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Certified Domination Number in Subdivision of Graphs

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ABSTRACT

A set S of taken vertices in that $G = (V,E)$ is called *dominating set* if for every vertex that not in the set S has minimum one neighbour in that S. A dominating set S *in* a given graph G is said be as a *certified dominating set* o G if any vertex in S has either beenzero or minimum two neighbours in the complement $V - S$. The *certified domination number*, $\gamma_{cer}(G)$ in that G is defined as the minimum number of certified dominating setin G. In this paper, we try to study some of the certified domination number of special Subdivision graphs of certain families of graphs.

Keywords: Dominating set, Certified key Dominating set, Certified Domination Number, Complement graph, Subdivision graphs.

1. Introduction

In this paper, graph $G=(V,E)$ we mean a simple, finite, connected, undi- rected graph with neither loops nor multiple edges. The *order* $|V(G)|$ is denoted by n. For graph theoretic terminology we refer to West[8]. The *open neighborhood* of any vertex v in G is $N(v) = \{x : xv \in E(G)\}$ and *closed neighborhood* of a vertex v in G is $N[v] = N(v) \cup \{v\}$. The *degree*of a vertex in the graph G is denoted by *deg(v)* and the maximum degree (minimum degree) in the graph G is denoted by $\Delta(G)$ ($\delta(G)$). For a set$S \subseteq V(G)$ the open (closed) neighborhood $N(S)$ ($N[S]$) in G is defined as $N(S) = \bigcup_{v \in S} N(v)$ $\left(N[S] = \bigcup_{v \in S} N[v]\right)$.

A walk in which all the vertices are distinct is called a *path*. A path on

n-vertices is denoted by P_n. A walk $(u_0, u_1, u_2, \dots, u_n)$ is called *closed walk* if $u_0 = u_n$. A closed walk in which $u_0, u_1, u_2, \dots, u_{n-1}$ are distinct is called a *cycle*. A cycle on n vertices is denoted by C_n. We write K_n for a *complete graph* of order n. A graph G is called a *bipartite graph* if the vertex set V can be partitioned into two distinct subsets V_1 and V_2 such that every edge of G joins a vertex of V_1 to a vertex of V_2. If G contains every edge joining the vertices of V_1 to the vertices of V_2, the G is called a *complete bipartite graph*. The complete bipartite graph with bipartition V_1, V_2 such that $|V_1| = p$ and $|V_2| = q$ is denoted by $K_{p,q}$. The graph $K_{1,p-1}$ is called a *star*. The complement of a graph G, denoted by $\overline{G}$, is a graph with the vertex set $V(G)$ such that for every two vertices v and w, $vw \in E(G)$ if and $vw \notin E(\overline{G})$. For G, the graph G^+ is obtained by joining exactly one leaf to each vertex of G. The graph $\overline{G}$ is obtained if the vertices in G one adjacent, then they are not adjacent in $\overline{G}$ and vertices. A vertex of degree 0 is called an *isolated vertex* and a vertex of degree 1 is called an *end vertex* or a *pendant vertex*. A vertex that is adjacent to a pendant vertex is called a *support vertex*.

A subdivision of an edge $e = uv$ of a graph G is the replacement of the edge e by a path (u, w, v). The graph G obtained from G by subdividing every edge e of G exactly one is called the subdivision graph of G and is denoted by $S(G)$.

The *corona* of two disjoint graphs G_1 and G_2 is defined to be the graph $G = G_1 \circ G_2$ formed from one copy of G_1 and $|V(G_1)|$ copies of G_2 where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2. In particular, the corona $G \circ K_1$ is the graph constructed from a copy of H, where for each vertex $v \in V(G)$, a new vertex v' and a pendant vv' are added.

The concept of certified domination in graphs was introduced by Dettlaff, Lemanska, Topp, Ziemann and Zylinski[3] and further studied in[2]. It has many application in real life situations. This motivated we to study the certified domination number of central graphs.

In [6], authors studied certified dominaiton number in graphs which isdefined as follows:

Definition 1.1. Let $G = (V, E)$ be any graph of order n. A subset $S \subseteq V(G)$ is called a *certified dominating set* of G if S is a dominating set of G and every vertex in S has either zero or at least two neighbours in $V - S$. The *certified domination number* denoted by $\gamma_{cer}(G)$ is the minimum cardinality of certified dominating set in G.

2. Known Results

Theorem 2.1. [2] For any graph G of order $n \geq 2$, every certified dominating set of G contains its support vertices.

Theorem 2.2. [2] For any graph G of order n, $1 \leq \gamma(G) \leq \gamma_{cer}(G) \leq n$.

Theorem 2.3. [2] For any graph G of order $n \geq 3$, $\gamma_{cer}(G) = 1$ if and only if G has a vertex of degree $n-1$.

3. Certified Domination in Subdivision Graphs

Observation 3.1. The certified domination number of subdivision of somestandard graphs can be easily found and are given as follows:

(i)For any path graph $P_n (n \geq 2)$, γ_{cer}.

(ii)For any cycle graph $C_n (n \geq 3)$, γ_{cer}.

(iii)For any complete graph $K_n (n \geq 2)$, $\gamma_{cer}(S(K_n)) = n-2$.

(iv)For any wheel graph $W_n (n \geq 4)$, γ_{cer}.

(v)For any fan graph $F_n (n \geq 4)$, γ_{cer}.

Theorem 3.2. For $n \geq 2$, $\gamma_{cer}\left(S(K_{1,n} \circ K_1)\right) = n+3$.

Proof. Let v be the central vertex of $K_{1,n}$ and $v_1, v_2, \ldots, v_n$ be the pendant vertices adjacent to v in $K_{1,n}$. Let $\{u, u_1, u_2, \ldots, u_n\}$ be the set of pendant vertices adjacent to $\{v, v_1, v_2, \ldots, v_n\}$, respectively to from $K_{1,n} \circ K_1$. Now let $v', v_1', v_2', \ldots, v_n'$ be the vertices that subdivide the edges $uv, u_1v_1, u_2v_1, \ldots, u_nv_n$ respectively. Let $u_1', u_2', \ldots, u_n'$ be the set of vertices that subdivide the edges $vv_1, vv_2, vv_3, \ldots, vv_n$ respectively. Then $S(K_{1,n} \circ K_1)$ has $(4n+3)$ −vertices and $(4n+2)$ −edges.

Let $S = \{v, v', v_1', v_2', \ldots, v_n'\}$ S dominates all the vertices of $S(K_{1,n} \circ K_1)$ and also S is a minimum dominating set of $S(K_{1,n} \circ K_1)$. Since $S - \{v\}$ is the set of all support vertices of $S(K_{1,n} \circ K_1)$ by Theorem 2.1, $\gamma_{cer}\left(S(K_{1,n} \circ K_1)\right) \geq |S| - 1 = n+1$. Further $|N(v') \cap (V(S(K_{1,n} \circ K_1)) - S)| = 1$, that S is not a certified dominating set of $S(K_{1,n} \circ K_1)$.

Take $S_1 = S \cup \{u\}$. Clearly $N[S_1] = V(S(K_{1,n} \circ K_1))$. Further we get

$|N(x) \cap (V(S(K_{1,n} \circ K_1)) - S_1)| \geq 2$ for every $x \neq u, v' \in S_1$. Also, we have

$|N(u) \cap (V(S(K_{1,n} \circ K_1)) - S_1)| = 0$ and $|N(v') \cap (V(S(K_{1,n} \circ K_1)) - S_1)| = 0$. Thus, every element in S_1 has either zero or greater than two neighbours in $V(S(K_{1,n} \circ K_1)) - S_1$. Therefore that S_1 is a certified dominating set of $S(K_{1,n} \circ K_1)$. Also, if we remove a vertex from S_1, it will not be a certified dominating set of $S(K_{1,n} \circ K_1)$. Moreover there does not exists a certified dominating set of cardinality less than S_1. Hence S_1 is a minimum certified dominating set of $S(K_{1,n} \circ K_1)$ and so $\gamma_{cer}(S(K_{1,n} \circ K_1)) = |S_1| = n+2+1 = n+3$.

Corollary 3.3. *For* $n \geq 2$, $\gamma(S(K_{1,n} \circ K_1)) = n+2$.

Theorem 3.4. *For* $n \geq 2$, $\gamma_{cer}(S(P_n \circ K_1)) = 2n-1$.

Proof. Let the vertex set and the edge set of P_n are $\{v_1, v_2, \ldots, v_n\}$ and $\{e_1, e_2, \ldots, e_{n-1}\}$, respectively. Let $\{u_1, u_2, ..., u_n\}$ be the set of pendant vertices adjacent to $v_1, v_2, ..., v_n$, respectively to the graph $P_n \circ K_1$, where the new edges v_iu_i denoted as e'_i for $1 \leq i \leq n$.

Now we subdivide the corona graph $P_n \circ K_1$. Let $v'_1, v'_2, ..., v'_{n-1}$ be the sub- divided vertices of the edges $e_1, e_2, ..., e_{n-1}$, respectively. Also, let $u'_1, u'_2, ..., u'_n$ be the subdivided vertices of the edges $e'_1, e'_2, ..., e'_n$, respectively in $P_n \circ K_1$.

Clearly the subdivided graph $S(P_n \circ K_1)$ contains $(4n - 1)$-vertices. In $S(P_n \circ K_1)$, $S = \{u'_1, u'_2, ..., u'_n\}$ be the set of all support vertices of $S(P_n \circ K_1)$. Therefore by Theorem 2.1, every certified dominating set must contain S.

Since each v'_i is not dominated by any vertex in S, that S itself is not a certified dominating set of $S(P_n \circ K_1)$. Since $deg(u'_i) = 2$ and $N(u'_i) = \{v_i, u_i\}$ for $1 \leq i \leq n$, it is clear that for every $v \in V(P_n)$, $S_1 = S \cup \{v\}$ is not a certified dominating set of $S(P_n \circ K_1)$. Otherwise, if $v_i \in S_1$, then u'_i has exactly one neighbour u_i in $V(S(P_n \circ K_1)) - S_1$. Also for $1 \leq i \leq n-1$, that $N(v'_i) = \{v_i, v_{i+1}\}$ and $v_i, v_{i+1} \notin S_1$, each $v'_i \in S_1$. Thus, $S_2 = S \cup \{v'_1, v'_2, ..., v'_{n-1}\}$ is a dominating set of $S(P_n \circ K_1)$. Further, every vertex in S_2 has exactly two neighbour in $V(S(P_n \circ K_1)) - S_2$. Therefore that S_2 is a certified dominating set of $S(P_n \circ K_1)$. Moreover, if we remove any vertex from S_2 we obtained that S_2 is not a certified dominating set of $S(P_n \circ K_1)$. Hence, S_2 is a minimum certified dominating set of $S(P_n \circ K_1)$ and so $\gamma_{cer}(S(P_n \circ K_1)) = |S_2| = |S| + n - 1 = n + n - 1 = 2n - 1$. ■

Theorem 3.5. *For* $n \geq 3$, $\gamma_{cer}(S(C_n \circ K_1)) = 2n$.

Proof. The proof is similar to Theorem 3.4. ■

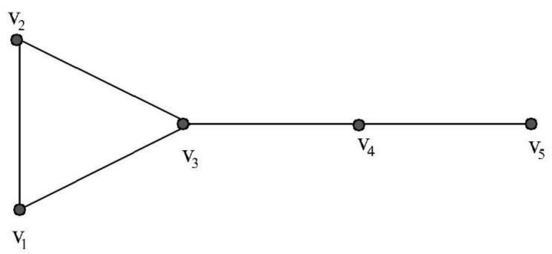
Theorem 3.6. *Let G be a connected* (n, m)*-graph. Then*

$\gamma_{cer}(S(G \circ K_1)) \leq n + m$.

Proof. Let G be a connected graph with n vertices as $v_1, v_2, ..., v_n$ and m edges. Construct the corona graph $G \circ K_1$ of G. Since each vertex of G has a pendant edge in $G \circ K_1$, the set of all vertices of G as the support vertices of $G \circ K_1$. Let $u_1, u_2, ..., u_n$ be the set of pendant vertices of $G \circ K_1$.

Construct the subdivision graph $S(G \circ K_1)$ of $G \circ K_1$. Let $u'_1, u'_2, ..., u'_n$ be the vertices that subdivide the edges $u_1v_1, u_2v_2, ..., u_nv_n$ of $G \circ K_1$, re- spectively. Let $w_1, w_2, ..., w_m$ be the subdivided vertices of $S(G \circ K_1)$ that subdivide the m edges of G. Then the $v_1, v_2, ..., v_n, u_1, u_2, ..., u_n$ are dominated by the vertices $u'_1, u'_2, ..., u'_n$ in $S(G \circ K_1)$ and $w_1, w_2, ..., w_m$ be the vertices that dominates the remaining vertices of $S(G \circ K_1)$. Then the set $S = \{u'_1, u'_2, ..., u'_n, w, w_2, ..., w_m\}$ be a dominating set of $S(G \circ K_1)$. Since $w_1, w_n, ..., w_m$ subdivide the edges that are incident to with the support vertices of $G \circ K_1$ and $u'_1, u'_2, ..., u'_n$ are the vertices that are adjacent with a support vertex and a pendant vertex of $G \circ K_1$, every vertex in S has at least two neighbours in $V(S(G \circ K_1)) - S$. Therefore that S is a certified dominating set of $S(G \circ K_1)$ and hence $\gamma_{cer}(S(G \circ K_1)) \leq |S| = n + m$. ■

Remark 3.7. The upper bound of $\gamma_{cer}(S(G \circ K_1))$ for the connected (n, m)- graph given by Theorem 3.6 is strict. For an example consider the graph G shown in Figure 3.1.

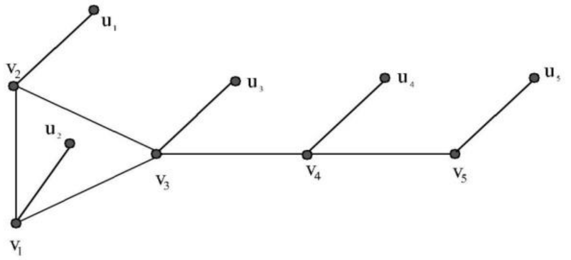


G

Figure 3.1

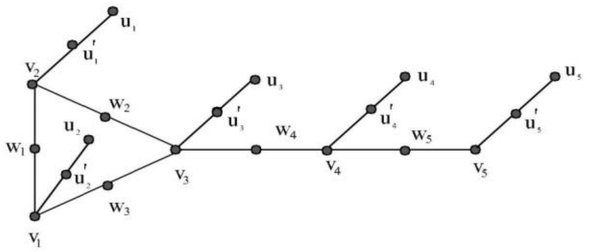
This is a connected $(5, 5)$-graph. Then for this G, $S(G \circ K_1)$ is shown in Figure 3.3.

Here $n = m = 5$ and $S = \{u'_1, u'_2, u'_3, u'_4, u'_5, w_1, v_2, u_3, w_5\}$ is a minimum certified dominating set of $S(G \circ K_1)$. Thus, $\gamma_{cer}(S(G \circ K_1)) = |S| = 9 < n+m$. Therefore $\gamma_{cer}(S(G \circ K_1)) < n + m$.



$G \circ K_1$

Figure 3.2



$S(G \circ K_1)$

Figure 3.3

Theorem 3.8. *Let G be a connected* (n, m)*-graph with maximum degree* Δ. *Then* $\gamma_{cer}(S(G \circ K_1)) = n + m$ *if and only if* $\Delta \leq 2$.

Proof. Let G be a connected (n, m)-graph with maximum degree Δ. Assume $\gamma_{cer}(S(G \circ K_1)) = n + m$. Let S be a minimum certified dominating set of $S(G \circ K_1)$. To prove $\Delta \leq 2$. Suppose $\Delta \geq 3$. Then there exists a vertex v_i such that $deg_G(v_i)$. Then $S \cup \{v_i, u_i\} - deg_G(v_i)$ is a certified dominating set of $S(G \circ K_1)$ and hence $\gamma_{cer}(S(G \circ K_1)) \leq |S| = n + m + 2 - deg_G(v_i) \leq n + m - 1 < n + m$, which is a contradiction. Thus, $\Delta \leq 2$.

Conversely, assume $\Delta \leq 2$. If $\Delta = 1$, then $G = P_2$. So that $S(G \circ K_1)$ ' P_7. Therefore by Observation 3.1(i), $\gamma_{cer}(S(G \circ K_1)) = 3$. Since any path is a tree, P_2 has two vertices and only one edge. Thus, $n + m = 2 + 1 = 3 = \gamma_{cer}(S(G \circ K_1))$. Suppose $\Delta = 2$. Let S be a minimum certified dominating set of $S(G \circ K_1)$. By Theorem 2.1, $\{u'_1, u'_2, ...u'_n\} \subseteq S$. If $S = \{u'_1, u'_2, ...u'_n, w_1, w_2, ..., w_m\}$, then $|S| = n + m$ and hence $\gamma_{cer}(S(G \circ K_1)) = n + m$. Now assume S containsa vertex v_i of G. Since $\Delta = 2$, each $v_i (1 \leq i \leq n)$ dominates atmost three vertices in $S(G \circ K_1)$. Also v_i is a support vertex of $G \circ K_1$, that v_i is a vertex that adjacent to a pendant vertex in $S(G \circ K_1)$. Without loss of generality, we assume v_i is adjacent with u'_j. Then it is easily vertified that $S_1 = S \cup \{v_i, u_j\} - deg_G(v_i)$ is a certified dominating set of $S(G \circ K_1)$. Also if we remove a vertex v from S, then $S_1 - \{v\}$ is not a certified dominating set of $S(G \circ K_1)$. Therefore S_1 is a minimum certified dominating set of $S(G \circ K_1)$ and hence $\gamma_{cer}(S(G \circ K_1)) = |S_1| = |S \cup \{v_i, v_j\}| - |deg_G(v_i)| = n + m + 2 - 2 = n + m$.

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