ISSN: 0974-5823 Vol. 6 No. 3 December, 2021

International Journal of Mechanical Engineering

A New Class of Nano Ideal Generalized Closed Set in Nano Ideal Topological Space

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Abstract

The aim of this paper is to introduce a new class of nano ideal generalized Cl.S., namely $nIs_{\alpha}g -$ Cl.S.s on nano ideal Tp.Sp. and studied their properties. Also, we have introduced a nano generalized Cl.S., namely $ns_{\alpha}g -$ Cl.S. in nano Tp.Sp. Further, we have investigated their interrelationship with other existing generalized Cl.S.s. In addition, we have given appropriate examples to understand the abstract concepts clearly.

Keywords: $ns_{\alpha}g -$ Cl.S., $nIs_{\alpha}g -$ Cl.S., $nIs_{\alpha}g -$ Cl.S..

1.INTRODUCTION

An ideal $\mathcal{J} \neq \emptyset$ is a collection of subsets of which satisfies (i) $\mathcal{H}_1 \in \mathcal{J}$ and $\mathcal{H}_2 \subset \mathcal{H}_2$ implies $\mathcal{H}_2 \in \mathcal{J}$ and (ii)$\mathcal{H}_1 \in \mathcal{J}$ and $\mathcal{H}_2 \in \mathcal{J}$ implies $\mathcal{H}_1 \cup \mathcal{H}_2 \in \mathcal{J}$.M.Lellis Thivagar[4] introduced the theory of nano topology as an extension of theory of sets in order to study the intelligent systems which are characterized by insufficient and incomplete information. Indeed, nano topology has several applications in the real-life problems. The notion of nano ideal topological space (briefly, $n\mathcal{J}-$ topological space) was introduced by Parimala et.al[9] and investigated its properties and characterizations. In 1970, Levine introduced the concept of generalized Cl.S.s as a generalization of Cl.S.s in Tp.Sp. M.Parimala et.al[6] introduced the concept of nano ideal generalized Cl.S.s in $n\mathcal{J} -$ Tp.Sp. In this paper, we introduce a new class of nano generalized Cl.S. known as $ns_{\alpha}g -$ Cl.S.s in nano Tp.Sp. and a nano ideal generalized Cl.S. known as $nIs_{\alpha}g -$ Cl.S.s on $n\mathcal{J} -$ Tp.Sp. Further, the characteristics and properties of $nIs_{\alpha}g -$ Cl.S. are discussed.

2. PRELIMINARIES

Definition 2.1[4] Let $\mathcal{U}$ be a nonempty finite set of objects called the universe and $\mathcal{R}$ be an equivalence relation on $\mathcal{U}$ named as indiscernibility relation. Then $\mathcal{U}$ is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(\mathcal{U}, \mathcal{R})$ is said to be an approximation space. Let $X \subseteq \mathcal{U}$. Then,

(i) The lower approximation of X with respect to $\mathcal{R}$ is the set of all objects which can be for certain classified as X with respect to $\mathcal{R}$ and is denoted by $L_{\mathcal{R}}(X)$. That is, $L_{\mathcal{R}}(X) = \cup \{\mathcal{R}(X) \subseteq X : x \in \mathcal{U}\}$ where $\mathcal{R}(X)$ denotes the equivalence class determined by $x \in \mathcal{U}$.

(ii) The upper approximation of X with respect to $\mathcal{R}$ is the set of all objects which can be possibly classified as X with respect to $\mathcal{R}$ and is denoted by $U_{\mathcal{R}}(X)$. That is, $U_{\mathcal{R}}(X) = \cup \{\mathcal{R}(X) : \mathcal{R}(X) \cap X \neq \emptyset, x \in \mathcal{U}\}$.

(iii) The boundary region of X with respect to $\mathscr{R}$ is the set of all objects which can be classified neither as X nor as not $-X$ with respect to $\mathcal{R}$ and is denoted by $B_{\mathcal{R}}(X)$. That is, $B_{\mathcal{R}}(X) = U_{\mathcal{R}}(X) - L_{\mathcal{R}}(X)$.

Definition 2.2 [4] Let $\mathcal{U}$ be a universe, $\mathcal{R}$ be an equivalence relation on $\mathcal{U}$ and $\tau_{\mathcal{R}}(X) = \{\mathcal{U}, \emptyset, L_{\mathcal{R}}(X), U_{\mathcal{R}}(X), B_{\mathcal{R}}(X)\}$, where $X \subseteq \mathcal{U}$, satisfies the following axioms:

(i) $\mathcal{U}, \emptyset \in \tau_{\mathcal{R}}(X)$.

(ii) The union of the elements of any sub-collection of $\tau_{\mathcal{R}}(X)$ is in $\tau_{\mathcal{R}}(X)$.

(iii) The intersection of the elements of any finite subcollection of $\tau_{\mathcal{R}}(X)$ is in $\tau_{\mathcal{R}}(X)$.

Therefore, $\tau_{\mathcal{R}}(X)$ is a topology on $\mathcal{U}$ called the nano topology on $\mathcal{U}$ with respect to X. We call $(\mathcal{U}, \tau_{\mathcal{R}}(X))$ as the nano topological space. The elements of $\tau_{\mathcal{R}}(X)$ are called nano Op.S.s (briefly, $n-$ Op.S.s). The complement of a nano Op.S. is called a nano Cl.S. (briefly, $n-$ Cl.S.).

Definition 2.3 [6] Let $(\mathcal{U}, \mathcal{N}, \mathcal{I})$ be a $n\mathcal{I}-$ topological space with an ideal $\mathcal{I}$ on $\mathcal{U}$ where $\mathcal{N} = \tau_{\mathcal{R}}(X)$ and $(.)_n^*$ be a set operator from $P(\mathcal{U})$ to $P(\mathcal{U})$, ($P(\mathcal{U})$ the set of all subsets of $\mathcal{U}$). For a subset $H \subset \mathcal{U}, \mathcal{H}_n^*(\mathcal{I}, \mathcal{N}) = \{x \in \mathcal{U}: G_n \cap H \notin \mathcal{I}$, for every $G_n \in G_n(x)\}$, where $G_n = \{G_n: x \in G_n, G_n \in \mathcal{N}\}$ is called the nano local function (briefly, $n-$ local function) of $\mathcal{H}$ with respect to $\mathcal{I}$ and $\mathcal{N}$. We will simply write $\mathcal{H}_n^*$ for $\mathcal{H}_n^*(\mathcal{I}, \mathcal{N})$.

Definition 2.4 [9] Let $(\mathcal{U}, \mathcal{N})$ be a nano topological space with an ideals $\mathcal{I}, \mathcal{I}'$ on $\mathcal{U}$ and H, L be subsets of $\mathcal{U}$.Then

(i) $H \subseteq L \Rightarrow \mathcal{H}_n^* \subseteq L_n^*$.

(ii) $\mathcal{I} \subseteq \mathcal{I}' \Rightarrow \mathcal{H}_n^*(\mathcal{I}') \subseteq \mathcal{H}_n^*(\mathcal{I})$.

(iii) $H_n^* = n-cl(H_n^*) \subseteq n-cl\,(H)$ (H_n^* is a nano Cl.S. subset of $n-cl\,(H)$).

(iv) $(\mathcal{H}_n^*)_n^* \subseteq \mathcal{H}_n^*$.

(v) $\mathcal{H}_n^* \cup L_n^* = (H \cup L)_n^*$.

(vi) $\mathcal{H}_n^* - L_n^* = (H-L)_n^* - L_n^* \subseteq (H-L)_n^*$.

(vii) $V \in \mathcal{N} \Rightarrow V \cap \mathcal{H}_n^* = V \cap (V \cap H)_n^* \subseteq (V \cap H)_n^*$.

(viii) $\mathcal{J} \in \mathcal{I} \Rightarrow (H \cup \mathcal{J})_n^* = \mathcal{H}_n^* = (H - J)_n^*$.

Theorem 2.5 [9] If $(\mathcal{U}, \mathcal{N}, \mathcal{I})$ is a $n\mathcal{I}-$ topological space $H \subseteq \mathcal{H}_n^*$, then $H_n^* = n-cl\,(H_n^*) = n-cl(H)$.

Definition 2.6 [9] Let $(\mathcal{U}, \mathcal{N}, \mathcal{I})$ be a $n\mathcal{I}-$ topological space. The set operator $n-cl^*$ is called a nano*-closure and is defined as $n-cl^*(H) = H \cup H_n^*$ for $H \subseteq \mathcal{U}$.

Definition 2.7 [9] A subset H of a $n\mathcal{I}-$ topological space $(\mathcal{U}, \mathcal{N}, \mathcal{I})$ is n^*-dense in itself (resp. n^*- perfect and n^*- Cl.S.) if $H \subseteq H_n^*$ (resp. $H = H_n^*$, $H_n^* \subseteq H$).

Lemma 2.8 [6] Let $(\mathcal{U}, \mathcal{N}, \mathcal{I})$ be a $n\mathcal{I}-$ topological space and $H \subseteq \mathcal{U}$. If H is n^*- dense in itself, then $H_n^* = n-cl(H_n^*) = n-cl(H) = n-cl^*(H)$.

Definition 2.9 [7] A subset H of a $n\mathcal{I}-$ topological space $(\mathcal{U}, \mathcal{N}, \mathcal{I})$ is said to be a nano ideal generalized Cl.S. (briefly, nI_g-Cl.S.) set if $H_n^* \subseteq K$, whenever $H \subseteq K$ and K is nano Op.S.

Definition 2.10 [5] A subset H of a $n-$ topological space $(\mathcal{U}, \mathcal{N})$ is said to be nano $\alpha-$ Op.S.if $H \subseteq n-int\left(n-cl\left(n-int(H)\right)\right)$.

Result 2.11 [9] Let $(\mathcal{U}, \mathcal{N}, \mathcal{I})$ be a $n\mathcal{I}-$ topological space and $H \subseteq \mathcal{U}$. If $H \subseteq H_n^*$ then $H_n^* = n-cl(H_n^*) = n-cl(H) = n-cl^*(H)$.

Definition 2.12 [8] A subset H of a $n-$ topological space $(\mathcal{U}, \mathcal{N})$ is said to be $ns_\alpha-$ Op.S. if there exists a $n\alpha-$ Op.S. $\mathcal{P}$ in $\mathcal{U}$ such that $\mathcal{P} \subseteq H \subseteq n-cl(\mathcal{P})$ or equivalently if $H \subseteq n-cl\left(n\alpha-int(\mathcal{P})\right)$.

Result 2.13.[9] Let $(\Gamma, \mathcal{N})$ be a nano topological space. Then

(i) Every $n-$ Op.S. is $ns_\alpha-$ Op.S.

(ii) Every $n\alpha-$ Op.S. is $ns_\alpha-$ Op.S.

3. $nIs_\alpha g-$ Closed Sets

Definition 3.1 (i) A subset H of a $n-$ topological space $(\Gamma, \mathcal{N})$ is said to be nano semi $\alpha-$ generalized Cl.S. (briefly, $ns_\alpha g-$ Cl.S.) if $n-cl(H) \subseteq \mathcal{G}$ whenever $H \subseteq \mathcal{G}$ and $\mathcal{G}$ is $ns_\alpha-$ Op.S.

(ii) A subset H of a $n\mathcal{I}-$ topological space $(\Gamma, \mathcal{N}, \mathcal{I})$ is said to be nano ideal semi α generalized Cl.S. (briefly, $nIs_\alpha g-$ Cl.S.) if $H_n^* \subseteq \mathcal{G}$ whenever $H \subseteq \mathcal{G}$ and $\mathcal{G}$ is $ns_\alpha-$ Op.S.

Example 3.2 Let $\Gamma = \{\nu_1, \nu_2, \nu_3, \nu_4\}$; $\Gamma/R = \{\{\nu_1, \nu_3\}, \{\nu_2\}, \{\nu_4\}\}$; $X = \{\nu_2, \nu_3\}$ and $\mathcal{J} = \{\emptyset, \{\nu_3\}\}$. $\mathcal{M} = \{\emptyset, \Gamma, \{\nu_2\}, \{\nu_1, \nu_2, \nu_3\}, \{\nu_1, \nu_3\}\}$.

(i) $ns_\alpha g-$ Cl.S.s are $\emptyset, \Gamma, \{\nu_4\}, \{\nu_1, \nu_4\}, \{\nu_2, \nu_4\}, \{\nu_3, \nu_4\}, \{\nu_1, \nu_2, \nu_4\}, \{\nu_1, \nu_3, \nu_4\}, \{\nu_2, \nu_3, \nu_4\}$.

(ii) $nIs_\alpha g-$ Cl.S.s are $\emptyset, \Gamma, \{\nu_3\}, \{\nu_4\}, \{\nu_1, \nu_4\}, \{\nu_2, \nu_4\}, \{\nu_3, \nu_4\}, \{\nu_1, \nu_2, \nu_4\}, \{\nu_1, \nu_3, \nu_4\}, \{\nu_2, \nu_3, \nu_4\}$.

Proposition 3.3 Every $nIs_\alpha g-$ Cl.S. is $nIg-$ Cl.S. in $(\Gamma, \mathcal{N}, \mathcal{I})$.

Proof: Let $(\Gamma, \mathcal{N}, \mathcal{I})$ be a $n\mathcal{I}-$ topological space and $H \subseteq \Gamma$ be a $nIs_\alpha g-$ Cl.S. Let $H \subseteq K$ and K is $n-$

Op.S. Since H is $nIs_{\alpha}g -$ Cl.S. and every $n -$ Op.S. is $ns_{\alpha} -$ Op.S., $H_n^* \subseteq K$ whenever $H \subseteq K$ and K is $ns_{\alpha} -$ Op.S. Therefore, $H_n^* \subseteq K$ whenever $H \subseteq K$ and K is $n -$ Op.S. Hence, the result.

Remark 3.4 The reverse implication of the preceding theorem need not be true as shown in the example below.

Example 3.5 Let $\Gamma = \{\nu_1, \nu_2, \nu_3, \nu_4\}$; $\Gamma/R = \{\{\nu_1\}, \{\nu_2, \nu_3\}, \{\nu_4\}\}$; $X = \{\nu_1, \nu_4\}$ and $\mathcal{J} = \{\emptyset, \{\nu_1\}\}$. $\mathcal{N} = \{\emptyset, \Gamma, \{\nu_1, \nu_4\}\}$. $nIs_{\alpha}g -$ Cl.S.s are $\emptyset, \Gamma, \{\nu_1\}$, $\{\nu_2, \nu_3\}, \{\nu_1, \nu_2, \nu_3\}, \{\nu_2, \nu_3, \nu_4\}$ and $nIg -$ Cl.S.s are $\emptyset, \Gamma, \{\nu_1\}, \{\nu_2\}, \{\nu_3\}, \{\nu_1, \nu_2\}, \{\nu_1, \nu_3\}, \{\nu_2, \nu_3\}, \{\nu_2, \nu_4\}, \{\nu_3, \nu_4\}, \{\nu_1, \nu_2, \nu_3\}, \{\nu_1, \nu_2, \nu_4\}$, $\{\nu_1, \nu_3, \nu_4\}, \{\nu_2, \nu_3, \nu_4\}$. Here $\{\nu_1, \nu_3, \nu_4\}$ is $nIg -$ Cl.S. but not $nIs_{\alpha}g -$ Cl.S.

Proposition 3.6 Every $n^* -$ Cl.S. is $nIs_{\alpha}g -$ Cl.S.

Proof: Let H be a $n^* -$ Cl.S. Then, $H_n^* \subseteq H$.Let $H \subseteq K$ and K is $ns_{\alpha} -$ Op.S. Hence, $H_n^* \subseteq K$ whenever $H \subseteq K$ and K is $ns_{\alpha} -$ Op.S. Therefore, H is $nIs_{\alpha}g -$ Cl.S.

Remark 3.7 The reverse implication of the above theorem is not valid as shown in the example given below.

Example 3.8 In Example 3.2. the set $\{\nu_1, \nu_4\}$, is $nIs_{\alpha}g -$ Cl.S. but not $n^* -$ Cl.S.

Theorem 3.9 If $(\Gamma, \mathcal{N}, \mathcal{J})$ is a $n\mathcal{J} -$ topological space, then H_n^* is always $nIs_{\alpha}g -$ Cl.S. for every subset $H \subseteq \Gamma$.

Proof: Let $H_n^* \subseteq K$ where K is $ns_{\alpha} -$ Op.S. Since $(H_n^*)_n^* \subseteq H_n^*, (H_n^*)_n^* \subseteq K$ whenever $H_n^* \subseteq K$ and K is $ns_{\alpha} -$ Op.S. Therefore, H_n^* is $nIs_{\alpha}g -$Cl.S.

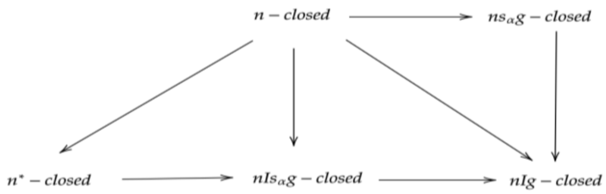
Proposition 3.10 Every $ns_{\alpha}g -$ Cl.S. is $nIs_{\alpha}g -$ Cl.S. in $n\mathcal{J} -$ topological space $(\Gamma, \mathcal{N}, \mathcal{J})$.

Proof: Let H be a $ns_{\alpha}g -$ Cl.S. Then $n - cl(H) \subseteq K$ whenever $H \subseteq K$ and K is $ns_{\alpha} -$ Op.S. Also, $n - cl^*(H) \subseteq n - cl(H) \subseteq K$ whenever $H \subseteq K$ and K is $ns_{\alpha} -$ Op.S.implies that H is $nIs_{\alpha}g -$ Cl.S.

Remark 3.11 The reverse implication of the preceding theorem is not valid as shown in the example below.

Example 3.12 In Example 3.2, $\{\nu_3\}$ is $nIs_{\alpha}g -$ Cl.S. but not $ns_{\alpha}g -$ Cl.S.

Remark 3.13 The following figure shows that the relationship of $nIs_{\alpha}g -$ Cl.S.s with some of the existing sets, which we have discussed in this section



These implications are not reversible.

Theorem 3.14 Let $(\Gamma, \mathcal{N}, \mathcal{J})$ be a $n\mathcal{J} -$ topological space. Then every subset of Γ is $nIs_{\alpha}g -$ Cl.S. if and only if every $ns_{\alpha} -$ Op.S. is $n^* -$ Cl.S.

Proof: Suppose every subset of Γ is $nIs_{\alpha}g -$ Cl.S. If K is $ns_{\alpha} -$ Op.S., then K is $nIs_{\alpha}g -$ Cl.S. so that $K_n^* \subseteq K$. Hence, K is $n^* -$ Cl.S. Conversely, suppose that every $ns_{\alpha} -$ Op.S. is $n^* -$ Cl.S. If K is $ns_{\alpha} -$ Op.S. such that $H \subseteq K \subseteq \Gamma$ then, $H_n^* \subseteq K_n^* \subseteq K$ so that H is $nIs_{\alpha}g -$ Cl.S.

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