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Bi-regular Bipolar Fuzzy Graphs

[1]T.Saratha Devi, [2]V.Swetha

[1] Assistant Professor, Department of Mathematics, Research Center, G. Venkataswamy Naidu College, Kovilpatti-628502, Tamil Nadu, India.

[2] Research Scholar, Manonmaniam Sundaranar University, Abishekapatti- 627012, Tirunelveli, Tamil Nadu, India.

Abstract

In this paper, bi-regular fuzzy graphs and totally bi-regular fuzzy graphs are defined. Comparative study between bi-regular fuzzy graph and totally bi-regular fuzzy graph is done. A necessary and sufficient condition under which they are equivalent is provided. Characterization of bi-regular fuzzy graph in which underlying crisp graph is a cycle is investigated. Also, whether the results hold for totally bi-regular fuzzy graphs is examined.

Key words: Bipolar fuzzy graph, degree of a vertex in fuzzy graph, regular fuzzy graph, totally regular fuzzy graph.

AMS subject classification : 05C12, 03E72, 05C72.

1 Introduction

In 1736, Euler first introduced the concept of graph theory. Graph theory is a very useful tool for solving combinatorial problems in different areas such as operations research, optimization, topology, geometry, number theory and computer science. Fuzzy set theory was first introduced by Zadeh in 1965. Fuzzy set theory has emerged as a potential area of interdisciplinary research and fuzzy graph theory is of recent interest. The first definition of fuzzy graph was introduced by Haufmann in 1973 based on Zadeh's fuzzy relations in 1971. In 1975, Rosenfeld introduced the concept of fuzzy graphs. Now, fuzzy graphs have been witnessing a tremendous growth and finds application in many branches of engineering and technology. A. Nagoorgani and K.Radha introduced the concept of regular fuzzy graphs in 2008 [5]. These motivates us to introduce an bi-regular bipolar fuzzy graphs and totally bi- regular bipolar fuzzy graphs and discussed some of its properties.

Throughout this paper, the vertices take the membership value $A = (m^+_1, m^-_1)$ and edges take the membership value $B = (m^+_2, m^-_2)$ where $m^+_1, m^+_2 \in [0, 1]$ and $m^-_1, m^-_2 \in [-1, 0]$.

2 Preliminaries

We present some known definitions related to fuzzy graphs and bipolar fuzzy graphs for ready reference to go through the work presented in this paper.

Definition 2.1. A fuzzy graph $G : (\sigma, \mu)$ is a pair of functions (σ, μ) , where $\sigma : V \rightarrow [0,1]$ is a fuzzy subset of a non empty set V and $\mu : V X V \rightarrow [0, 1]$ is a symmetric fuzzy relation on σ such that for all u, v in V , the relation $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ is satisfied. A fuzzy graph G is called complete fuzzy graph if the relation $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ is satisfied.

Definition 2.2. A bipolar fuzzy graph with an underlying set V is defined to be the pair (A, B) , where $A = (m_1^+, m_1^-)$ is a bipolar fuzzy set on V and $B = (m_2^+, m_2^-)$ is a bipolar fuzzy set on E such that $m_2^+ (x, y) \leq min\{(m_1^+(x), m_1^+ (y)\}$ and $m_2^- (x, y) \geq max\{m_1^- (x), m_1^- (y)\}$ for all $(x, y) \in E$. Here, A is called bipolar fuzzy vertex set on V and B is called bipolar fuzzy edge set on E.

Definition 2.3. The strength of connectedness between two vertices u and v is defined as $\mu^\infty(u, v) = sup\ \{\mu^k(u, v) : k = 1, 2, ...\}$, where $\mu^k(u, v) = sup\ \{\mu(u, u_1) \wedge \mu(u_1, u_2) \wedge ... \wedge \mu(u_{k-1}, v) : u, u1, u2, ..., uk-1, v$ is a path connecting u and v of length k }.

Definition 2.4. The positive degree of a vertex $u \in G$ is defined as $d^+(u) = \sum m_2^+ (u, v)$, for $uv \in E$.

The negative degree of a vertex $u \in G$ is defined as $d^{-}(u) = \sum m_2^{-}(u, v)$, for $uv \in E$ and $m_2^{+}(uv) = m_2^{-}(uv) = 0$ if uv not in E. The degree of a vertex u is defined as $d(u) = (d+(u), d-(u))$.

Definition 2.5. Let $G : (A, B)$ be a bipolar fuzzy graph, where $A = (m_1^{+} , m_1^{-})$ and $B = (m_2^{+} , m_2^{-})$ be two bipolar fuzzy sets on a non empty set V . Then, G is said to be regular bipolar fuzzy graph if all the vertices of G has same degree (c_1, c_2) .

Definition 2.6. The total degree of a vertex $u \in V$ is denoted by $td(u)$ and is defined as $td(u) = (td+(u), td-(u))$, where $td^{+}(u) = \sum m_2^{+}(u, v) + (m_1^{+}(u))$ and $td^{-}(u) = \sum m_2^{-}(u, v) + (m_1^{-}(u))$.

Definition 2.7. Let $G : (A, B)$ be a bipolar fuzzy graph, where $A = (m_1^{+} , m_1^{-})$ and $B = (m_2^{+} , m_2^{-})$ be two bipolar fuzzy sets on a non empty set V . Then, G is said to be totally regular bipolar fuzzy graph if all the vertices of G has same total degree (c_1, c_2).

Definition 2.8. Let $G : (A, B)$ be a bipolar fuzzy graph, where $A = (m_1^{+}, m_1^{-})$ and $B = (m_2^{+} , m_2^{-})$ be two bipolar fuzzy sets on a non empty set V. Then, G is said to be fuzzy cycle if there does not exist unique edge xy such that $B(xy) = \wedge\{B(uv) : uv \in E\}$.

Definition 2.9. Let $G : (A, B)$ be a bipolar fuzzy graph, where $A = (m_1^{+} , m_1^{-})$ and $B = (m_2^{+} , m_2^{-})$ be two bipolar fuzzy sets on a non empty set V. Then, G is said to be fuzzy bridge if the removal of any edge may reduces the strength of connectedness.

3 Bi-Regular Bipolar Fuzzy Graphs

In this section Bi-regular bipolar fuzzy graphs and totally Bi-regular bipolar fuzzy graphs are introduced.

Definition 3.1. Let G be a bipolar fuzzy graph on $G^{*}(V, E)$. G is said to be bi-regular bipolar fuzzy graph if the degree of each vertex is either (k_1, k_2) or (k_3, k_4) . Then G is said to be $((k_1, k_2),(k_3, k_4))$ -bi-regular bipolar fuzzy graph.

Example 3.2. Consider a bipolar fuzzy graph on $G^{*}(V, E)$.

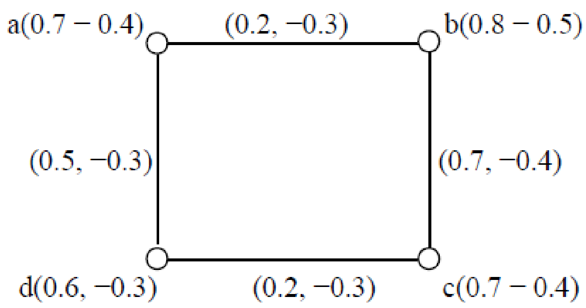

Figure.1

Here, d(a) = (0.7, −0.6) , d(b) = (0.9, −0.7) , d(c) = (0.9, −0.7) , d(d) = (0.7, −0.6). This graph is ((0.7, −0.6),(0.9, −0.7)) -bi-regular bipolar fuzzy graph.

Definition 3.3. Let G be a bipolar fuzzy graph on $G^{*}(V, E)$. G is said to be totally bi-regular bipolar fuzzy graph if the total degree of each vertex is either (c_1, c_2) or (c_3, c_4). Then G is said to be $((c_1, c_2),(c_3, c_4))$ - totally bi-regular bipolar fuzzy graph.

Example 3.4. Consider a bipolar fuzzy graph on $G^{*}(V, E)$.

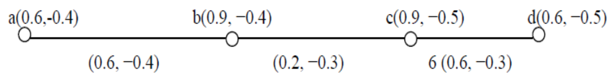

Figure.2

Here , td(a) = (1.2, −0.8) , td(b) = (1.7, −1.1) , td(c) = (1.7, −1.1) , td(d) = (1.2, −0.8). This graph is ((1.2, −0.8),(1.7, −1.1)) -totally bi-regular bipolar fuzzy graph.

Remark 3.5. A bi-regular bipolar fuzzy graph need not be a totally bi- regular bipolar fuzzy graph.

Example 3.6. Consider a bipolar fuzzy graph on $G^{*}(V, E)$.

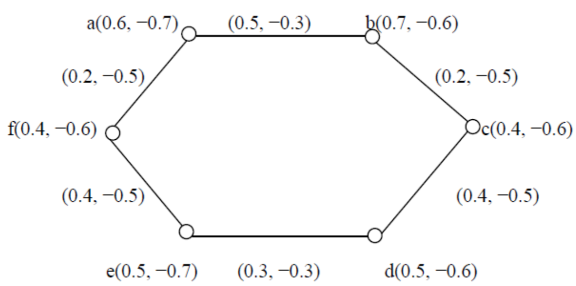

Figure.3

Here d(a) = (0.7, −0.8) , d(b) = (0.7, −0.8) , d(c) = (0.6, −1) , d(d) = (0.7, −0.8) , d(e) = (0.7, −0.8), d(f) = (0.6, −1) and td(a) = (1.3, −1.5), td(b) = (1.4, −1.4) , td(c) = (1, −1.6) , td(d) = (1.2, −1.4) , td(e) = (1.2, −1.5) , td(f) = (1, −1.6) . This graph is ((0.6,

$-1),(0.7, -0.8))$ -bi-regular bipolar fuzzy graph. But this is not totally bi-regular bipolar fuzzy graph.

Remark 3.7. A totally bi-regular bipolar fuzzy graph need not be a bi- regular bipolar fuzzy graph.

Example 3.8. Consider a bipolar fuzzy graph on $G^*(V, E)$.

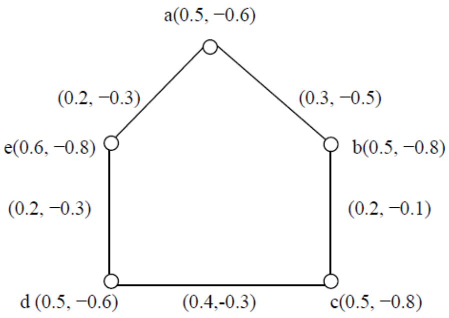

Figure.4

Here, $d(a) = (0.5, -0.8)$, $d(b) = (0.5, -0.6)$, $d(c) = (0.6, -0.4)$, $d(d) = (0.6, -0.6)$, $d(e) = (0.4, -0.6)$ and $td(a) = (1, -1.4)$,$td(b) = (1, -1.4)$,$td(c) = (1.1, -1.2)$, $td(d) = (1.1, -1.2)$,$td(e) = (1, -1.4)$. This graph is $((1, -1.4),(1.1, -1.2))$ - totally bi-regular bipolar fuzzy graph. But this is not bi-regular bipolar fuzzy graph.

Remark 3.9. A bi-regular bipolar fuzzy graph is also a totally bi- regular bipolar fuzzy graph.

Example 3.10. Consider a bipolar fuzzy graph on $G^*(V, E)$.

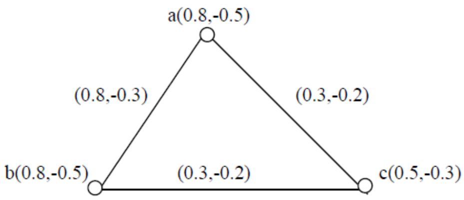

Figure.5

Here $d(a) = (1.1, -0.5)$, $d(b) = (1.1, -0.5)$, $d(c) = (0.6, -0.4)$. and $td(a) = (1.9, -1)$,$td(b) = (1.9, -1)$,$td(c) = (1.1, -0.7)$.

This graph is $((1.1, -0.5),(0.6, -0.4))$ -bi-regular bipolar fuzzy graph and $((1.9,-1),(1.1, -0.7))$ - totally bi-regular bipolar fuzzy graph.

Theorem 3.11. Let $G : (A, B)$ be a bipolar fuzzy graph on $G^*(V, E)$. Then A is a constant function if and only if the following are equivalent.

(i) G is a bi-regular bipolar fuzzy graph.

(ii) G is a totally bi-regular bipolar fuzzy graph.

Proof. Suppose that A is a constant function. Let $A(u) = (c_1, c_2)$ be a constant for all $u \in V$. Assume that G is a $((k_1, k_2),(k_3, k_4))$ bi-regular bipolar fuzzy graph. Then, let $d(u) = (k_1, k_2)$ and $d(w) = (k_3, k_4)$, for some $u, w \in V$. Now, $td(u) = d(u) + A(u)$ for all $u \in V \Rightarrow td(u) = (k_1, k_2) + (c_1, c_2)$, for some $u \in V$ and $td(w) = d(w) + A(w)$, for some $w \in V \Rightarrow td(w) = (k_3, k_4) + (c_1, c_2)$, for some $w \in V$. Hence G is totally-bi-regular bipolar fuzzy graph.

Thus (i) $\Rightarrow$ (ii) is proved.

Now Suppose G is a $((k_1, k_2),(k_3, k_4))$ -totally bi-regular bipolar fuzzy graph. Then $td(u) = d(u)+A(u) = (k_1, k_2)$, for some $u \in V \Rightarrow d(u)+A(u) = (k_1, k_2)$, for some $u \in V \Rightarrow d(u) + (c_1, c_2) = (k_1, k_2)$, for some $u \in V \Rightarrow d(u) = (k_1, k_2)-(c_1, c_2)$, for some $u \in V$ and $td(w) = d(w)+A(w) = (k_3, k_4)$, for some $w \in V \Rightarrow d(w)+A(w) = (k_3, k_4)$, for some $w \in V \Rightarrow d(w)+(c_1, c_2) = (k_3, k_4)$, for some $w \in V \Rightarrow d(w) = (k_3, k_4) - (c_1, c_2)$, for some $w \in V$ and so G is bi-regular bipolar fuzzy graph. Thus (ii) $\Rightarrow$(i)is proved. Hence (i) and (ii) are equivalent .

Conversely, assume that (i) and (ii) are equivalent. That is G is bi-regular bipolar fuzzy graph if and only if G is totally bi-regular bipolar fuzzy graph. Suppose A is not a constant function, then $A(u) \neq A(v) \neq A(w)$ for some vertices $u, v, w \in V$. Let G be a $((k_1, k_2),(k_3, k_4))$ -bi-regular bipolar fuzzy graph. Then $d(u) = (k_1, k_2)$, $d(v) = (k_1, k_2)$ and $d(w)=(k_3, k_4)$ and so $td(u) = d(u) + A(u) = (k_1, k_2) + A(u)$,$td(v)=d(v)+A(v)=(k_1, k_2) + A(v)$ and $td(w) = d(w) + A(w) = (k_3, k_4) + A(w)$. Since $A(u) \neq A(v) \neq A(w)$,we have $td(u) \neq td(v) \neq td(w)$ for some vertices u, v, w . So G is not totally bi-regular bipolar fuzzy graph. which is a contradiction to our assumption.

Now let G be a totally bi-regular bipolar fuzzy graph. Then let, $td(u) = td(v) = (c_1, c_2)$ and $td(w) = (c_3, c_4) \Rightarrow d(u) + A(u) = d(v) + A(v) = (c_1, c_2)$ and $d(w) + A(w) = (c_3, c_4) \Rightarrow d(u) = (c_1, c_2) - A(u)$, $d(v) = (c_1, c_2) - A(v)$ and $d(w) = (c_2, c_4) - A(w)$. Suppose $A(u) \neq A(v) \neq A(w)$ then G is not bi-regular bipolar fuzzy graph which is contradiction to our assumption. Hence A is a constant function.

Example 3.12. Consider a fuzzy bipolar graph on $G^*(V, E)$.

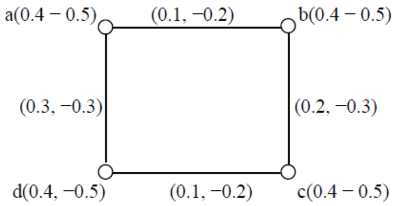

Figure.6

Here, d(a) = (0.4, −0.5), d(b) = (0.3, −0.5), d(c) = (0.3, −0.5), d(d) = (0.4, −0.5) and td(a) = (0.8, −1.0), td(b) = (0.7, −1.0), td(c) = (0.7, −1.0), td(d) = (0.8, −1.0). Here A is constant and G is ((0.4, −0.5),(0.3, −0.5)) -bi-regular bipolar fuzzy graph. Also, G is ((0.8, −1.0),(0.7,−1.0)) -totally bi-regular bipolar fuzzy graph.

Theorem 3.13. If a bipolar fuzzy graph G is both bi-regular bipolar and totally bi-regular bipolar. Then A is a constant function.

Proof. Let G be a $((k_1, k_2),(k_3, k_4))$ -bi-regular bipolar and $((k_5, k_6),(k_7, k_8))$ - totally bi-regular bipolar fuzzy graph . So $d(u) = (k_1, k_2)$, for some $u \in V$, $d(w) = (k_3, k_4)$, for some $u \in V$ and $td(u) = (k_5, k_6)$, for some $u \in V$, $td(w) = (k_7, k_8)$, for some $u \in V$. Now $td(u) = (k_5, k_6)$, for some $u \in V \Rightarrow d(u) + A(u) = (k_5, k_6)$, for some $u \in V \Rightarrow (k_1, k_2) + A(u) = (k_5, k_6)$, for some $u \in V \Rightarrow A(u) = (k_5, k_6) - (k_1, k_2)$, for all $u \in V$. Hence A is a constant

function.

Remark 3.14. Converse of the above theorem need not be true.

Example 3.15. Consider a bipolar fuzzy graph on $G^*(V, E)$.

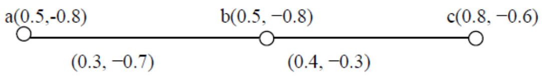

Figure.7

Here, A is a constant function. But d(a) = (0.3, −0.7), d(b) = (0.7, −1), d(c) = (0.4, −0.3) . Also td(a) = (0.8, −1.5), td(b) = (1.2, −1.8), td(c) = (0.9, −1.1) . So G is neither bi-regular bipolar nor totally bi-regular bipolar fuzzy graph.

Theorem 3.16. Let G be a bipolar fuzzy graph on $G^*(V, E)$, a cycle on n vertices. If A is constant function and $B(e_i)$

$$= \begin{cases} (c_1, c_2) & i = 1,2, \dots, n-1 \\ (c_3, c_4) & i = n \end{cases}$$

Then G is both bi-regular bipolar and totally bi-regular bipolar fuzzy graph.

Proof. Let $e_1, e_2, \dots, e_n$ be the edges of the cycle C_n . Let $A(u) = (k_1, k_2)$, for all $u \in V$ be a constant function.

Let $B(e_i) = \begin{cases} (c_1, c_2) & i = 1,2, \dots, n-1 \\ (c_3, c_4) & i = n \end{cases}$

Then $d(v_1) = ((c_1, c_2) + (c_3, c_4)) = d(v_n)$.

Also $d(v_i) = 2(c_1, c_2)$ for $i = 1, 2, \dots, n-1$.

So, G is $((c_1, c_2) + (c_3, c_4), 2(c_1, c_2))$ bi-regular bipolar fuzzy graph. Also, $td(v_1) = (c_1, c_2) + (c_3, c_4) + (k_1, k_2) = td(v_n)$ and $td(v_i) = 2((c_1, c_2) + (k_1, k_2))$, for $i = 2, 3, \dots, n-1$. Hence G is $(((c_1, c_2) + (c_3, c_4) + (k_1, k_2)), 2(c_1, c_2) + (k_1, k_2))$ totally bi-regular fuzzy graph. Hence G is both bi-regular bipolar and totally bi-regular bipolar fuzzy graph.

Example 3.17. Consider a bipolar fuzzy graph on $G^*(V, E)$.

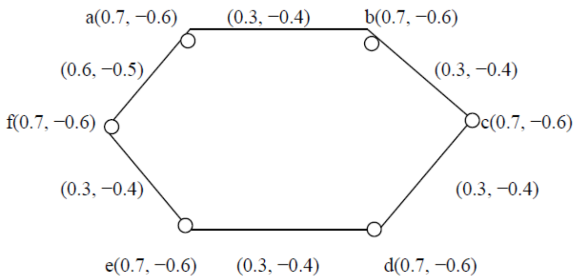

Figure.8

Here d(a) = (0.9, −0.9), d(b) = (0.6, −0.8), d(c) = (0.6, −0.8), d(d) = (0.6, −0.8), d(e) = (0.6, −0.8), d(f) = (0.9, −0.9) and td(a) = (1.6, −1.5), td(b) = (1.3, −1.4), td(c) = (1.3, −1.4), td(d) = (1.3, −1.4), td(e) = (1.3, −1.4), td(f) = (1.6, −1.5) .

Here A is constant function and $B(e_1) = B(e_2) = B(e_3) = B(e_4) = B(e_5) = (0.3, -0.4)$ and $B(e_6) = (0.6, -0.5)$.

Then G is both bi-regular bipolar and totally bi-regular bipolar fuzzy graph.

Theorem 3.18. Let G be a bipolar fuzzy graph $G^*(V, E)$, a path on $2n$ vertices. If B is constant function or alternative edges take same membership values then G is bi-regular bipolar fuzzy graph.

Proof. Let $e_1, e_2, \ldots, e_{2n-1}$ be the edges of the path.

Case (i): B is constant function. Then $B(e_i) = (c_1, c_2)$, for all i.

$$d(v_i) = \begin{cases} (c_1, c_2) & i \text{ is pendant vertex} \\ 2(c_1, c_2) & i \text{ is internal vertex} \end{cases}$$

Hence G is $((c_1, c_2), 2(c_1, c_2))$ bi-regular bipolar fuzzy graph.

Case(ii): Alternate edge take same membership values.

Let the edges $e_1, e_3, \ldots, e_{2n-1}$ takes the membership values (c_1, c_2) and the edges $e_2, e_4, \ldots, e_{2n-2}$ takes the membership values (c_3, c_4) .

Then $d(v_i) =$

$$\begin{cases} (c_1, c_2) & i \text{ is pendant vertex} \\ (c_1, c_2) + (c_3, c_4) & i \text{ is internal vertex} \end{cases}$$

Hence G is $((c_1, c_2),((c_1, c_2) + (c_3, c_4)))$ bi-regular bipolar fuzzy graph.

Remark 3.19. The above theorem does not hold for totally bi-regular bipolar fuzzy graph.

Example 3.20. Consider a bipolar fuzzy graph on $G^*(V, E)$.

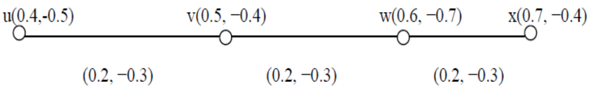

Figure.9

d(u) = (0.2, −0.3), d(v) = (0.4, −0.6), d(w) = (0.4, −0.6), d(x) = (0.2, −0.3) td(u) = (0.6, −0.8), td(v) = (0.9, −1), td(w) = (1, −1.3), td(x) = (0.9, −0.7) Here B is constant function. But G is not totally bi-regular bipolar fuzzy graph.

Theorem 3.21. Let G be a bipolar fuzzy graph on $G^*(V, E)$ a cycle on n vertices. Let A be a constant function and alternative vertices takes same membership values then G is totally bi-regular bipolar fuzzy graph.

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices of the cycle C_n and

$$A(v_i) = \begin{cases} (k_1, k_2) & i \text{ is odd} \\ (k_3, k_4) & i \text{ is even} \end{cases}$$

Since B is constant $B(e_i) = (c_1, c_2)$, for all i. Then $d(v_i) = 2(c_1, c_2)$ and

$td(v_i) = d(v_i)+A(v_i) =$

$$\begin{cases} 2(c_1, c_2) + (k_1, k_2) & i \text{ is odd} \\ 2(c_1, c_2) + (k_3, k_4) & i \text{ is even} \end{cases}$$

Hence G is $(2(c_1, c_2) + (k_1, k_2), 2(c_1, c_2) + (k_3, k_4))$ be a totally bi-regular bipolar fuzzy graph.

Example 3.22. Consider a bipolar fuzzy graph on $G^*(V, E)$.

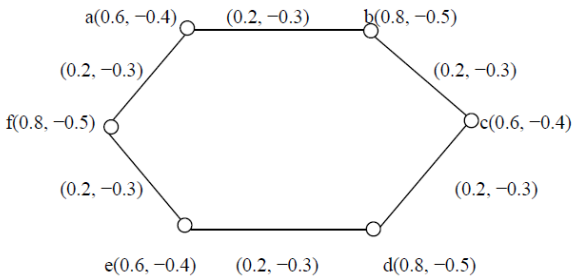

Figure.10

d(a) = d(b) = d(c) = d(d) = d(e) = d(f) = (0.4, −0.6) and td(a) = td(c) = td(e) = (1, −1), td(b) = td(d) = td(f) = (1.2, −1.1). Here B is constant and A takes alternate membership values then G is ((1, −1),(1.2, −1.1)) -totally bi-regular bipolar fuzzy graph.

Remark 3.23. The above theorem does not hold for bi-regular bipolar fuzzy graph, since B is constant function, $d(u) = (k_1, k_2)$ for all $u \in V$.

Proof. If B is constant function, then $d(u)$ is constant for all $u \in V$. Suppose alternate edges takes same membership values then d(u) is also constant for all $u \in V$. Hence G is a regular bipolar fuzzy graph but not bi-regular bipolar fuzzy graph.

Theorem 3.24. Let G be a bipolar fuzzy graph on $G^*(V, E)$ an even cycle on n vertices. If alternate vertices takes same membership values and alternate edges takes same membership values then G is totally bi-regular bipolar fuzzy graph.

Proof. Let $u_1, u_2, \ldots, u_n$ be the vertices of the cycle C_n and

Let $$A(u_i) = \begin{cases} (c_1, c_2) & i \text{ is odd} \\ (c_3, c_4) & i \text{ is even} \end{cases}$$

Since alternate edges takes same membership values and G is an even cycle, $d(u_i) = (k_1, k_2)$, for all i .

$$td(u_i) = \begin{cases}(c_1, c_2) + (k_1, k_2) & i \text{ is odd}\\ (c_1, c_2) + (k_1, k_2) & i \text{ is even}\end{cases}$$

So, G is $((c_1, c_2) + (k_1, k_2),(c_3, c_4) + (k_1, k_2))$ - totally bi-regular bipolar fuzzy graph.

Theorem 3.25. If the ladder on n vertices take constant vertex membership value (c_1, c_2) and constant edge membership value (k_1, k_2) . Then G is both bi-regular bipolar and totally bi-regular bipolar fuzzy graph.

Proof. Let $u_1, u_2, \ldots, u_{2n}$ be the vertices of the ladder and $e_1, e_2, \ldots e_{3n-2}$ be the edges of the ladder.

Let $A(u_i) = (c_1, c_2)$, for all i and $B(e_i) = (k_1, k_2)$, for all i

Then $d(u_i) = \begin{cases}2(k_1, k_2) & i \text{ is end vertex}\\ 3(k_1, k_2) & i \text{ is internal vertex}\end{cases}$

So, G is $(2(k_1, k_2), 3(k_1, k_2))$ -bi-regular bipolar fuzzy graph. Now, $td(u_i)$

$$= \begin{cases}2(k_1, k_2) + (c_1, c_2) & i \text{ is end vertex}\\ 3(k_1, k_2) + (c_1, c_2) & i \text{ is internal vertex}\end{cases}$$

Hence G is $((2(k_1, k_2)+(c_1, c_2), 3(k_1, k_2)+(c_1, c_2))$ - totally bi-regular bipolar fuzzy graph.

Theorem 3.26. Let $G : (A, B)$ be a bipolar fuzzy graph on $G^*(V, E)$, a cycle on n vertices. If

$$B(e_i) = \begin{cases}(c_1, c_2) & i = 1,2,\ldots,n-1\\ (c_3, c_4) & i = n\end{cases}$$ where

$(c_1, c_2) < (c_3, c_4)$, then G is a fuzzy cycle.

Proof. Let $e_1, e_2, \ldots, e_n$ be the edges of the cycle C_n .

Assume that $B(e_i) = \begin{cases}(c_1, c_2) & i = 1,2,\ldots,n-1\\ (c_3, c_4) & i = n\end{cases}$

where $(c_1, c_2) < (c_3, c_4)$.

So there does not exists unique edge e_n such that $B(e_n) = \wedge\{B(e_i) : e_i \in E\}$. Hence G is a fuzzy cycle.

Theorem 3.27. Let G be a bipolar fuzzy graph on $G^*(V, E)$, a path on n vertices. If B is constant function, then G does not have fuzzy bridge.

Proof. Assume that $G^*(V, E)$ is a path on n vertices. Since B is constant function, G is bi-regular bipolar fuzzy graph. Also, Since B is constant, removal of any edge does not reduce the strength of connectedness. Hence G does not have fuzzy bridge.

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