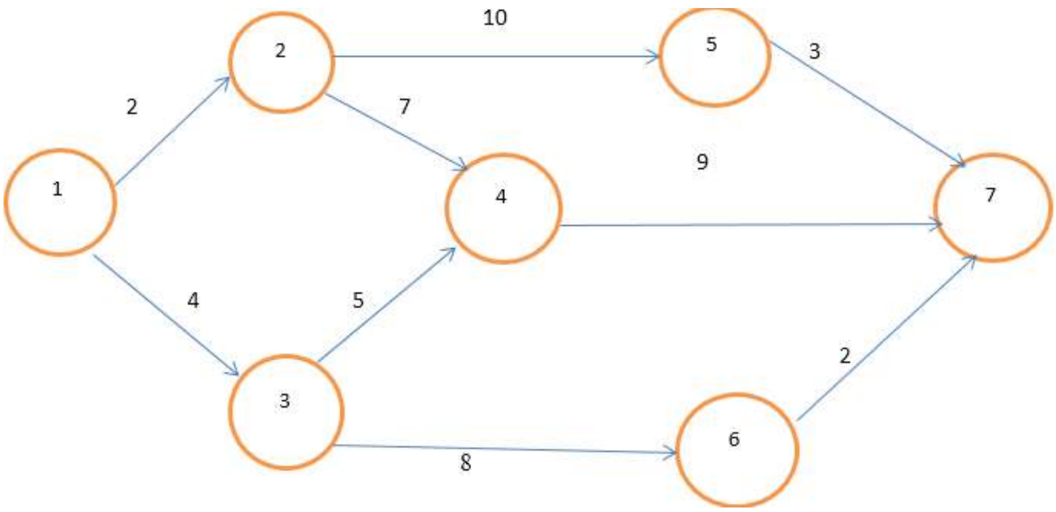
The edge (3- 5) is the largest value of the path then the edge (1- 3) = 8+4 = 12

Path 3: 1- 4- 5

The edge (4- 5) is the largest value of the path then the edge (1- 4) = 4+3 = 7

The Maximum flow is 20+12+7= 39

Example 5: Find the maximum flow of the following graph:



Path 1: 1- 2- 5- 7

The edge (2- 3) is the largest value of the path then the edge (1- 2) = 2+8 = 10 and (5- 7) = 3+7 = 10

Path 2: 1- 2- 4- 7

The edge (4- 7) is the largest value of the path then the edge (1- 2) = 2+7= 9 and (2- 4) =7+2 = 9

Path 3: 1- 3- 4- 7

The edge (4- 7) is the largest value of the path then the edge (1- 3) =5+4 = 9 and (3- 4) = 4+5 = 9

Path 4: 1- 3- 6- 7

The edge (3- 6) is the largest value of the path then the edge (1- 3) = 5+3 = 8 and (6- 7) = 2+6 = 8

The Maximum Flow is 10+9+9+8 = 36

IMPROVEMENT MINIMUM CUT

We have previously mentioned the problem of minimum cutting and this problem has been clarified through an example from real life which is the example of war and how to deliver equipment and weapons from one area to another through a network of roads and cutting one of these roads leads to the failure of the set plan by the enemy, and we can suggest an improvement for this problem, which is that instead of cutting the path with the least capacity we cut the path that, accommodates the most capacity in order to ensure more safety and to prevent the enemy from reaching its goal provided that the cost is increased to small extent. For examples:

Example 6: Find the Minimum cut of the following graph (same graph of example 1):

We can send on (1- 2) = 9 cars

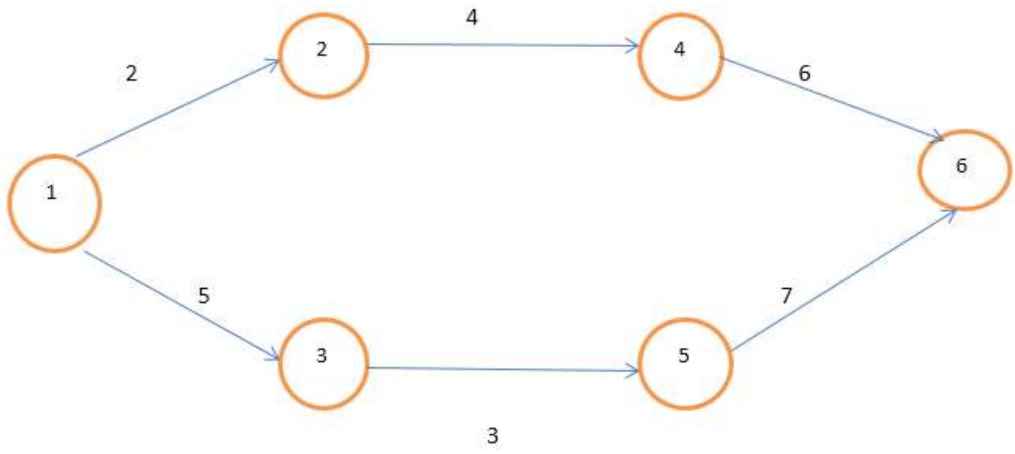
We can send on (2- 3) = 5 cars

We can send on (3- 4) = 7 cars

We must cross the edge with the largest capacity in order to increase the chance of the enemy not reaching the other side.

We could destroy the edge (1- 2) with largest capacity, Min cut = 9 = Max flow of example 1.

Example 7: Find the minimum cut of the following graph (same graph of example 2):



Path1: 1- 2- 4- 6

We must cross the edge with the largest capacity in order to increase the chance of the enemy not reaching the other side.

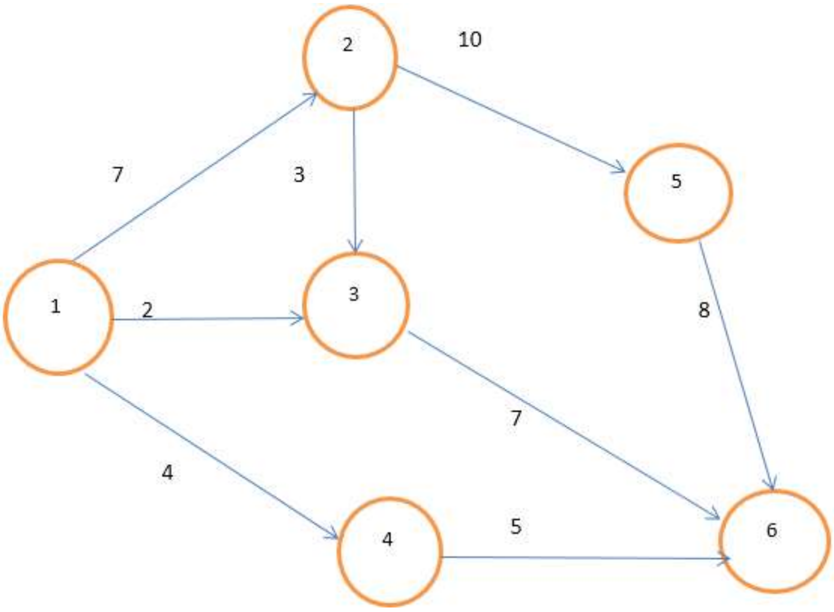
We can destroy the edge (4- 6) with largest capacity in this path.

Path 2: 1- 3- 5- 6

We could destroy the edge (1- 3) with largest capacity in this path.

Min cut is 6+7= 13 = Max flow of example 2.

Example 8: Find the Minimum cut of the following graph (same graph of example 3):



Path 1: 1- 2- 5- 6

We must cross the edge with the largest capacity in order to increase the chance of the enemy not reaching the other side.

We could destroy the edge (2- 5) with largest capacity in this path.

Path 2: 1- 2- 3- 6

We could destroy the edge (1- 2) or (3- 6) with largest capacity in this path.

Path 3: 1- 3- 6

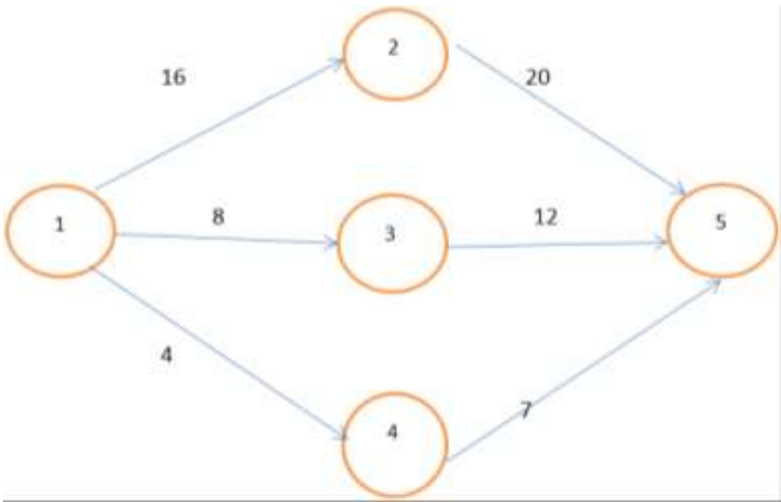
We could destroy the edge (3- 6) with largest capacity in this path.

Path 4: 1- 4- 6

We could destroy the edge (4- 6) with largest capacity in this path.

Min cut is 10+7+7+5 = 29 = Max flow of example 3.

Example 9: Find the Minimum cut of the following graph (same graph of example 4):



Path 1: 1- 2- 5

We could destroy the edge (2- 5) with largest capacity in this path.

Path 2: 1- 3- 5

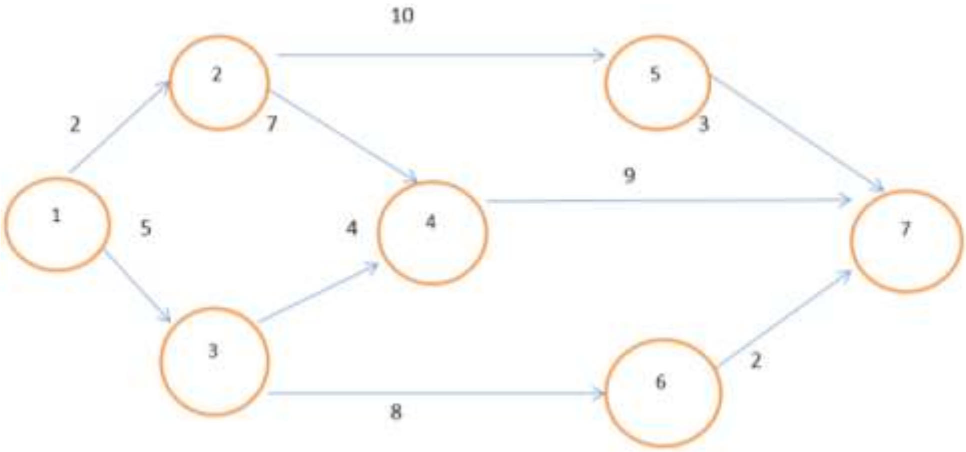
We could destroy the edge (3- 5) with largest capacity in this path.

Path 3: 1- 4- 5

We could destroy the edge (4- 5) with largest capacity in this path.

Min cut is 20+12+7 = 39 = Max flow of example 4.

Example 10: Find the Minimum cut of the following graph (same graph of example 5):



Path 1: 1- 2- 5- 7

We could destroy the edge (2- 5) with largest capacity in this path.

Path 2: 1- 2- 4- 7

We could destroy the edge (4- 7) with largest capacity in this path.

Path 3: 1- 3- 4- 7

We could destroy the edge (4- 7) with largest capacity in this path.

Path 4: 1- 3- 6- 7

We could destroy the edge (3- 6) with largest capacity in this path.

The Minimum cut is 10+9+9+8 = 36 = Max flow of example 5.

CONCLUSION

In this paper we tried to find an improvement of maximum flow method and the minimum cut method by finding a relationship between them. We found the equality relationship between them, that is, in all of the above ten examples (1 and 6), (2 and 7), (3 and 8), (4 and 9), and (5 and 10), the values of the maximum flow are equal to the values of the minimum cut.

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