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Mapping of vegetation cover hotspot using the multifractal singularity of NDVI applied to the fog oasis of Lúcumo, Lima, Peru

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Abstract. The study's objective was to verify the capacity of the multispectral singularity index (MSI) to delimit the sites of more excellent vegetation, of the fog oasis Loma de Lúcumo (LL), of the city of Lima, Peru. This ecosystem is affected by industrial and mining activities, as well as by informal urbanization. The methodology used was an adaptation of the method applied by Q. Cheng, in 2001, for the analysis of singularity in image processing. Normalized Difference Vegetation Index (NDVI) data derived from satellite images of different spatial resolutions and dates were used. To test the suitability of the MSI, its performance was visually compared with orthoimages from drones. It was evidenced and verified in the field that the MSI map detects the patches with the highest vegetation cover of the LL. The said map does not replace an NDVI map but complements it by adding information on spatial association and detection of particular sites. It is indicated that both the NDVI and the MSI are insufficient to prepare a vegetation map of the LL since a floristic and vegetation inventory is required as a necessary condition. It is recommended that a detailed vegetation map of the LL be developed with the support of NDVI and MSI. Likewise, the methodology used for multifractal modeling and spatial analysis with GIS is recommended.

Keywords. Fog oasis, Singularity; NDVI, GIS, vegetation mapping

1. Introduction

The Loma de Lúcumo fog oasis has been considered by the Peruvian State as a "Fragile Ecosystem" [1] to protect its ecological integrity threatened by industrial and mining activities, as well as by the informal urbanization that occupies lands around it. It is even perceived that the roads enabled within the foggy oasis for its environmental interpretation cross patches of dense vegetation that should be more protected. Therefore, detailed vegetation maps of a said oasis of fog must be drawn up to redesign and improve the location of civil structures and reorder human recreational activities in said space.

The plant communities of the fog oases are markedly seasonal; There are even years, mainly very dry, in which vegetation does not appear in large sectors, reappearing in average years [2] [3] [4]. Such communities are predominantly made up of annual herbaceous species that reach their most extraordinary vegetative development between June to October (wet season), covering considerable extensions on plains and hillsides facing the sea. In this wet season, an impressive exuberance of vegetation is observed that attracts visitors who come to these places for recreation purposes. But not the full extension of the foggy oasis presents a great intensity of vegetation cover since not all the sites are homogeneously humidified. Some sites offer better conditions for capturing the moisture from the mist coming from the sea, due to its natural condition as a mist-catcher, in which the vegetation develops with greater vigor and is made up of permanent species, with their adaptations to the water stress that comes with the overwhelming dry season, of the months from January to May. Although all the sites are essential, as they are part of the functioning dynamics of the fog oasis ecosystem, there are relevant sites that maintain their vegetation cover throughout the year, constituting patches of vegetation that serve as a natural reservoir. In the ecological landscape, for which they acquire a particular relevance or uniqueness. Therefore, the location and mapping of these patches or hotspots of vegetation cover are necessary for the effective management of the ecosystem.

On the other hand, it is of great help, for the maintenance of the vegetal cover of the fog oases, to recognize the patches of more fabulous vegetal cover since it can be inferred that there the seed bank would be more abundant and that although in some years vegetation is not presented, in the same way, they should be protected; because when the years are more humid, it will have been ensured that the area is available for the growth of plants, especially in places of marked seasonality or erratic behavior of the foggy oasis. On the other hand, it is essential to recognize the limits or maximum area up to which the vegetation will extend in the years of extraordinary humidity, which implies making periodic cartographies of the vegetation cover since the oasis of fog is also dependent on the global process of the El Niño Southern Oscillation (ENSO).

There is also a practical problem in preparing vegetation maps in Peru, referring to the fact that too many times, the non-existence of an oasis of fog is assumed in areas where this ecosystem should potentially be present. Although the solution could be straightforward from reviewing the time series of freely available images and obtaining the Normalized Difference Vegetation Index (NDVI), this action is not always carried out. However, it is suggested in the technical guides [5] [6]. The elaboration of the vegetation cover maps becomes critical in developing the environmental impact studies of projects since, from their cartography, the ecosystem services of the place that could be affected by a project are evaluated. On the other hand, the vegetation map units come to be considered ecological units due to the condition of indicator or integrating variables [7]. On the other hand, as the NDVI is very easy to obtain, it creates the illusion that the vegetation has been characterized from contours or isolines by ranges of NDVI. Categorize them into ovens.

Generally, the NDVI is widely used and simplified for the delimitation of the vegetation cover without considering the spatial variability of the NDVI, their association, and their local singularities. In this context, to improve the mapping of the vegetation cover, in particular of the fog oases, the present study was developed whose objective was to verify the capacity of the multispectral singularity index developed by Qiuming Cheng in 1999 [8], to delimit the sites with the highest vegetation cover in a foggy oasis, based on NDVI data. Although the MSI has been widely used in the analysis of geochemical data for the prospecting of valuable minerals [9], insofar as it is sensitive to detect particular sites with a higher concentration of information on a variable under study, which could be nested due to the general trend marked by the typical gradient of the variable, the question arose about the ability of said MSI to detect the sites with the highest concentration of vegetation based on their spectral responses or NDVI.

Regarding the MSI, it is possible to affirm that in the analysis of geochemical data, in mining prospecting, there is extensive experience in using this index [8] [9]. In geological exploration, the enrichment of mineral-forming elements can be described as a singular process and, therefore, can be detected by the particular mapping technique [10] [11]. In this field, Cheng, 1999, [8] proposed a new multifractal concept called "singularity," a unique phenomenon accompanied by an anomalous energy release or accumulation of material within narrow space-time intervals.

Uniqueness is a property of many different types of non-linear natural processes, including the formation of clouds, rains, hurricanes, floods, landslides, earthquakes, forest fires, and mineralization [12]. Multifractal models can characterize continuous physical and chemical values and spatial objects, whose spatial patterns of concentration values satisfy a multifractal model. It is possible to point out that the fractal and multifractal models implemented with GIS will be consolidated, and their uses will expand in geosciences and other fields [10]. However, no works have been found in ecology and vegetation mapping using the singularity map, therefore the interest of the present investigation.

The method developed by Cheng [8] maintains the properties of the local structure when applied to 1-D data interpolation as it integrates both the spatial association and the local singularity. That is, it combines the semivariogram that quantifies the spatial association and a singularity index to characterize the local structure of the data. The method is also extended to 2-D cases to create surfaces from the interpolation of 2-D point data [13] [14]

Spatial association indices (autocorrelation, covariance, and variogram) have been commonly used to characterize the local structure of spatial patterns, and the information derived from these indices can be used for data interpolation (i.e., Kriging). Defined singularity in the multifractal context is another index to measure the scale-invariant property of spatial patterns. Compared to spatial association indices, singularity is widely used in geochemical data analysis to prospect valuable minerals. Still, it is not yet recognized or generally used in digital image processing and geostatistics in mapping: vegetation and ecology [8].

The fractal structure of NDVI is calculated from satellite images. Around each point in the picture, and NDVI window average is calculated, and a record is plotted as a function of the number of pixels. This is repeated for windows of many sizes. This produces a logarithmic graph where you can fit a straight line. The slope of the line is related to the fractal dimension. This procedure applied to each point of the image produces a fractal dimension map.

The local structural property is often considered in image enhancement with various high/low pass filters. Spatial association indices (autocorrelation, covariance, and variogram) have been commonly used to characterize local surface structure for data interpolation in kriging. Singularity is another index that measures the scale-invariant property of measurements from a multifractal point of view. Compared to spatial association indices, the singularity has yet to be generally understood by the image processing community.

The concentration of NDVI is given by

$X = c\ \varepsilon$

where X is the concentration of NDVI, is the singularity, $2 - \alpha$ is the fractal dimension, ε is the scale size, and c is the density. The singularity is a number close to 2 and is related to the concentration of some quantity. If $\alpha < 2$, there is a lack of concentration, and if $\alpha > 2$, there is an increase in concentration.

Applying the concept of singularity to the analysis of a map, the positive singularity (a <2) generally corresponds to concentration values of "enriched" or high elements, while the negative singularity is to "exhausted" values. Therefore, the estimation of singularities of a map can be applied to characterize the patterns with concentration values of particular elements that could provide helpful information to interpret geochemical anomalies related to mineralization activities or local structures.

The singularity index generally has finite values around 2. For a conservative multifractal measure, the dimension of the set with $\alpha = 2$ is close to 2 (box count dimension), which means that the areas with non-singular values occupy the most significant part of the map. The fractal spectrum function f gives the dimensions of the other regions with $\alpha \neq 2$ (α) <2.

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2. Materials and Method

The study was carried out in the fragile ecosystem of Loma de Lúcumo [1]. The southern sector of the said ecosystem was prioritized as it is the area where ecotourism activities are carried out with the most incredible intensity. Therefore, the results of this study could contribute to its management. Figure 1 shows the location of the study area.

Figure 1. Location map of the fog oasis Loma de Lúcumo

To obtain the singularity map, an adaptation of the method applied by Qiuming Cheng in 2001 was used in the "Singularity analysis for image processing and improvement of anomalies," from which the algorithm was developed to obtain the map of singularity, in the Matlab computer program. [14] [15], [16], [17], [18], [19], [20], [21], [22], [23], [24]

<u>Algorithm to obtain the singularity index map</u>

Singularity is a measure of how a certain quantity varies with changes in scale. For example, the average value of a property will depend on the neighborhood's size used to calculate that average at specific points. In other cases, the standard will be independent of the size of the community. The singularity is a value close to 2. Where $\alpha < 2$, there is an increase of concentration, and where $\alpha > 2$, there is a decrease of concentration.

The algorithm to perform fractal interpolation using Matlab 2001a is as follows:

a) Reading the image: The image is loaded into memory using the imread function.

b) Running averages: A filter is applied to the image to obtain the average of a pixel within a neighborhood. The filter is nothing other than an array like the following

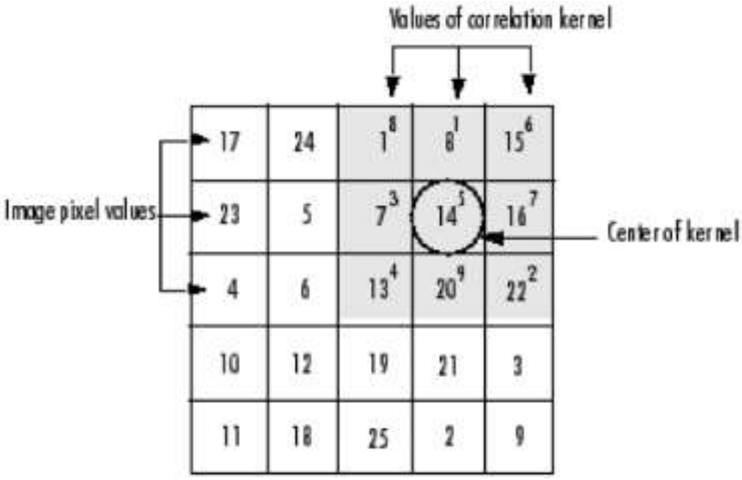
$$k = \begin{bmatrix} 8 & 1 & 6 \\ 7 & 5 & 7 \\ 4 & 9 & 2 \end{bmatrix}$$

This array is often called a kernel. The filter is nothing more than a weighted average of the pixels that are within a neighborhood. In this case, the weights of the pixels are given by the values of the elements of the matrix k. The image shows how the average is obtained for the pixel whose value is 14. The details of the matrices are multiplied, element by element, the products are added and divided by the number of factors.

$$A_{24} = \frac{1}{9}(1*8+8*1+15*6+7*3+14*5+16*7+13*4+20*9+22*2)$$

In the case of fractal interpolation, the kernels are arrays of ones. Different size kernels are applied to each pixel. To obtain the value of the singularity, a line is fitted to the graph of the average given by each grain against the size of each grain. The seeds used like the following:

$$k_{2,2} = \frac{1}{4}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$



The kernels used are square matrices whose size is powers of 2, 2x2, 4x4, 8x8, 16x16, etc.

c) Calculation of singularity: The base ten logarithms of the running averages and the number of data must have a linear relationship to obtain the singularity. In case the relationship is not linear, then there is no singularity. For each pixel of the image, a graph like the following is obtained. On the X-axis is the number of kernel elements and on the Y-axis are the running averages. Suppose the correlation coefficient is more significant than a predetermined value close to 1. In that case, the relationship between standards and the number of elements is considered to be linear, and the singularity will be given by $m = \alpha - 2$. Where m is the slope of the line and α is the value of the singularity.

If the correlation coefficient is not greater than the default value, then $\alpha = 2$ is defined. The value of the ordinate at the origin corresponds to the importance of c (xi) in the model $Z(x) = c(x)\, \varepsilon^{\alpha - 2}$, proposed by Cheng. These c (xi) are used to calculate the λ needed for interpolation. In the case that $\alpha = 2$, the value c (xi) coincides with the pixel value.

d) Map of singularities. Once the singularity has been obtained for each pixel, then the singularity map can be made. Singularities are values close to 2. If the singularity is greater than 2, then the pixel lacks concentration; if it is less than 2, then there is an excess of attention.

About satellite images

Data from the NDVI derived from satellite images of different spatial resolutions and due dates in the ecosystem dynamics of the Loma de Lúcumo fog oasis were used. To test the suitability of the multispectral singularity index, it was compared with drone orthoimages. There was a video of the drone overflight over the study sector, in addition to the field survey, in the dry and wet season in several years (five) until 2019.

Obtaining satellite images relevant for the study was problematic, mainly because the Loma de Lúcumo foggy oasis remains totally or partially covered with clouds during winter, the season in which they present their best development. Such limitation makes it impossible to provide feasible images (without cloud cover) to attend an investigation on the behavior of the vegetation cover and its NDVI at the time when the foggy oasis presents its highest vegetation cover.

There was a historical SPOT image, with a spatial resolution of 20 m, from November 30, 1998. In that year, the vegetation cover of the hills reached an extraordinary development; for this reason, that date is critical to carry out retrospective studies on the mountain of Atocongo [25]. Likewise, another historical IKONOS image was located, dated March 10, 2002, with 4 m spatial resolution. A GE1 image, dated January 3, 2015, with a resolution of 50 cm was also included. A Landsat image was also selected, dated April 16, 2020. From each selected image, the normalized difference indices (NDVI) were obtained according to the standard methodology.

3. Results

The combined use of the NDVI and MSI have allowed us to recognize the sites with the most significant vegetation cover in the Loma de Lúcumo foggy oasis. These sites must be conserved with priority to maintain the characteristics of the ecosystem since there the vegetation presents outstanding features in terms of more excellent plant cover, biomass, productivity, vigor, among other traits closely correlated with the NDVI. Figures 2 and 3 show representative views of the landscape of the Lúcumo fog oasis. Figure

2 presents a picture of Q. Lúcumo from the mid-altitude level towards the lower part of the creek. Figure 3 shows a view of the Q. Verde from the altitudinal deck towards the lower leg.

Figure 2. Dry season landscape, with trees of *Vachellia macracantha*

Figure 3. Wet season landscape, with Quebrollo scrub, *Acnistus arborescent*

The substantial seasonal influence of the climatic conditions of the place is confirmed since the NDVI values of the patches shown in Figure 4 vary widely. In the humid season, patches of NDVI values greater than 0.40 are located, which is a high value for an oasis of fog. On the other hand, it is possible to find fewer dry patches in the dry season, with NDVI values of 0.14. In general, the NDVI values are higher in the months of higher humidity and lower temperature (July, August, September) about the hottest and less humid months (January, February, March). One aspect to consider is that the patches shown in Figure 4 have been obtained through NDVI contours, not MSI. Therefore, the present objective of incorporating the MSI in the vegetation cover studies would facilitate the patches' location with the highest vegetation cover intensity.

It is usual for ecological and environmental study purposes to develop vegetation maps based on NDVI. Figure 5 exemplifies these types of maps that may well be expeditious, but they are not maps that reflect a physiognomy or floristics of the oasis of fog. Said Figure 5 would be a base map of NDVI since it is made from the NDVI of a SPOT image of November 1998, a year not yet exceeded, to date, in terms of the intensity of vegetation cover on hills [25]. The 1997/98 El Niño was followed by a prolonged period of cold La Niña conditions from mid-1998 to early 2001.

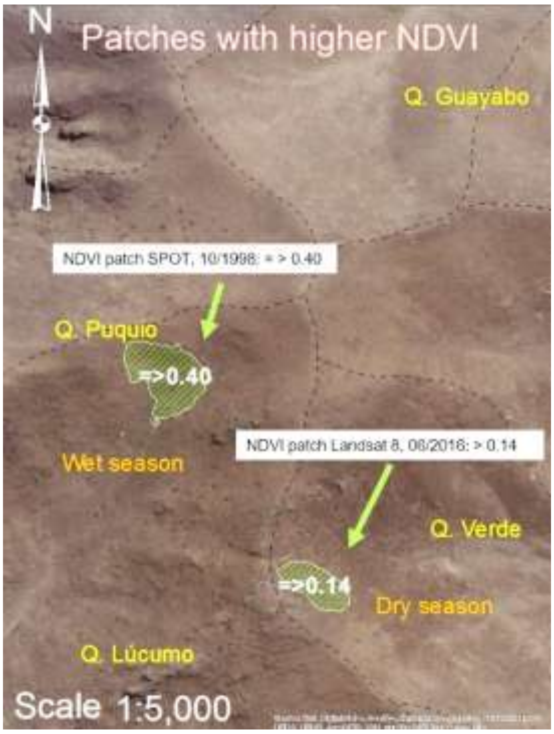


Figure 4. Patches with the highest NDVI of the foggy oasis by season

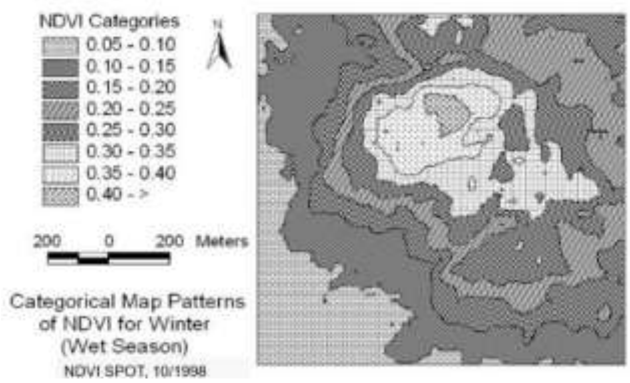


Figure 5. NDVI map for the wet season, SPOT 1998

Figure 6 also shows an NVDI category map, for the year 2002, from Ikonos NDVI. That year was also important as the first El Niño of the 21st century occurred in 2002/03. According to standard ENSO indices, this event was of moderate-intensity and less dramatic than the 1997/98 El Niño or the major 1982/83 El Niño. Figure 6 shows dry season conditions that differ from Figure 5, which shows wet season conditions. What is also interesting about Figure 6 is that it presents a comparison with a singularity map obtained from the same image. In the colored map, the same lines shown in the NDVI category map can be seen in blue stripes. Inside the blue lines, it is possible to observe patches in green tonality that correspond to the sites of most extraordinary singularity (MSI values less than 2, according to the methodology presented, Cheng 2001). The cherry-colored patches correspond to areas without vegetation cover. That is to say, said colored Map gives the patches with the most significant vegetation cover for that month and that year; and, in turn, shows the advantage of obtaining a singularity map from NDVI.

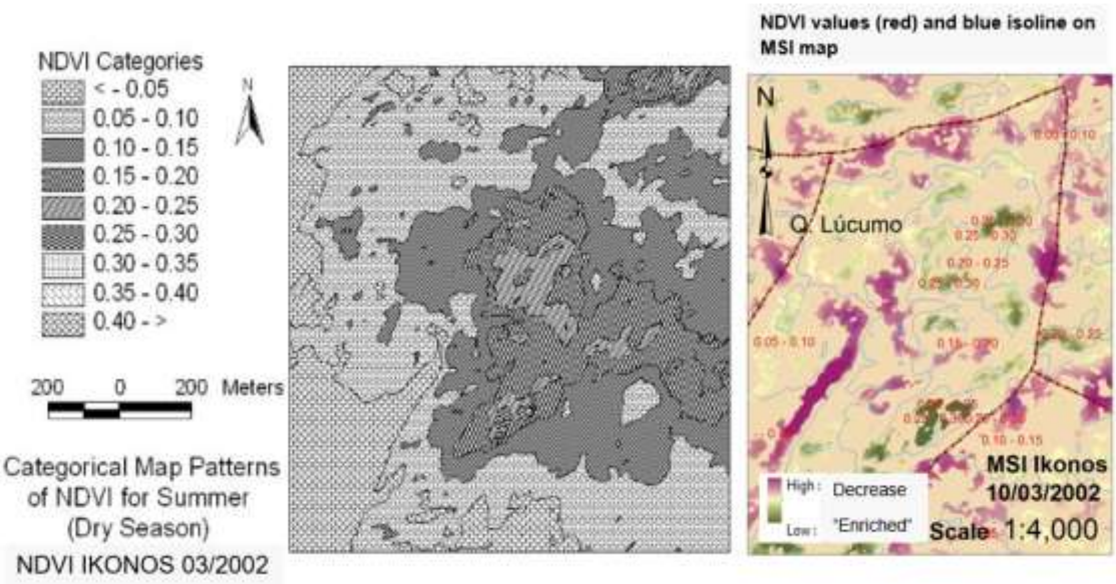


Figure 6. Comparative NDVI and MSI maps for the dry season, IKONOS 2002

Map 7 corresponds to a singularity map obtained from a Landsat image from April 2020. In April and May, the dry season ends, and the hills begin to get wet. Thus, the patches in green tonality are precisely the mountain sectors in which some of the vegetation covers were maintained. Based on this information, botanical biologists can concentrate their samplings on these patches to learn about plant communities' physiognomy, floristics, and other characteristics. In all cases, the resolution of the image and the date are decisive for obtaining the singularity maps.

Figure 8 shows comparatively maps of NDVI and MSI. It is possible to point out that the sequence of obtaining the NDVIs and then getting the MSIs is a recommended step as this singularity map will show the places where the NDVIs are highest and most concentrated and, therefore, the most significant vegetation cover in the area. The patches in green tonality with the indication "less than 2" correspond to these sites.

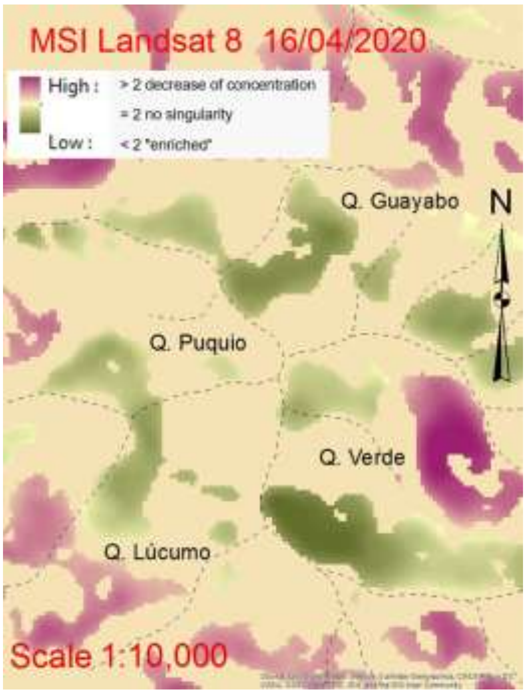


Figure 7. MSI Map for the Dry Season, Landsat 8, 2020

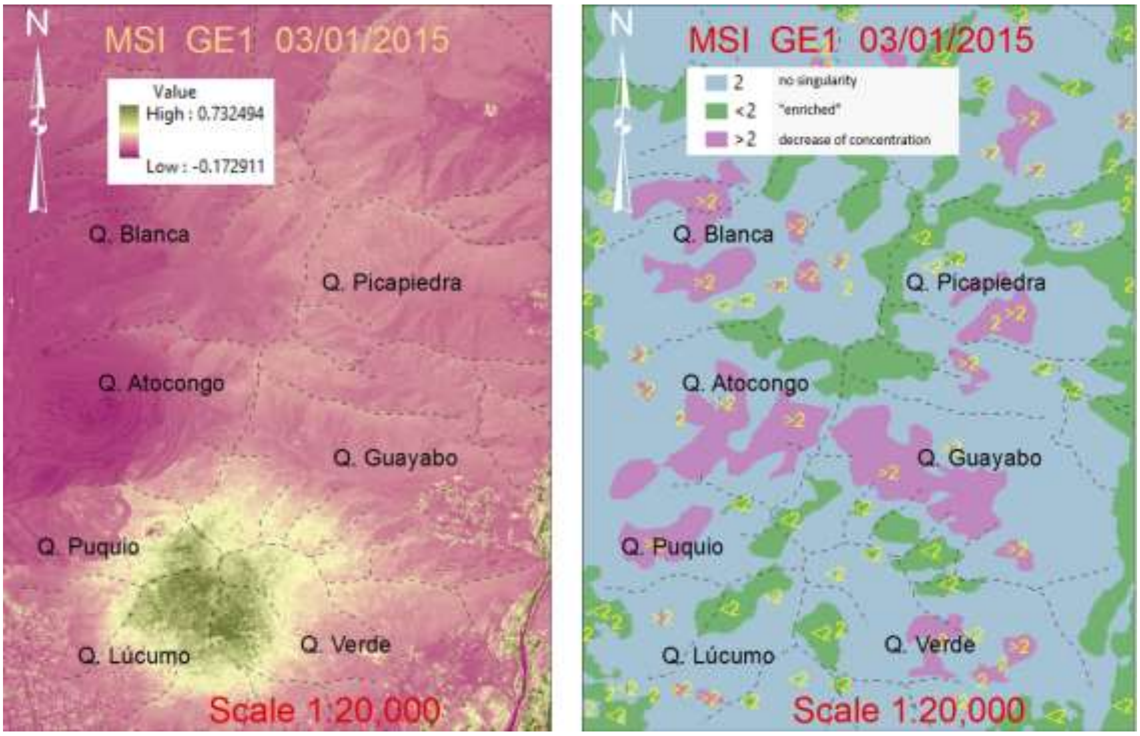


Figure 8. Comparative NDVI and MSI Maps, GE1 2015

4. Discussion

The NDVI multispectral singularity map (Figure 8) shows the sites with the highest concentration of highest NDVI values. Since the GE satellite image, from which the NDVIs were derived, corresponded to January 3, 2015, the respective map (Figure 8) reflects the constant minimum condition of the patches with the most extraordinary singularity of NDVI (wet season and dry season). The map's validity was verified through field surveys and the video of the drone overflight, which showed that these places of most extraordinary singularity correspond mainly to plant communities of *Acnistus arborescent*, a deciduous perennial shrub, called "quebrollo" in the wet season (see Figure 3). The presence of this species is notorious for its abundant foliage in the places that receive more humidity in the more humid months, so much so that in the less damp areas in that season, its absence is evident. As for the dry season, in the dampest patches of this time, the presence of "quebrollo" is barely visible because there are no leaves. Likewise, it is evident that the nuclei of more excellent vegetation (See Figure 8) do not occur "cross-country" but in the sites influenced by rocky outcrops. In this dry season, these patches of more excellent vegetation present lower NDVI values, and the most notable species are *Croton alnifolius* "Croton" and *Ophryosporus pubescens* "Piqueria"; and in the lower ranges, *Trixis cacalioides* "Trixis", perennial shrubs. [26]

These nuclei with more excellent vegetation cover occur more or less in the same sectors, and in the wet season, the NDVI values are higher than in the dry season. Field observation also recognized that in the humid season an annual, climbing plant, called wild “Caigua” *Sicyos baderoa*, develops with great vigor in many places. This plant explains the high NDVI values of winter images. That is, not all sites with high NDVI values correspond to a shrub or wooded communities. This observation should be considered at all times when mapping the vegetation of the fog oasis.

Another aspect that must be considered is that in the said map (Figure 8), there is no wooded thicket in the middle and upper part of the Q. Puquio. Therefore, on the map, it does not appear as a singularity of the NDVI since the vegetation cover is very similar to the area of its surroundings. Therefore the NDVI is more or less equal. However, the presence of scattered arboreal specimens of *Vachellia macracantha* "Huarango" or "Promo" do not affect the elevation of the NDVI, although they are relevant in said plant community since the appearance changes to a wooded scrub and the floristic composition varies.

Figures 5 and 7 show maps of vegetation cover only of Q. Lúcumo, one of the 08 streams that make up the foggy oasis under study. Figure 5 shows a map (based on a 1998 spot image) that corresponds to the wet season, and Figure 7 shows another map (based on a 2002, Ikonos image) that reflects the conditions of the dry season. It is more or less in this way as it is customary to present the vegetation cover maps based on NDVI ranges and on which their limitation had been indicated.

In general, the NDVI maps show the values of the spectral response of the vegetation, without more; therefore, the highest and lowest values will be observed, even if it corresponds to a single pixel. The contours that are presented are the result of a simple interpolation. On the other hand, the singularity map (MSI) does not show the NDVI values but the areas in which the values are highest and most concentrating (MSI values <2) that result from incorporating a spatial correlation analysis and an analysis of multifractal uniqueness to NDVI data; that is, only the vegetation patches with the highest NDVI and the highest concentration of such values, or hotspot of vegetation cover, are shown. Therefore, the complementary use of NDVI maps and NDVI singularity maps is possible. However, it must be taken into account that both the NDIV and MSI maps are not sufficient conditions to prepare the map as the necessary condition is required, which would be the inventory of the flora and vegetation of the place; Therefore, both maps (NDVI and MSI) only reflect an intensity of vegetation cover but do not automatically reflect the types of floristic community, nor do they even reflect the physiognomy of the vegetation and the habitats or types of fog oases, as explained.

It is recommended that fog oasis vegetation monitoring programs consider a singularity map to establish the sampling points, the cores with the highest coverage, and the driest cores. Currently, the programs in question consider the fog oases as a homogeneous area (or as a large unit), which is not wise since it is possible to delimit well-differentiated sectors in terms of relief and more physical-geographic characteristics that determine the presence and composition. We have called the vegetation cover hotspots of plant communities, although it is also feasible to say that they are unique areas. The methodological process followed is also recommended, based on by Qiuming Cheng, 2001 [14], "Singularity analysis for image processing and improvement of anomalies," from which the algorithms were generated to obtain the singularity map.

5. Conclusions

It has been shown that the multispectral singularity map delimits the sites with the most intense coverage.

The areas of the most extraordinary singularity or hotspot of vegetation cover do not necessarily correspond to a specific physiognomy or floristic composition.

The algorithm built in a commercial computer program was straightforward and did not require ample machine resources and person-hours to obtain the singularity map.

Commercial GIS programs have not incorporated multispectral singularity index estimation and mapping

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