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Fair dominating sets and Fair domination polynomial of a Cycle Graph

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ABSTRACT

Let $G = (V, E)$ be taken simple graph. A set $S \subseteq V$ is a fair as dominating set of G, if any vertex not in S is adjacent to only one or more vertices in S. A dominating set S of G is a fair as dominating set if every two vertices $u, v \in V(G) - S$ are dominated by same number of vertices from S. The smaller number taken over all fair as dominating sets in G is called the fair as domination number of G denoted by $\gamma_f(G)$. Let C_n cycle graph of order n. Let $D_f(C_n, i)$ be the family of all fair as dominating sets of a wheel C_n with number i, and let $d_f(C_n, i) = |D_f(C_n, i)|$. In this paper, we try to explore the fair as domination polynomial cycle graph and also more properties are consider in it.

Key words: dominating sets, domination as polynomial, fair as dominating sets, fair domination as polynomial.

1. Introduction

Consider the graph as $G = (V, E)$ as an undirected graph, where $|V(G)| = n$ take the cardinality of vertices and $|E(G)| = m$ often the number of edges of G. For undefined term refer Harary [9].

A set $S \subseteq V(G)$ is a dominating as set if any vertex not in S is adjacent to one or so many vertices in S. The number minimum taken over all dominating sets in G is called domination number of G and is often called the domination number of G and denoted by $\gamma(G)$.

A dominating set S as fair as dominating set if any two vertices $u, v \in V(G) - S$ are dominated by the same number of vertices from S. The smaller number taken as all of asover all fair dominating sets in G is called the fair as domination number of G and denoted $\gamma_f(G)$.

A domination as polynomial of graph G is the polynomial $D(G, x) = \sum_{i=1}^{n} d(G, i)x^i$, where $d(G, i)$ number of dominating sets of G of number i.

Analogously, a fair as domination polynomial of a graph G of order n is the polynomial $D_f(G, x) = \sum_{i=\gamma_f(G)}^{n} d_f(G, i)x^i$, where $d_f(G, i)$ number of fair as dominating sets of G of number i.

An element a as shown to be a zero polynomial $f(x)$ if $f(x) = 0$. An element a called zero polynomial of multiplicity m if $(x - a)^m / f(x)$ and $(x - a)^{m+1}$ not a divisor of $f(x)$.

2. Fair Domination Polynomial of a Cycle Graph

In this section, we consider to study the fair as dominating sets and fair as domination polynomial of cycle graph C_n.

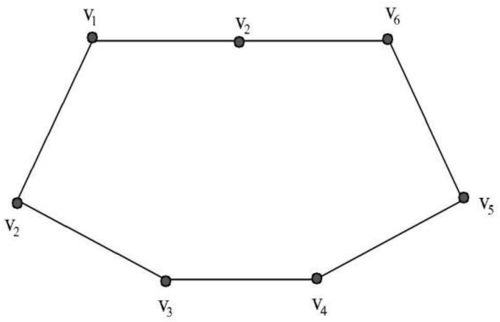
Definition 2.1. Let C_n be consider cycle graph of order n. Let $D_f(C_n, i)$ the family of fair as dominating sets of G with number i. The fair as domination polynomial of C_n the polynomial

$$D_f(C_n, x) = \sum_{i=\gamma_f(C_n)}^{n} d_f(C_n, i)x^i,$$ where

$d_f(C_n, i)x^i$ the number of fair as dominating sets of C_n of number i.

Example 2.2.

Consider cycle graph C_7 vertex set taken as $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ given in Fig 2.1.



C_7

Figure 2.1

Here $\gamma_f(C_7) = 3$.

$D_f(C_7, 3)$
$= \{\{v_1, v_4, v_5\}, \{v_2, v_5, v_6\}, \{v_3, v_6, v_7\}, \{v_4, v_1, v_7\}, \{v_1, v_2, v_5\},$
$\{v_1, v_3, v_6\}, \{v_3, v_4, v_7\}\}$

$D_f(C_7, 4)$
$= \{\{v_1, v_3, v_5, v_6\}, \{v_2, v_4, v_6, v_7\}, \{v_1, v_3, v_5, v_7\}, \{v_1, v_2, v_4, v_6\},$
$\{v_2, v_3, v_5, v_7\}, \{v_1, v_3, v_4, v_6\}, \{v_1, v_2, v_4, v_5\}\}$

$D_f(C_7, 5)$
$= \{v_1, v_2, v_3, v_4, v_5\}, \{v_2, v_3, v_4, v_5, v_6\}, \{v_3, v_4, v_5, v_6, v_7\},$
$\{v_1, v_4, v_5, v_6, v_7\},$
$\{v_1, v_2, v_3, v_6, v_7\}, \{v_1, v_2, v_3, v_4, v_7\}, \{v_1, v_2, v_5, v_6, v_7\},$
$\{v_1, v_2, v_3, v_4, v_6\},$
$\{v_2, v_3, v_4, v_5, v_7\}, \{v_1, v_3, v_4, v_5, v_6\}, \{v_2, v_4, v_5, v_6, v_7\},$
$\{v_1, v_3, v_5, v_6, v_7\},$
$\{v_1, v_2, v_4, v_6, v_7\}, \{v_1, v_2, v_3, v_5, v_7\}, \{v_1, v_2, v_3, v_5, v_6\},$
$\{v_2, v_3, v_4, v_6, v_7\},$
$\{v_3, v_4, v_5, v_6, v_7\}, \{v_1, v_2, v_4, v_5, v_6\}, \{v_2, v_3, v_5, v_6, v_7\},$
$\{v_1, v_3, v_4, v_6, v_7\},$
$\{v_1, v_2, v_4, v_5, v_7\}$

$D_f(C_7, 6) =$
$\{v_1, v_2, v_3, v_4, v_5, v_6\}, \{v_2, v_3, v_4, v_5, v_6, v_7\}, \{v_1, v_3, v_4, v_5, v_6, v_7\},$
$\{v_1, v_2, v_4, v_5, v_6, v_7\}, \{v_1, v_2, v_3, v_5, v_6, v_7\}, \{v_1, v_2, v_3, v_4, v_6, v_7\},$
$\{v_1, v_2, v_3, v_4, v_5, v_7\}\}$

$D_f(C_7, 7) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$

Now,

$$D_f(C_7, x) = \sum_{i=\gamma_f(C_7)}^{|V(C_7)|} d_f(C_7, i)x^i$$

$$\sum_{i=3}^{7} d_f(C_7, i)x^i$$

$$d_f(C_7, 3)x^3 + d_f(C_7, 4)x^4 + d_f(C_7, 5)x^5 + d_f(C_7, 6)x^6 + d_f(C_7, 7)x^7$$

$$7x^3 + 7x^4 + 7x^4 + 21x^5 + 7x^6 + x^7$$

Hence,

$$D_f(C_7, x) = 7x^3 + 7x^4 + 7x^4 + 21x^5 + 7x^6 + x^7.$$

To prove over main results we need the following lemma.

Lemma 2.3. For any cycle graph $C_n (n \geq 5)$,

$$\gamma_f(C_n) = \begin{cases} \lceil \frac{n}{3} \rceil if n \equiv 0 \vee 1 (mod 3) \\ \lceil \frac{n}{3} \rceil + 1 if n \equiv 2 (mod 3) \end{cases}$$

Theorem 2.4. For any cycle graph C_n with n vertices,

$$d_f(C_n, i) = \emptyset \text{ if } 1 < \lceil \frac{n}{3} \rceil + 1 \text{ or } i > n.$$

Proof: Let C_n be a cycle with n vertices

We know that any member of $D_f(C_n, i)$ contains atmost n vertices.

Therefore, we have $d_f(C_n, i) = \emptyset$for $i > n$.

Also, since $\lceil \frac{n}{3} \rceil$ or $\lceil \frac{n}{3} \rceil + 1$ is minimum cardinality of a fair dominating set, there is no fair dominating set of cardinality less than $\lceil \frac{n}{3} \rceil$.

Therefore, $D_f(C_n, i) = \emptyset$ if $1 < \lceil \frac{n}{3} \rceil$.

Hence, $D_f(C_n, i) = \emptyset$ if $i > n$ or $i < \lceil \frac{n}{3} \rceil$.

■

Theorem 2.5. For $n \geq 3$, a star graph C_{3n} may not have a fair dominating set of cardinality $n + 1$.

Proof: Consider C_{3n} where $n \geq 3$. We shall find a fair dominating set S of cardinality $n + 1$ in C_{3n}. Since $n + 1 < \lfloor \frac{n}{2} \rfloor$, not every element in $V(C_{3n}) - S$ are independent. Then $V(C_{3n}) - S$ contains at least two adjacent vertices. Since S is a fair dominating set of C_{3n}, that $V(C_{3n}) - S$ does not contain more than two adjacent vertices. We consider the following two cases:

Case (i): If every vertices in $V(C_{3n}) - S$ forms induced union of path P_2. Then it is clear that S contains exactly $n -$ vertices.

Hence this case fails.

Case (ii): If every vertices in $V(C_{3n}) - S$ need not forms induced union of path P_2. This means that $V(C_{3n}) - S$ contains an induced path P_1. Assume v be the vertex of P_1. Then the vertices adjacent to v in $V(C_{3n}) - S$ is dominated by two vertices of S and the remaining vertices in $V(C_{3n}) - S$ are dominated by exactly one vertex from S. So that S is not a fair dominating set.

Hence we cannot find a fair dominating set of cardinality $n+2$ for a star graph C_{3n} for $n \geq 3$.

■

Theorem 2.6 For $n \geq 9$, a cycle graph C_n not every power of x exists in a fair domination polynomial.

Proof: Consider a cycle graph C_{3n} with $n \geq 3$ vertices. By Theorem 2.5, a cycle graph C_{3n} may not have a fair dominating set of particular cardinality. Hence the result follows.

■

Lemma 2.7. For any cycle graph C_n with n vertices,

i. $d_f(C_n, n) = 1$

ii. $d_f(C_n, n-1) = n$

iii. $d_f(C_n, n-2) = \binom{n}{2}$.

iv. for $k \geq 2, d_f(C_{3k}, k) = 3$.

v. for ≥ 3, $d_f(C_{3k}, k+1) = 0$.

vi. for ≥ 3, $d_f(C_{3k+1}, k+1) = 3k+1$.

vii. for $k \geq 3$, $d_f(C_{3k+2}, k+2) = 6k+4$

viii. $d_f(C_n, i)$ is always a positive integer.

Proof. i. For any graph G with n vertices, we have $d_f(G, n) = 1$.

Hence $d_f(C_n, n) = 1$.

ii. For any graph C_n with n vertices, $V(C_n)$ is the unique fair dominating set of cardinality n.

Therefore, we have $d_f(C_n, n-1) = n$.

iii. By the definition, we can choose a fair dominating set of cardinality $n-2$ in C_n as $\binom{n}{2}$ different ways.

Hence, $d_f(C_n, n-2) = \binom{n}{2}$.

iv. Consider the cycle graph C_{3k}, where $k \geq 2$. Then it has $3k$ vertices. The fair dominating sets of C_{3k} of cardinality k are $\{1,4,7,\dots,3k-2\}, \{2,5,8,\dots,3k-1\}$ and $\{6,9,\dots,3k\}$.

Therefore we have 3 fair dominating sets of C_{3k} of cardinality k.

Hence $d_f(C_{3k}, k+1) = 3$.

v. This follow from Theorem:2.

vi.Consider the wheel graph C_{3k+1}. Then it has $3k+1$ vertices. The fair dominating set of C_{3k+1} of cardinality $k+1$ are $\{1,2,5,\dots,3k-1\}, \{2,3,6,\dots,3k\}, \{3,4,7,\dots,3k+1\}, \dots, \{3k+1,1,4,7,\dots,3k-2\}$.

Therefore we have $3k+1$ fair dominating sets of C_{3k+1} cardinality $k+1$. Hence $d_f(C_{3k+1}, k+1) = 3k+1$.

vii.Consider the cycle graph C_{3k+2}. Then it has $3k+2$ vertices. The fair dominating sets of C_{3k+2} of cardinality $k+2$ are $\{1,2,5,6,9,\dots,3k\}, \{2,3,6,7,\dots,3k+1\}, \{3,4,7,8,\dots,3k+2\}, \dots, \{3k+2,1,4,5,8,\dots,3k+1\}, \{1,2,3,6,9,\dots,3k\}, \{2,3,4,7,10,\dots,3k+1\}, \{3,4,5,8,11,\dots,3k+2\}, \dots, \{3k+1,3k+2,1,4,7,\dots,3k-2\}$.

Therefore we have $3k+2+3k+2$ fair dominating sets of cardinality $k+2$. Hence $d_f(C_{3k+2}, k+2) = 3k+2+3k+2 = 6k+4$.

viii. Clearly $d_f(C_n, i)$ is the cardinality of total collection of fair dominating sets of cardinality i. Hence $d_f(C_n, i)$ has to be a positive integer including zero.

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