

APPLICATION OF RELIABILITY GROWTH MODEL IN MILITARY

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ABSTRACT

The process of gathering, modeling, analyzing, and interpreting data from the reliability growth development test program is known as reliability growth analysis (development testing). Additionally, reliability growth analysis utilizing field data is possible (fielded systems). Additionally, fielded systems can look through data from highly technical repairable equipment. There are a variety of models that can be used (or developed) to evaluate the growth processes, depending on the metric(s) of interest and the data collection technique. In this study, we use the AMSAA reliability growth model and statistical testing to a newly developed military tank system. During the last test phase, data on five tanks' system failures was gathered.

Keywords: Reliability Growth Analysis, Reliability Growth Model, Cramer-Von Mises Fit, AMSAA Model

INTRODUCTION

According to **T. Auld et al. (2013)**, the DOD's Energy Siting Clearing House, which was created to assess wind farm projects that had been delayed, has improved the permitting process by bringing much-needed cooperation. Using the Integrated Noise Model, **G. Licitra et al. (2014)** calculated the airport's noise impact and assessed the population exposed to it (INM). To increase the goodness of fit of previously published conventional SRGMs and ANN based combination models, **I. Lakshmanan and S. Ramasamy (2015)** proposed a new neural network combination model based on the dynamically evaluated weights. To predict software system flaws and assist researchers and the software industry in creating highly reliable software products, **B.B. Sagar et al. (2016)** provided the best software reliability growth model containing features of both Weibull distribution and inflection S-shaped SRGM. In their **2017 article**, **A. Fortier**

and M. G. Pecht examined several points of view on the most recent IPC study on lead-free electronics in military and aerospace applications. **G. P. et al., Pandian (2018)** talked about the problems that can occur when handbook-based techniques are used in both commercial and military avionics applications. There is also discussion of several reliability design alternatives, such as testing, physics-of-failure, similarity analysis, and data analytics for prognostics and systems health management. A review of the RAS development for platform-centric earthworks, as well as an analysis of the technical viability, maturity, key technical challenges, and future directions for the application of RAS technologies to earthmoving tasks of interest to the army, was presented by **Q. P. Ha et al., (2019)**. The software reliability growth model put forth by **Z. Hui and X. Lui (2020)**, which is based on the Gaussian new distribution makes it easier to examine the errors produced during the software development process and lessens the uncertainty brought on by subjective human variables.

MILITARY TANKS

One of the world's most powerful armies is that of India. It engages in warfare with a variety of vehicles. Main Battle Tank is one of them. Tanks are mobile platforms for land weaponry that are substantially armored. They have a big tank cannon that is positioned in a revolving gun turret. Machine guns and other long-range weapons, such as rocket launchers and guided anti-tank missiles, are used in addition to this.

Powerful engines and tracks are employed. These offer good mobility in a variety of environments, such as mud, snow, and ice, where a wheeled vehicle would not be able to function as effectively.

An important acquisition choice made during the 20-month-long military standoff in Eastern Ladakh between the Chinese and Indian forces is for the

Indian Army to equip itself with locally made light tanks to operate in high-altitude terrain.

1. T-90M Bhishma



- Main Battle Tank (MBT)
- The Russian T-90 Main Battle Tank was adapted for the Indian Army as the T-90M Bhishma.

2. DRDO Arjun (Lion)



- Main Battle Tank (MBT)
- The primary objective of the Arjun Mk II is to significantly enhance the original Arjun MBT's overall capability.

3. DRDO Tank EX / MBT Ex (Karna)



- Main Battle Tank (MBT) Project
- It is a significant fighting tank of the Indian Army.

4. BMD-2 (Boyevaya Mashina Desanta)



- It is also known as an infantry fighting vehicle and an airborne amphibious light tank.
- The BMD-2 is an infantry fighting vehicle that can be airdropped.
- It is the natural continuation of the BMD line of automobiles.

5. BMP-2 (Boyevaya Mashina Pekhoty)



- Infantry Combat vehicle
- The Soviet Union first used the amphibious infantry fighting vehicle, or ICV BMP II, in the 1980s.

6. T-54 MBT



- It is Medium Tank / Main Battle Tank (MBT)
- The legendary Soviet T-34 Medium Tank of World War II was superseded by the T-54 Main Battle Tank.

7. T-72 (Ajeya)



- It is also known as **Ajeya** Main Battle Tank (MBT)
- The T-72 Main Battle Tank replaced the T-54/T-55 family of platforms and is still in use today..

RELIABILITY GROWTH ANALYSIS

Typically, the initial iterations of a complicated new system under development will have design, manufacturing, and/or technical flaws. These flaws may cause the prototypes' initial dependability to fall short of the system's reliability objective or requirement. The prototypes are frequently put through a rigorous testing regimen in order to find and fix these flaws. Problem areas are found during testing, and the proper corrective steps (or redesign) are then implemented. Reliability growth refers to increase in a product's (component, subsystem, or system's) reliability over time as a result of modifications made to its design and/or manufacturing procedure.

The idea of reliability growth goes beyond the abstract and the absolute. The management method for taking corrective action, the efficacy of the fixes, reliability requirements, the initial reliability level, reliability funding, and competitive variables are all related to reliability growth. For instance, one management team might rectify 90% of the testing-related failures, whereas another management team using the identical design and test data would only do so for 65% of the testing-related failures. With the same fundamental design, different management approaches may result in varying dependability values. When compared to the initial reliability at the start of testing, the effectiveness of the remedial activities is also based on a relative standard.

Corrective measures that increase reliability by 400% for equipment with an initial reliability aim of 10% are not as meaningful as those that increase reliability by 50% for a system with an initial reliability goal of 50%.

RELIABILITY GROWTH MODELLING

A reliability growth model describes how the system's dependability evolves over time while being tested. The reliability of the system should increase during system testing and debugging as the underlying defects causing these failures are fixed as they are found. The conceptual reliability growth model must next be converted into a mathematical model in order to predict dependability.

In reliability growth modeling, reliability is assessed at various periods in time and compared to known functions that indicate potential reliability changes. An equal step function, for instance, implies that a system's reliability rises linearly with each release. It is feasible to forecast the system's dependability at a later time by comparing the observed reliability growth with one of these functions. Thus, reliability growth models can be utilized to assist in project planning.

CROW-AMSAA RELIABILITY GROWTH MODEL

Dr. Larry H. Crow stated that the Duane model may be stochastically represented as a Weibull process in "Reliability Analysis for Complex, Repairable Systems" (1974), enabling statistical methods to be applied in this model's application in reliability increase. As a result of this statistical expansion, the Crow-AMSAA (N.H.P.P.) model was created. The U.S. Army Materiel Systems Analysis Activity used this technique first (AMSAA). On systems where utilization is gauged on a continuous scale, it is widely used. It can also be applied for high reliability, a large number of trials and one-shot items. Typically, test procedures are carried out phase by phase. Instead of tracking reliability across test phases, the Crow-AMSAA model is developed for tracking reliability within a single test phase.

The empirical link established by J. T. Duane serves as the foundation for the Crow-AMSAA model. It is equivalent to an NHPP model with a Weibull intensity function (non-homogeneous Poisson process). While the length of the test phases within a reliability program might vary, the Crow-AMSAA model focuses on the reliability growth within a certain phase. Assume that a specific program phase starts at $t = 0$ & let $0 < A_1 < A_2 < \dots < A_n$ be the

instances when adjustments are made to the equipment during the testing phase. The failure intensity, λ_i can be assumed constant between the times $[A_{i-1}, A_i]$ when design changes are made on the system. Therefore, the number of failures, N_i during the i th time period has the Poisson distribution with mean $\lambda_i(A_i - A_{i-1})$.

$$P(N_i = n) = \frac{[\lambda_i(A_i - A_{i-1})]^n \cdot e^{-\lambda_i(A_i - A_{i-1})}}{n!}, n = 0, 1, 2, \dots$$

The constant failure intensity, λ_i assumes that the times between successive failures for this interval follow the exponential distribution.

$$F(x) = 1 - e^{-\lambda_i x}, x > 0$$

Let $N(T)$ be the total number of failures for the entire test period, T . $N(T)$ has the Poisson distribution with mean λT if λ is constant. If λ varies, then $N(T)$ is the sum of the failure rates for the first interval and the second interval, between A_1 and T . The mean of $N(T)$, which is λT , is defined as follows when failure rate is λ_1 for the first interval and failure intensity is λ_2 for the second period:

$$\phi(T) = \lambda_1 A_1 + \lambda_2 (T - A_1)$$

If the failure intensity is homogenous (constant) throughout the test intervals, $N(T)$ follows a homogeneous Poisson process with mean λT . If the failure intensity is non-homogeneous, that is, if it differs between $[S(i-1), S(i)]$ & $[S(i-2), S(i-1)]$, $N(T)$ follows a non-homogeneous Poisson process. Here is how the mean value function looks.

$$\phi(T) = \int_0^T \rho(y) dy \quad \dots (1)$$

where $\rho(y) = \lambda$

$$y \in [A_{i-1}, A_i]$$

then for any T , $P[N(T) = n] = \frac{[\phi(T)]^n \cdot e^{-\phi(T)}}{n!}, n = 0, 1, 2, \dots$

A non-homogeneous Poisson process with an intensity function is an integer-valued process $[N(T), T > 0]$, (T) . If ΔT is infinitesimally small, then $(\Delta T)T$ roughly represents the likelihood of a system failure in the range $(T, T + \Delta T)$. The Weibull failure rate function is assumed to be a reasonable approximation of (T) in the Crow-AMSAA model.

$$\rho(T) = \frac{\beta}{\eta^\beta} \cdot T^{\beta-1}$$

Therefore, if $\lambda = \frac{1}{\eta^\beta}$, the intensity function, $\rho(T)$ or the instantaneous failure intensity, $\lambda_i T$ is defined as

$$\lambda_i(T) = \lambda \beta T^{\beta-1}, T > 0, \lambda > 0 \text{ \& } \beta > 0$$

From Eqn. (1), the average number of failures by time T becomes:

$$\phi(T) = \int_0^T \lambda_i(T) dT = \int_0^T \lambda \beta T^{\beta-1} dT = \lambda T^\beta$$

The cumulative failure intensity, λ_c is

$$\lambda_c = \lambda T^{\beta-1}$$

Therefore, the cumulative MTBF_c is

$$MTBF_c = \frac{1}{\lambda} T^{1-\beta}$$

PARAMETER ESTIMATION USING MAXIMUM LIKELIHOOD

The probability density function (*pdf*) of the i th event given that the $(i-1)$ th event occurred at T_{i-1} is

$$f(T_i | T_{i-1}) = \frac{\beta}{\eta} \left(\frac{T_i}{\eta}\right)^{\beta-1} \cdot e^{-\frac{1}{\eta^\beta}(T_i^\beta - T_{i-1}^\beta)}$$

The likelihood function is

$$L = \lambda^n \beta^n e^{-\lambda T^* \beta} \prod_{i=1}^n T_i^{\beta-1}$$

where T^* is the termination time and is given by:

$$T^* = \begin{cases} T_n & \text{if the test is failure terminated} \\ T > T_n & \text{if the test is time terminated} \end{cases}$$

Taking the natural log on both sides

$$\ln L = \ln \Lambda = n \ln \lambda + n \ln \beta - \lambda T^* \beta + (\beta - 1) \sum_{i=1}^n \ln T_i \quad \dots (2)$$

and differentiating with respect to λ we get

$$\frac{\partial \Lambda}{\partial \lambda} = \frac{n}{\lambda} - T^* \beta$$

set equal to zero and solve for λ we get

$$\hat{\lambda} = \frac{n}{T^* \beta}$$

now differentiate equation (2) with respect to β we get

$$\frac{\partial \Lambda}{\partial \beta} = \frac{n}{\beta} - \lambda T^* \beta \ln T^* + \sum_{i=1}^n \ln T_i$$

set equal to zero and solve for β we get

$$\hat{\beta} = \frac{n}{n \ln T^* - \sum_{i=1}^n \ln T_i}$$

NUMERICAL

A new military tank system is under development. System failure data has been collected on five tanks during the final test phase. Plot and track reliability growth of tanks based on a series of failure times as 2.4, 24.9, 52.5, 53.4, 54.7, 57.2, 118.6, 140.2, 185, 207.6, 293.9, 322.3, 365.9, 366.8, 544.8, 616.8, 627.5, 646.8, 664, 738.1, 764.7, 765.1, 779.6, 799.9,

852.9, 1116.3, 1161.1, 1257.1, 1276.3, 1308.9, 1340.3, 1437.3, 1482, 1489.9, 1715.1, 1828.9, 1971.5, 2303.4, 2429.7, 2457.4, 2535.2, 2609.9, 2674.2, 2704.8, 2849.6, 2923.5 one per line. It is given that total accumulated test time (unit-hours) for a tank is 3000. Consider level of significance 10% for "no growth" test of hypothesis and for Cramer-von Mises goodness of fit test. Evaluate Chart based on data fitted to AMSAA mode and Chart based on calculated instantaneous value at each failure tank. Calculate reliability of tank at test time (unit-hours) 200.

Failure No.	Cumulative Unit-Hours of Test Time at Failure (X _i , Hours)	Ln (T/X _i)	MTBF ₉₀ % LCL	Instantaneous MTBF (Hours)	MTBF ₉₀ % UCL	FR _{90%} LCL	Instantaneous Failure Rate (Failures/Hour)	FR _{90%} UCL
1	2	7.1309		7			0.1457	
2	25	4.7915	3	17	651	0.2969	0.0594	0.0015
3	52	4.0456	6	22	218	0.1696	0.0446	0.0046
4	53	4.0286	7	23	134	0.1420	0.0443	0.0075
5	55	4.0045	8	23	103	0.1247	0.0439	0.0097
6	57	3.9598	9	23	87	0.1121	0.0432	0.0115
7	119	3.2306	13	31	101	0.0792	0.0326	0.0099
8	140	3.0633	14	33	97	0.0702	0.0306	0.0103
9	185	2.7860	17	36	100	0.0602	0.0275	0.0100
10	208	2.6708	18	38	98	0.0553	0.0263	0.0102
11	294	2.3231	21	43	106	0.0468	0.0230	0.0095
12	322	2.2309	23	45	104	0.0439	0.0222	0.0096
13	366	2.1040	25	47	105	0.0407	0.0212	0.0095
14	367	2.1016	25	47	102	0.0397	0.0212	0.0098

15	545	1.7059	30	55	115	0.033 4	0.0182	0.008 7
16	617	1.5818	32	58	117	0.031 2	0.0173	0.008 5
17	628	1.5646	33	58	115	0.030 5	0.0172	0.008 7
18	647	1.5343	34	59	114	0.029 6	0.0170	0.008 8
19	664	1.5081	35	59	112	0.028 9	0.0169	0.008 9
20	738	1.4023	37	62	115	0.027 4	0.0162	0.008 7
21	765	1.3669	38	63	114	0.026 7	0.0160	0.008 7
22	765	1.3664	38	63	113	0.026 3	0.0160	0.008 9
23	780	1.3476	39	63	112	0.025 9	0.0158	0.009 0
24	800	1.3219	39	64	111	0.025 4	0.0157	0.009 0
25	853	1.2577	41	65	112	0.024 5	0.0153	0.008 9
26	1,116	0.9886	46	72	123	0.021 9	0.0138	0.008 1
27	1,161	0.9492	47	74	124	0.021 4	0.0136	0.008 1
28	1,257	0.8698	49	76	126	0.020 6	0.0132	0.007 9
29	1,276	0.8546	49	76	126	0.020 3	0.0131	0.008 0
30	1,309	0.8294	50	77	126	0.020 0	0.0130	0.008 0
31	1,340	0.8057	51	78	126	0.019 7	0.0129	0.008 0
32	1,437	0.7358	53	80	128	0.019 0	0.0125	0.007 8
33	1,482	0.7052	54	81	128	0.018 7	0.0124	0.007 8
34	1,490	0.6999	54	81	128	0.018 5	0.0124	0.007 8
35	1,715	0.5591	57	85	134	0.017 4	0.0117	0.007 5
36	1,829	0.4949	59	88	136	0.016 9	0.0114	0.007 3

37	1,972	0.4198	61	90	139	0.016 3	0.0111	0.007 2
38	2,303	0.2642	65	96	147	0.015 3	0.0105	0.006 8
39	2,430	0.2108	67	98	149	0.014 9	0.0102	0.006 7
40	2,457	0.1995	68	98	148	0.014 8	0.0102	0.006 7
41	2,535	0.1683	69	99	149	0.014 5	0.0101	0.006 7
42	2,610	0.1393	70	100	150	0.014 3	0.0100	0.006 7
43	2,674	0.1150	71	101	151	0.014 1	0.0099	0.006 6
44	2,705	0.1036	71	102	151	0.014 0	0.0098	0.006 6
45	2,850	0.0514	73	104	153	0.013 7	0.0096	0.006 5
46	2,924	0.0258	74	105	154	0.013 5	0.0095	0.006 5
Test end (T)	3,000		75	106	155	0.006 4	0.0095	0.013 4
N = 46 failure s		Sum = 74.618 9						

Table: 1

$$\hat{\beta} = \frac{n}{n \ln T^* - \sum_{i=1}^n \ln T_i} = \mathbf{0.6165}$$

$$\hat{\lambda} = \frac{n}{T^* \hat{\beta}} = \mathbf{0.3305 \text{ failures/hour}}$$

The instantaneous MTBF at 3,000 hours is

$$MTBF_c(3000) = \frac{1}{\lambda} T^{1-\beta} = \mathbf{106 \text{ hours}}$$

$$MTBF_c(200) = \mathbf{37 \text{ hours (Projected /estimated MTBF based on AMSAA model)}}$$

The instantaneous failure rate (ρ) at 3,000 hours is

$$\rho(T) = \frac{\beta}{\eta \hat{\beta}} \cdot T^{\beta-1} = \mathbf{0.0095 \text{ failures/hour}}$$

Statistical Test for Trend

For a time terminated test, the test statistic is

$$\chi_{2n}^2 = \frac{2n}{\hat{\beta}} = \mathbf{149}$$

Under the null hypothesis of exponential times to failure (i.e., "no growth", or constant failure rate), χ_{2n}^2 has a chi-square distribution with $2n$ degrees of freedom. The statistic $\hat{\beta}$ estimates the growth parameter β . **Three possibilities exist:**

- i. **No growth:** In the case of no growth, β is equal to 1.
- ii. **Positive reliability growth:** For positive reliability growth β is less than 1.
- iii. **Negative reliability growth:** For negative growth (reliability degradation) β is greater than 1.

For large or small values of χ_{2n}^2 , the null hypothesis of "no growth" is rejected.

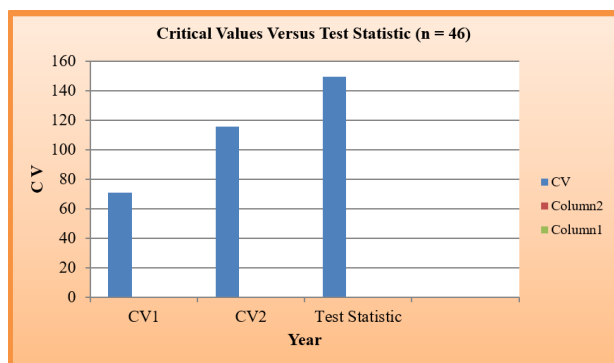
If the test statistic is greater than CV2, positive reliability growth is taking place.

If it is less than CV1, negative reliability is taking place.

If the test statistic is between CV1 and CV2, then no growth is occurring, or any growth that may be taking place is inconclusive given the significance level selected and more testing is needed to prove the null hypothesis.

For the $n = 46$ failure times entered, $\hat{\beta}$ is 0.6165, indicating significant reliability growth (growth rate of 0.3835).

To test the null hypothesis of "no growth", the statistic χ^2_{2n} , for a time truncated test, can be used. Under the null hypothesis, this statistic is chi-square with $2N = 92$ degrees of freedom. At the 10% significance level, the appropriate critical values (CV) found in a table of chi-square percentiles for 92 degrees of freedom are $CV1 = 71$ and $CV2 = 115$. The test statistic is $\chi^2_{2n} = 149$. Since the value of the test statistic (149) is greater than $CV2$ (115), the null hypothesis of "no growth" is rejected at the 10% significance level.



Cramer-von Mises Statistic (Model Fit Test)

The unbiased maximum likelihood estimate (MLE) for the shape parameter β for a time terminated test is

$$\bar{\beta} = \frac{n-1}{n} \hat{\beta} = 0.6031$$

The Cramer von-Mises statistic is given by

$$C_m^2 = \frac{1}{12m} + \left[\sum_{j=1}^m \left(\frac{X_j}{T} \right)^{\bar{\beta}} - \frac{2j-1}{2m} \right]^2$$

where $m = \text{total failure count}$

$j = \text{failure number}$

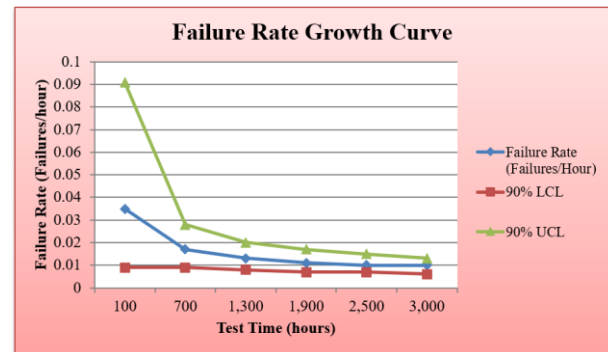
$T = \text{total test time}$

$$\Rightarrow C_m^2 = 0.0429$$

Critical value for Cramer-von Mises goodness of fit test is **0.1725**

At the 10% significance level, the critical value for the Cramer-von Mises goodness of fit test is 0.1725. Since 0.0429 is less than 0.1725,

the **AMSAA model is accepted** as being compatible with the data.



CONCLUSION

As we can see in our numerical that the value of the test statistic of reliability growth model is greater than $CV2$ which means that positive reliability growth for the military tank is taking place. Also since the data is accepted by AMSAA model which means the design of the military tank is also compatible accordingly to the given data.

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