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Application of Queueing Theory for the Improvement of Railway Services

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Abstract - In most railway stations, the lines of waiting passengers are always very long. The M/M/K/∞: FCFS model is changed into M/M/1/∞: FCFS to identify which is more efficient, a single line or several lines. Here, we developed an optimization technique for queuing to satisfy the customers demand. The outcome of analysis was effective and realistic of these factors.

Keywords **-** $M/M/K/\infty$ model, Optimal service stations, Queueing model.

INTRODUCTION

Operations research [2] gives a scientific approach to decision – making that involves the operations of organizational system. Queueing theory is the study of customers demand, satisfaction and improving the service facility of servers under various circumstances. One of the major problems in queuing systems that we come across in our daily lives is business service systems, in which customers obtain service from commercial organisations. Many organisations provide direct service at a fixed location, such as barbershop, gas station, bank etc. and many commercial organisations such as railways, hospitals that have made tremendous attempts to enhance service efficiency to meet the customer satisfaction, since the majority of them still have long lines. Customers wait in a line at the bank due to the inefficiency of the queuing system, which demonstrates a lack of customer-centric business strategies and a low system service rate. Customers form the line because a consumer's service may not be given as soon as the customer arrives at the service facility. Customers would build a queue if the service facility were not sufficient. The only approach to readily fulfil service demand is to increase service capacity while simultaneously improving the efficiency of existing capacity.

Toshiba Sheikh, Sanjay Kumar Singh, Anil Kumar Kashyap [3] studied the application of queuing theory in banking sectors. Many authors [1],[4],[5],[6],[7] have developed the optimization technique to overcome the difficulty of long queues. In this paper a new optimization technique is developed to reduce the waiting time of the customer. The determination to reduce customer waiting time is then obtained using queuing theory to address the problem of long customer waiting lines to satisfy the goal of people centred with maximum effectiveness.

FORMULATION OF QUEUEING SYSTEM

A. *Queuing Theory*

Queuing theory is a study of analysing waiting lines. Many authors have studied different types of queuing systems with various models. The Queue model is a powerful tool for figuring out how to handle a queueing system in the most effective manner. Behaviour problems, optimization problems, and statistical inference of queuing systems are at the base of queuing theory, which is also known as random system theory.

B. *Terminology and Notations*

In order to formulate and calculate the model, the following terminology and notations are used:

N =Number of Customers in the System.

P_n =Probability of exactly n customers in the system.

L_s = Average number of customers in the system.

L_q = Average number of customers in the queue.

W_s =Waiting time of customers in the system.

W_q =Waiting time of customers in the queue.

λ_n =The mean arrival rate (expected number of arrivals per unit time) are new customers are in systems.

μ_n =The mean service rate for overall systems (expected number of customers completing service per unit time) when n customers are in systems.

The mean arrival rate is constant for all n, and the mean service rate per busy server is constant for all $n \geq 1$, which is denoted by μ. And when $n \geq K$ that is all K servers are busy, $\mu = s\mu$. Under this condition, the expected inter-arrival time is $\frac{1}{\lambda}$ and the expected service time is $\frac{1}{\mu}$. The utilization factor for the service facility is $\rho = \frac{\lambda}{\mu k}$. i.e., the expected fraction of time as the individual servers are busy because $\frac{\lambda}{\mu k}$ represents the fraction of the service capacity $(k\mu)$ of the system that is average by arriving customers λ.

C. $M/M/1/\infty$ *Model*

This is the easiest queueing method to comprehend. In the system, there is just one server. Arrivals are distributed according to a Poisson distribution with a mean arrival rate of λ, whereas service time is distributed according to an exponential distribution with an average service rate of μ. $P_n = p(N = n)$ (n=0,1, 2,…) is the probability distribution of the queue length. i.e., the percentage of time when servers are busy: $\rho = \frac{\lambda}{\mu}$

Expected number of customers in the system $L_s = \frac{\rho}{1-\rho}$

Expected number of customers waiting on the queue: $L_q = \frac{\rho^2}{1-\rho}$

Expected waiting time of customers in the queue: $W_q = \frac{\rho}{\mu-\lambda}$

Expected waiting time of customers in the queue: $W_s = \frac{1}{\mu-\lambda}$

D. $M/M/k/\infty$ *Model*

This model considers the case where many service stations are open at the same time and each consumer in line can be served by more than one station channel. Consider an M/M/k queue with z servers, an arrival rate λ, and a service rate μ. The ratio is used to define the traffic intensity.

$$\rho_k = \frac{\rho}{k} = \frac{\lambda}{k\mu}$$

The following is a study of the steady distribution of a queuing system: When the number of system servers is k, P n=p(N=n) (n=0,1, 2,...) is the probability distribution of the queue length N in steady state. Then we have $\lambda_n = \lambda$, n = 0,1,2,… The following two possibilities can occur if there are n customers in the line at any given time:

(i) There will be no queue if $n < k$ (the number of customers in the system is smaller than the number of servers). The $(k - n)$ number of servers, on the other hand, will not be overcrowded. $n < k$; $\mu_n = n\mu$.

(ii) If $n \geq k$ (the number of customers in the system is greater than or equal to the number of servers), all servers will be busy with the maximum number of customers in the queue being $(n - k)$. The combined service rate will then be: $\mu_n = k\mu$; $n \geq k$.

The probability of having n consumers in the system is calculated using the model as follows:

$$\rho_k = \frac{\lambda}{k\mu}$$

$$P_0 = \left[\sum_{n=0}^{z-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{k!}\left(\frac{\lambda}{\mu}\right)^k \frac{k\,\mu}{k\,\mu - \lambda}\right]^{-1}$$

$$P_n = \begin{cases} (\rho^n/n!)P_0 \; n \leq k \\ \rho^n/(k!\,k^{n-k})P_0 \; n > k \end{cases}$$

When $n \geq k$, it means that the number of clients in the system is greater than the number of servers, which means that the following consumers will have to wait.

$$C(k,\rho) = \sum_{n=k}^{\infty} P_n = \frac{\rho^k}{k!\,(1-\rho_k)} P_0$$

Where

$$\rho_k = \frac{\rho}{k} = \frac{\lambda}{k\mu}$$

Expected number of customers waiting on the queue:

$$L_q = \left[\frac{1}{(k-1)!}\left(\frac{\lambda}{\mu}\right)^z \frac{\mu\lambda}{(k\mu - \lambda)^2}\right] P_0$$

Expected number of customers in the system:

$$L_s = L_q + \frac{\lambda}{\mu}$$

Expected waiting time of customers in the queue:

$$W_q = \frac{L_q}{\lambda}$$

Expected time a customer spends in the system:

$$W_s = \frac{L_s}{\lambda}$$

OPTIMIZATION IN RAILWAY STATION QUEUE

The Railway Station Queuing Problem is investigated in terms of the following issues using the queuing theory:

A. One Line or More

In reality of course, the railway station has waiting time, and there are multiple service stations. A queue or a waiting line can be found at every service station. If each service station has a queue according to their agenda, the arriving customers join each line with probability 1/2, known as the two scheduled queues. When there are two lines, for example, the system can be thought of as two distinct M/M/1 systems, with arrival rate of each service station is $\lambda = \lambda/2$. If there is a line, the system will be M/M/2, L, L_q, W, and W_q will be computed and compared to see which is more efficient, and we will analyse it from a technical standpoint as follows:

When there is a line k=2, $\lambda = 70$, $\mu = 60$, $\rho = 7/6$

$$P_0 = \left[\sum_{n=0}^{z-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{k!}\left(\frac{\lambda}{\mu}\right)^k \frac{k\,\mu}{k\,\mu - \lambda}\right]^{-1} = 0.3798$$

$$L_q = \left[\frac{1}{(k-1)!}\left(\frac{\lambda}{\mu}\right)^k \frac{\mu\lambda}{(k\mu - \lambda)^2}\right] P_0 = \ 0.0012$$

$$L_s = L_q + \frac{\lambda}{\mu} = 1.168$$

$$W_q = \frac{L_q}{\lambda} = 1.714$$

$$W_s = \frac{L_s}{\lambda} = 0.017$$

When there are two lines $\lambda = \lambda/2 = 35$, $\mu = 60$, $\rho = 7/12$

$$L_s = \frac{\rho}{1-\rho} = 1.398$$

$$L_q = \frac{\rho^2}{1-\rho} = 0.815$$

$$W_s = \frac{1}{\mu - \lambda} = 0.04$$

$$W_q = \frac{\rho}{\mu - \lambda} = 0.023$$

When there are three lines $\lambda = \lambda/3 = 23.333$, $\mu = 60$, $\rho = 23.333/60$

$$L_s = \frac{\rho}{1-\rho} = 0.637$$

$$L_q = \frac{\rho^2}{1-\rho} = 1.884$$

$$W_s = \frac{1}{\mu - \lambda} = 0.027$$

$$W_q = \frac{\rho}{\mu - \lambda} = 0.011$$

When there are four lines $\lambda = \lambda/4 = 17.5$, $\mu = 60$, $\rho = 17.5/60$

$$L_s = \frac{\rho}{1-\rho} = 0.412$$

$$L_q = \frac{\rho^2}{1-\rho} = 0.120$$

$$W_s = \frac{1}{\mu-\lambda} = 0.024$$

$$W_q = \frac{\rho}{\mu-\lambda} = 0.007$$

When there are five lines $\lambda = \lambda/5 = 14$, $\mu = 60$, $\rho = 14/60$

$$L_s = \frac{\rho}{1-\rho} = 0.304$$

$$L_q = \frac{\rho^2}{1-\rho} = 0.070$$

$$W_s = \frac{1}{\mu-\lambda} = 0.022$$

$$W_q = \frac{\rho}{\mu-\lambda} = 0.005$$

When there are n lines, it indicates that there are n service stations, each with a queue based on its schedule, and each arriving customer enters each queue with a probability of 1/n. It's referred to as planned n queues. The average rate of arrival is λ /n, while the average rate of service is μ.

Expected number of customers in the system: $L_s = \frac{\lambda/n}{\mu-\lambda/n} = \frac{\lambda}{\mu-\lambda}$

Expected number of customers waiting on the queue: $L_q = \frac{\lambda L}{n\mu} = \frac{\lambda^2}{n\mu(n\mu-\lambda)}$

Expected waiting time of customers in the queue: $W_q = \frac{nL_q}{\lambda} = \frac{\lambda}{\mu(n\mu-\lambda)}$

Average time a customer spends in the system: $W_s = \frac{1}{\mu-\lambda/n} = \frac{n}{n\lambda-\lambda}$

Table I shows that the system's waiting time is 0.04 in the case of two lines and 0.023 in the case of one line. The amount of time spent waiting is visibly decreasing, as is the length of the queue. This demonstrates that in railway services, whether the idea of justice is “first come, first served,” or if technically, a line is better than multiple lines, station management should pay attention to this issue.

TABLE 1

CHARACTERISTICS OF QUEUEING SYSTEMS

Lines	$\boldsymbol{\mu}$	$\boldsymbol{\lambda}$	$\boldsymbol{L_s}$	$\boldsymbol{L_q}$	$\boldsymbol{W_s}$	$\boldsymbol{W_q}$
1	60	70	1.168	0.0012	0.017	1.714
2	60	35	1.398	0.815	0.04	0.023
3	60	23.333	0.637	1.884	0.027	0.011
4	60	17.5	0.412	0.120	0.024	0.007
5	60	14	0.304	0.070	0.022	0.005

CONCLUSION

The following three measures improve the efficiency of railway stations: the queuing number, the number of service stations, and the ideal service rate are explored using queuing theory. The outcomes are successful and practical, as demonstrated by the example. The amount of time customers has to wait is reduced. The level of customer satisfaction has improved. It was demonstrated that this optimal queuing model is feasible.

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