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On Solving Fuzzy Transportation Problem Using Fuzzy One Point Method

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***Abstract* -** In this article, a procedure based on fuzzy one point method is proposed to solve fuzzy transportation problems where all the parameters are taken as Pentagonal fuzzy numbers. Numerical Examples are given to illustrate the proposed approach in detail.

Keywords - Fuzzy one point method, Pentagonal fuzzy numbers, Pentagonal fuzzy transportation problem, Ranking technique.

INTRODUCTION

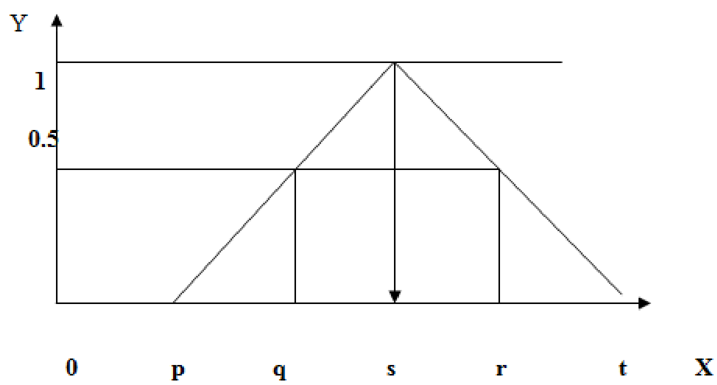
A Fuzzy transportation problem (FTP) is a transportation problem in which the transportation cost, supply and demand quantities are fuzzy quantities. The objective of the fuzzy transportation cost problem is to determine the minimal optimal solution. The basic transportation problem was coined by Hitchcock [6]. The idea of decision making in fuzzy situation was scheduled by Bellman and Zadeh [2], many authors revealed an examination about FTP namely; Basir zadeh [1], Chanas and Kucht [3], Gani and Razak [5], Kaur and Kumar [7][8][9], Zimmerman [14]. Pandiyan and Natarajan [11] developed fuzzy zero point method for finding a fuzzy optimal solution for fuzzy transportation problems. Elizabeth and Sujatha [12] developed fuzzy one point method for finding fuzzy optimal solution for fuzzy transportation problems. In this article, a pentagonal fuzzy transportation problemswas solved by using a ranking technique and one point method were also used to find the optimal solution.

PRELIMINARIES

2.1. Fuzzy Sets[13]: A fuzzy set is an extension of a classical set, whose elements have a degree of membership, that is, it permits the gradual assessment of the membership of elements in a set. If X is the universal set, then a fuzzy set '$\tilde{A}$' in X is defined as a set of ordered pairs of x in $\tilde{A}$ where $\mu_{\tilde{A}}: X \to [0,1]$.

2.2 Fuzzy Numbers[13]: Among the various types of fuzzy sets, of special significance are fuzzy numbers that are defined on the set R of real numbers. Membership functions of these sets, which have the form $\mu_{\tilde{A}}: R \to [0,1]$ is viewed as fuzzy numbers.

2.3Pentagonal Fuzzy Numbers: A fuzzy number $\tilde{A} = (p,q,r,s,t)$ is called a pentagonal fuzzy number when the membership function has the form where the middle point r has the grade of membership 1 and w1, w2 are the respective grades of points q,s. Note that every PFN is associated with two weights w1 and w2.



$$\mu_{\tilde{A}}(X) = \begin{cases} \frac{1}{2}\left(\frac{x-p}{q-p}\right), p \le x \le q \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-q}{r-q}\right), q \le x \le r \\ 1 - \frac{1}{2}\left(\frac{x-r}{s-r}\right), \quad s \le x \le r \\ \frac{1}{2}\left(\frac{t-x}{t-s}\right), \quad r \le x \le t \\ 0, \quad x < p \; and \; x > t \end{cases}$$

1. Arithmetic Operations on Pentagonal Fuzzy Number

Let $\widetilde{A} = \{p_1, q_1, r_1, s_1, t_1\}$ and $\widetilde{B} = \{p_2, q_2, r_2, s_2, t_2\}$ be two pentagonal fuzzy numbers.

(i) Addition

$$\tilde{A}+\tilde{B} = \{p_1 + p_2, q_1 + q_2, r_1 + r_2, s_1 + s_2, t_1 + t_2\}$$

(ii) Subtraction

$$\tilde{A}-\tilde{B} = \{p_1 - p_2, q_1 - q_2, r_1 - r_2, s_1 - s_2, t_1 - t_2\}$$

(iii) Multiplication

$$\tilde{A} \times \tilde{B} = \{p_1 p_2, q_1 q_2, r_1 r_2, s_1 s_2, t_1 t_2\}$$

RANKING OF PENTAGONAL FUZZY NUMBERS

Let $\tilde{A} = (p, q, r, s, t)$ is called a pentagonal fuzzy number. Then the ranking of Pentagonal fuzzy number is

$$R(\tilde{A}) = \frac{(p + q + r + s + t)}{5}$$

MATHEMATICAL FORMULATION FOR BALANCED FTP

$$Z = Min\, Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} . x_{ij} \qquad (1)$$

Subject to the constraints

$$\sum_{j=1}^{n} x_{ij} = a_i\, , i = 1\ to\ m\, ;$$

$$\sum_{i=1}^{m} x_{ij} = b_j\, , j = 1\ to\ n\, ;$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j\ ;\ x_{ij} \geq 0\ \forall\ i, j \qquad (2)$$

Mathematical Formulation for Unbalanced FTP: [12]

$$Z = Min\, Z = \left(\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} . x_{ij}\right) - \left(\sum_{i=1}^{m} x_{ij}\right),$$

$$where\ j = n\ is\ a\ dummy\ column \qquad (3)$$

$$Z = Min\, Z = \left(\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} . x_{ij}\right) - \left(\sum_{i=1}^{n} x_{ij}\right),$$

$$where\ i = m\ is\ a\ dummy\ row \qquad (4)$$

TABLE 1

MATHEMATICAL FORMULATION OF FUZZY TRANSPORTATION PROBLEM

	D_1	…………	D_n	FS
S_1	$\tilde{c}_{11}$	…………	$\tilde{c}_{1n}$	$\tilde{a}_1$
⋮	⋮	⋮	⋮	⋮
S_m	$\tilde{c}_{m1}$	…………	$\tilde{c}_{mn}$	$\tilde{a}_m$
FD	$\tilde{b}_1$	…………	$\tilde{b}_n$	

ALGORITHM FOR FUZZY TRANSPORTATION PROBLEM

Step-1: The fuzzy transportation cost are given in terms of pentagonal fuzzy numbers.

Step-2: The pentagonal fuzzy numbers are defuzzified by using the ranking technique.

Step-3: Check that the given problem is a balanced one. If it is not balanced, convert the problem into the balanced one by establishing the dummy column or dummy row with cost entry as one.

Step-4: In transportation table, each row entries should be divided by the row minimum that is, if u_i is the minimum of the i^{th} row of the table$[\tilde{c}_{ij}]$then thei^{th}row entries should be divided by u_i, then the resulting table will be$[\tilde{c}_{ij}/u_i]$.

Step-5: After using Step-4, each column entries of the table $[\tilde{c}_{ij}/u_i]$ should be divided by the column minimum that is, if $v_j$is the minimum of j^{th} column of the table $[\tilde{c}_{ij}/u_i]$ then j^{th} column entries should be divided by v_j, then the resulting table will be$[(\tilde{c}_{ij}/u_i)/v_j]$. It is noted that $(\tilde{c}_{ij}/u_i)/v_j \geq 1$for all i, j. In the resulting table $[(\tilde{c}_{ij}/u_i)/v_j]$, each row and each column should have at least one fuzzy one entry.

Step-6: The column or row that has only one fuzzy one should be selected and the minimum of source and demand should be allotted to that corresponding cell. If the fuzzy supply points are fully used and all fuzzy demand points are fully received go to Step-8. If not, go to Step-7.
Step-7: Draw a minimum number of lines horizontally and vertically to cover them all (all the ones). Then select the least unveiled element and divide all the unveiled elements with it and multiply it at the intersection of the lines, leaving the other elements uninterrupted. Now check if each row and column contains at least one fuzzy input. If so, go to step 6; otherwise go to step 4, step 5, then go to step 6.
Step-8: This mapping provides an optimal solution for the FTP given with the objective function

$$Min\ Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}.x_{ij}$$

if PFTP is balanced, without establishing a dummy column or a dummy row (or),

$$Z = Min\ Z = \left(\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}.x_{ij}\right) - \left(\sum_{i=1}^{m} x_{ij}\right)$$

if the PFTP is balanced by establishing a dummy column $j = n$(or),

$$Z = Min\ Z = \left(\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}.x_{ij}\right) - \left(\sum_{i=1}^{n} x_{ij}\right)$$

if the PFTP is balanced by establishing a dummy row $i = m$.
All these objective functions are subject to the conditions

$$\sum_{j=1}^{n} x_{ij} = a_i\ , i = 1\ to\ m\ ; \sum_{i=1}^{m} x_{ij} = b_j\ , j = 1\ to\ n\ ; \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j\ ;\ x_{ij} \geq 0\ \forall\ i,j.$$

Finally we get the fuzzy transportation cost.
Note: Consider a fuzzy transportation table (FTT)$[\tilde{c}_{ij}]$ with m sources S_i , $i = 1\ to$ m and n destinations D_j , $j = 1\ to\ n$. Let $\tilde{a}_i = (a_{1i}, a_{2i}, a_{3i}, a_{4i}, a_{5i})$ be the fuzzy supply (FS) at source S_i and $\tilde{b}_j = (b_{1j}, b_{2j}, b_{3j}, b_{4j}, b_{5j})$ be the fuzzy demand (FD) at destination D_j . Let $\tilde{c}_{ij} = (c_{1ij}, c_{2ij}, c_{3ij}, c_{4ij}, c_{5ij})$ be the fuzzy transportation cost (FTC) of FTT $[\tilde{c}_{ij}]$ from source S_i to destination D_j .The problem then is to determine a feasible way to transport the quantity available at each source to satisfy the demand at each destination, so that the total cost of transport is minimized in a fuzzy environment. This problem can be represented as follows:

NUMERICAL EXAMPLES

(a) Let us consider the pentagonal fuzzy transport problem with three sources, namely S_1, S_2, S_3 and three destinations D_1, D_2, D_3. The costs of transporting a unit of goods from the ith source to the jth destination, the elements of which are pentagonal fuzzy numbers, are shown in Table 2. Determine the minimum cost of all fuzzy transportation.

TABLE 2
PENTAGONAL FUZZY TRANSPORTATION TABLE

	D_1	D_2	D_3	**Supply**
S_1	(1,3,9,7,10)	(6,4,5,9,11)	(5,4,8,13,15)	(20,40,60,70,80)
S_2	(1,3,6,8,7)	(3,5,7,10,15)	(2,4,7,9,13)	(25,35,45,85,90)
S_3	(2, 4, 7, 10, 12)	(4,10,9,12,15)	(3,5,6,8,13)	(30,50,60,70,90)
Demand	(25,35,45,65,80)	(40,50,55,70,85)	(35,40,55,75,95)	

Approximate the given pentagonal fuzzy supply, cost and demand using ranking technique, now we get the crisp transportation table

TABLE 3
CRISP TRANSPORTATION TABLE

	D_1	D_2	D_3	**Supply**
S_1	6	7	9	54
S_2	5	8	7	56
S_3	7	10	7	60
Demand	50	60	60	170

Now applying step-4 and step-5 we get the following table,

TABLE 4
RESULTING TABLE WITH AT LEAST ONE FUZZY ONE ENTRY IN EACH ROW AND COLUMN

	D_1	D_2	D_3	**Supply**
S_1	1	1	1.5	54
S_2	1	1.37	1.4	56
S_3	1	1.22	1	60
Demand	50	60	60	170

Allot the minimum of source and demand to the cell in the row or column with only one fuzzy one.

TABLE 5
FUZZY ONE ALLOTMENT TABLE

	D_1	D_2	D_3	**Supply**
S_1	*	*54		54
S_2	*50			56
S_3	*		*60	60
Demand	50	60	60	170
'*' denotes the place of 1's				

Here, fuzzy supply points are not fully utilized and fuzzy demand points are not fully received.
Go to step 7 and repeat the process until all fuzzy mount points have been fully utilized and all fuzzy demand points have been fully received.

TABLE 6
FINAL OPTIMAL TABLE

	D_1	D_2	D_3	**Supply**
S_1		*54		54
S_2	*50	*6		56
S_3			*60	60
Demand	50	60	60	170

The minimum transportation cost is

$$Min\ Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}.x_{ij}$$

$$Z = (54 \times 7) + (50 \times 5) + (6 \times 8) + (60 \times 7)$$
$$= 378 + 250 + 48 + 420$$

$= 1096.$

(a) Consider afuzzy transportation problem where supply at sources, demand at destination, and fuzzy transportation costs, etc., are taken as pentagonal fuzzy numbers.

TABLE 7
PENTAGONAL FUZZY TRANSPORTATION TABLE

	D_1	D_2	D_3	**Supply**
S_1	(5, 10, 13,14, 18)	(1, 2, 3, 4, 5)	(2, 6, 8, 10, 14)	(2, 11, 23, 34, 45)
S_2	(3, 4, 5, 6, 7)	(1, 5, 6, 7, 11)	(1, 4, 5, 9, 16)	(10, 47, 52, 65, 76)
S_3	(3, 6, 9, 12, 15)	(2, 5, 7, 8, 8)	(1, 1, 1, 1, 1)	(3, 18, 56, 76, 87)
Demand	(11,16, 51, 67, 75)	(20, 40, 60, 80, 100)	(15, 30, 45, 75, 110)	

The given fuzzy transportation problem is unbalanced. Now, convert the unbalanced fuzzy transportation problem into a balanced one.

TABLE 8
BALANCED FUZZY TRANSPORTATION TABLE

	D_1	D_2	D_3	**Supply**
S_1	(5, 10, 13,14, 18)	(1, 2, 3, 4, 5)	(2, 6, 8, 10, 14)	(2, 11, 23, 34, 45)
S_2	(3, 4, 5, 6, 7)	(1, 5, 6, 7, 11)	(1, 4, 5, 9, 16)	(10, 47, 52, 65, 76)
S_3	(3, 6, 9, 12, 15)	(2, 5, 7, 8, 8)	(1, 1, 1, 1, 1)	(3, 18, 56, 76, 87)
S_4	(1, 1, 1, 1, 1)	(1, 1, 1, 1, 1)	(1, 1, 1, 1, 1)	(31, 10, 25, 47, 77)
Demand	(11, 16, 51, 67, 75)	(20, 40, 60, 80, 100)	(15, 30, 45, 75, 110)	

Approximate the given pentagonal fuzzy supply, cost and demand using ranking technique, now we get the crisp transportation table.

TABLE 9
CRISP TRANSPORTATION TABLE

	D_1	D_2	D_3	**Supply**
S_1	12	3	8	23
S_2	5	6	7	50
S_3	9	6	1	48
S_4	1	1	1	38
Demand	44	60	55	159

Now applying step-4 and step-5 we get the following table.

TABLE 10
RESULTING TABLE WITH AT LEAST ONE FUZZY ONE ENTRY IN EACH ROW AND COLUMN

	D_1	D_2	D_3	**Supply**
S_1	4	1	2.67	23
S_2	1	1.2	1.4	50
S_3	9	6	1	48
S_4	1	1	1	38
Demand	44	60	55	159

Allot the minimum of source and demand to the cell in the row or column with only one fuzzy one.

TABLE 11
FUZZY ONE ALLOTMENT TABLE

	D_1	D_2	D_3	**Supply**
S_1		*23		23
S_2	*44			50
S_3			*48	48
S_4	*	*31	*7	38
Demand	44	60	55	159
'*' denotes the place of 1's				

Here, fuzzy supply points are not fully utilized and fuzzy demand points are not fully received.
Go to step 7 and repeat the process until all fuzzy mount points have been fully utilized and all fuzzy demand points have been fully received.

TABLE 12
FINAL OPTIMAL TABLE

	D_1	D_2	D_3	**Supply**
S_1		*23		23
S_2	*44	*6		50
S_3			*48	48
S_4		*31	*7	38
Demand	44	60	55	159

The fuzzy optimal cost is given by

$$Z = Min\ Z = \left(\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}.x_{ij}\right) - \left(\sum_{j=1}^{n} x_{ij}\right)$$

$$Z = \{(23 \times 3) + (44 \times 5) + (6 \times 6) + (48 \times 1) + (31 \times 1) + (7 \times 1)\} - \{31 + 7\}$$

$$= (69 + 220 + 36 + 48 + 31 + 7) - (38)$$

$$= 411 - 38 = 373$$

CONCLUSION

In this article, a systematic procedure is developed for solving balanced and unbalanced pentagonal fuzzy transportation problems using ranking technique for the representative value of the pentagonal fuzzy numbers. Fuzzy one point method is used to find the optimal solution that is the minimum transportation cost.

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