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D-K iteration Robust Controller Design for DIPC with Parametric Uncertainty

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ABSTRACT

The Double Inverted Pendulum on a Cart (DIPC) is an interesting control plant which resembles many features found in, for example, walking robots, robot gymnast (Robogymnast) and flexible space structures, and many other industrial and robotic applications. In practice, there are many uncertainty in the pendulum parameters which influences straight the system dynamics. In this paper a D-K iteration robust controller is designed for position tracking control of a double inverted pendulum on a cart (DIPC), with considering of limition control effort. Relative parametric uncertainties are considered for every masses. So the proposed controller is capable to do the position tracking control for different materials in especial density range. It is show that because of special relation between masses, moments of inertia and lengths, the relative uncertainties in moments of inertia and lengths are considered indirectly too. The robustness of the proposed control is confirmed by simulation results.

Key words: *D-K iteration; parametric uncertainty; robust performance; convex optimization.*

INTRODUCTION

This DIPC is a controllable and observable SIMO, nonlinear time invariant system with structured parameter uncertainty. It also seems to have been one of the attractive tools for testing linear and nonlinear control laws, Because DIPC is one of the complex nonlinear systems [1].

Many researchers worked on this plant, for example in [2] Fuzzy control research for stabilization a double inverted pendulum at an upright position is proposed based on weight variable fuzzy inputs, In [3] interval type2 fuzzy logic (IT2FL) and PID controller is designed for swing-up position control of DIPC, [4] has designed a controller based on neuro-fuzzy methods by using feedback-error-learning and [5] makes a study of application of μ synthesis with regional pole assignment for stabilizing a DIPC with some parametric uncertainties and designs state-feedback controller by solving a multi-objective controller synthesis problem.

In real cases, most uncertainties of such a pendulum plant may have more reasonably structures. For example, because the moments of inertia are difficult to be estimated precisely, it inducts ours to assume unknown deviations in this parameter. Also, we'd like to design the closed loop control system more robust against those parameters which have more serious influence on the system behaviour. For instance, the variation in masses of pendulums may have effects on the controllability of the Linear model. Hence it would be important to treat uncertainties in such parameter individually rather than congregate them in an overall, unstructured uncertainty of the system dynamics. Consequently, it may be more suitable to apply the μ-synthesis technique which may lead to a less conservative design to meet tighter design specifications [7]

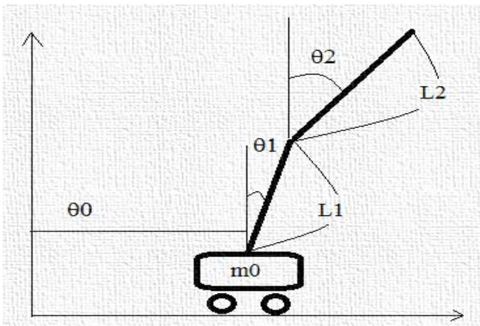

FIG. 1 DOUBLEINVERTED PENDULUM ON A CART

μ synthesis problem

The Structured Singular Value (SSV) μ is a very powerful tool for the analysis of robustness of any kind performance for a controller. However, it makes sense to seek to find the controller that minimizes a given μ-condition: this methodology is the μ-synthesis problem [2]. DK-iteration combines H_∞ synthesis and μ-analysis, and often yields good results.

For starting a short introduction to this method we have to bring a property of SSV as Improved upper bound. Define $\mathrm{D_d}$ to be the set of matrices D which commute with Δ (i.e. satisfy ΔD=DΔ). Then:

$$\mu(M) \leq \min_{D \in D} \bar{\sigma}(DMD^{-1}) \quad (1)$$

Then we have an optimization problem. This optimization is convex in D (i.e. has only one global minimum) and you know that the inequality is in fact an equality if there are 3 or fewer blocks in Δ [10]. And for upper order (4+) worst known example has an upper bound which is about 15% larger than μ [6].

Now the idea is to find the controller that minimizes the peak value over frequency of this upper bound, in this way:

$$\min_k \left(\min_{D \in D_s} \| DN(k) D^{-1} \|_\infty \right) \quad (2)$$

It means we have an alternating minimization for $\| DN(k) D^{-1} \|_\infty$ with respect to k and D(respect one while holding the another one fixed).The D-K iteration has three steps in brief:

1. **K-step**. Synthesize a H_∞ controller for the scaled problem, $\min_k (\| DND^{-1} \|_\infty)$ with fixed D(s).

2. **D-step**. Find D(jω) to minimize at each frequency $\bar{\sigma}(DMD(j\omega)^{-1})$ with fixed N.

3. Fit the magnitude of each element of D($j\omega$) to a stable and minimum phase transfer function D(s) and go to Step 1.

Usually it's desired to have a low order for D(s). One reason for preferring a low-order fit is that this reduces the order of the H_∞ problem, which usually improves the numerical properties of the H_∞ optimization k-Step and also yields a controller of lower order as mentioned earlier.

In the k-step where the controller is synthesized, it is often desirable to use a slightly sub-optimal controller (e.g. with a H_∞ norm, γ, which is 5% higher than the optimal value, γ_{min}). This yields a blend of H_∞ and H_2 optimality with a controller which usually has a steeper high-frequency roll-off than the H_∞ optimal controller.

Mathematical Model of DIPC

The parameters of double inverted pendulum system are shown in TABLE I. The DIPC system is graphically depicted in Fig. 1. Using Lagrange equation is one of the usual ways to derive the equations of motion for DIPC system.

TABLE 1 Nomenclature (system parameters)

Symbols (unit)	**Parameter**
m_0(kg)	mass of the cart
m_i (kg)	mass of pendulum i (i=1,2)
l_i(m)	distance from a pivot joint to the i-th pendulum link center of mass
Li(m)	length of an i-th pendulum link
θ_0(m)	wheeled cart position
θ_1, θ_2(rad)	pendulum angles
Ii	moment of inertia of i-th pendulum link w.r.t. its center of mass ($I_1 = I_2$)
G	gravity constant
U	control force
P	potential energy
L	Lagrangian
kg = kilogram	s = second ,m = meter,

Differentiating of the Lagrangian function by $\dot{\theta}$ and θ, yields the explicit Lagrange equation (3) as [11]:

$$u = \left(\sum m_i\right)\ddot{\theta}_0 + (m_1 l_1 + m_2 L_1)\cos(\theta_1)\ddot{\theta}_1 + m_2 l_2 \cos(\theta_2)\ddot{\theta}_2 - (m_1 l_1 + m_2 L_1)\sin(\theta_1)\dot{\theta}_1^2 - m_2 l_2 \sin(\theta_2)\dot{\theta}_2^2$$

$$0 = (m_1 l_1 + m_2 L_1)\cos(\theta_1)\ddot{\theta}_0 + \left(m_1 l_1^2 + m_2 L_1^2 + I_1\right)\ddot{\theta}_1 + m_2 L_1 l_2 \cos(\theta_1 - \theta_2)\ddot{\theta}_2 + m_2 L_1 l_2 \sin(\theta_1 - \theta_2)\dot{\theta}_2^2 - (m_1 l_1 + m_2 L_1) g \sin\theta_1$$

$$0 = m_2 l_2 \cos(\theta_2)\ddot{\theta}_0 + m_2 L_1 l_2 \cos(\theta_1 - \theta_2)\ddot{\theta}_1 + \left(m_2 l_2^2 + I_2\right)\ddot{\theta}_2 - m_2 L_1 l_2 \sin(\theta_1 - \theta_2)\dot{\theta}_1^2 - m_2 l_2 g \sin\theta_2$$

If we add special disturbance (d) to our model then it yields the following nonlinear vector-matrix differential equation:

$$D_{nl}(\theta)\ddot{\theta} + Q_{nl}(\theta, \dot{\theta}) = Hu + Td$$

Where in that we have:

$$D_{nl}(\theta) = \begin{pmatrix} d_1 & d_2 \cos\theta_1 & d_3 \cos\theta_2 \\ d_2 \cos\theta_1 & d_4 & d_5 \cos(\theta_1 - \theta_2) \\ d_3 \cos\theta_2 & d_5 \cos(\theta_1 - \theta_2) & d_6 \end{pmatrix}$$

$$Q_{nl} = \begin{pmatrix} -d_2 \sin(\theta_1 - \theta_2)\dot{\theta}_1^{\,2} - d_3 \sin(\theta_2)\dot{\theta}_2^{\,2} \\ d_5 \sin(\theta_1 - \theta_2)\dot{\theta}_2^{\,2} - f_1 \sin(\theta_1) \\ -d_5 \sin(\theta_1 - \theta_2)\dot{\theta}_1^{\,2} - f_2 \sin(\theta_2) \end{pmatrix}$$

$$H = (1 \quad 0 \quad 0)^T$$

$$T = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

Assuming that centers of mass of the pendulums are located in the geometrical center of the links, which are solid rods, we have: $l_i = L_i/2, I_i = m_i L_i^{\,2}/12$. then we get:

$$d_1 = m_0 + m_1 + m_2$$

$$d_2 = m_1 l_1 + m_2 L_1 = \left(\frac{1}{2}m_1 + m_2\right)L_1$$

$$d_3 = m_2 l_2 = \frac{1}{2}m_2 L_2$$

$$d_4 = m_1 l_1^{\,2} + m_2 L_1^{\,2} + I_1 = \left(\frac{1}{3}m_1 + m_2\right)L_1^{\,2}$$

$$d_5 = m_2 L_1 l_2 = \frac{1}{2}m_2 L_1 L_2$$

$$d_6 = m_2 l_2^{\,2} + I_2 = \frac{1}{3}m_2 L_2^{\,2}$$

$$f_1 = (m_1 l_1 + m_2 L_1)g = \left(\frac{1}{2}m_1 + m_2\right)L_1 g$$

$$f_2 = m_2 l_2 g = \frac{1}{2}m_2 L_2 g$$

Linearized Model of DIPC

As the proposed plan to design the controller can be done only by using linear model of the plant, so the above set of equations should be linearized about the equilibrium point. It means $\theta_1 = \theta_2 = 0$, $\sin\theta_1 = \theta_1$, $\sin\theta_2 = \theta_2$, $\sin(\theta_1 - \theta_2) = \theta_1 - \theta_2$, $\cos(\theta_1 - \theta_2) = 1$, $\cos(\theta_1) = 1$, $\cos(\theta_2) = 1$ and $\dot{\theta}_1 = \dot{\theta}_2 = 0$.

After linearization of Equation under the assumptions of small deviations of the pendulum from the vertical position and of small velocities, one obtains the following equation:

$$\ddot{\theta} = -D^{-1}Q\,\theta + D^{-1}Hu + D^{-1}Td$$

where,

$$D = \begin{pmatrix} d_1 & d_2 & d_3 \\ d_2 & d_4 & d_5 \\ d_3 & d_5 & d_6 \end{pmatrix}$$

$$Q = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -f1 & 0 \\ 0 & 0 & -f2 \end{pmatrix}.$$

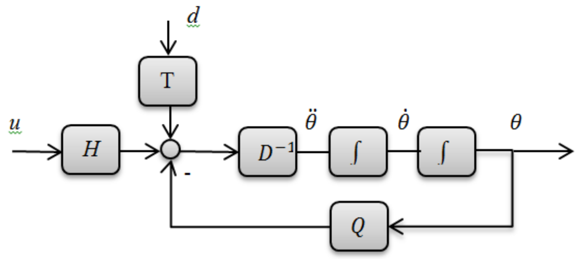

Fig. 2 BLOCK DIAGRAM OF THE DIPC SYSTEM

Modeling of Uncertainties

Assume that the masses m0, m1, m2 are constants but with possible relative error of 10%, then:

$$m_i = \overline{m}(1 + p_i\delta_{mi}), i = 0,1,2 \qquad (4)$$

$$p_i = 0.10$$

As mentioned earlier because of assuming that the centers of mass of the pendulums are located in the geometrical center of the links, which are solid rods, $l_i = L_i/2, I_i = m_i L_i^{\,2}/12$, we can state that this relation convey us to this point that in practice there is this kind of uncertainty for moments of inertia too. So in real a relative uncertainty is also considerd for moments of inertia.

In "Equation (4)" m_i is the nominal value of the corresponding masses, pi = 0.1 is the maximum relative uncertainty in each of these masses and $-1 < \delta_{mi} < 1$; i=0, 1, 2. As a result the matrix D is obtained in the form

$$D = \overline{D} + \Delta_D.$$

Where the elements of $\overline{D}$are determined by the nominal values of the moments of inertia,

$$D = \begin{pmatrix} d_1 & d_2 & d_3 \\ d_2 & d_4 & d_5 \\ d3 & d5 & d6 \end{pmatrix} +$$

$$\begin{pmatrix} \overline{m_0}p_0\delta_{m0} + \overline{m_1}p_1\delta_{m1} + \overline{m_2}p_2\delta_{m2} & 0.5L_1\overline{m_1}p_1\delta_{m1} + L_1\overline{m_2}p_2\delta_{m2} & 0.5L_2\overline{m_2}p_2\delta_{m2} \\ 0.5L_1\overline{m_1}p_1\delta_{m1} + L_1\overline{m_2}p_2\delta_{m2} & L_1^{\,2}\overline{m_1}p_1\delta_{m1}/3 + L_1^{\,2}\overline{m_2}p_2\delta_{m2} & 0.5L_1L_2\overline{m_2} \\ 0.5L_2\overline{m_2}p_2\delta_{m2} & 0.5L_1L_2\overline{m_2} & L_2^{\,2}\overline{m_2}p_2\delta_{m2}/3 \end{pmatrix}$$

The matrix $\Delta_D$may be represented in below format:

$$\Delta_D = D_1\Delta_m D_2$$

That this type of uncertainty formulation have presented in some example of [9], where

$$D_1 =$$

$$\begin{pmatrix} m_0 s_0 & m_1 s_1 & .5L_1 m_1 s_1 & m_2 s_2 & L_1 m_2 s_2 & .5L_2 m_2 s_2 \\ 0 & .5L_1 m_1 s_1 & (L_1^{\,2} m_1 s_1)/3 & L_1 m_2 s_2 & L_1^{\,2} m_2 s_2 & .5L_1 L_2 m_2 s_2 \\ 0 & 0 & 0 & .5L_2 m_2 s_2 & .5L_1 L_2 m_2 s_2 & (L_2^{\,2} m_2 s_2)/3 \end{pmatrix}$$

$$D_2 = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}^T$$

$$\Delta_m = \begin{pmatrix} \delta_{m0} & 0 & 0 & 0 & 0 & 0 \\ 0 & \delta_{m1} & 0 & 0 & 0 & 0 \\ 0 & 0 & \delta_{m1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta_{m2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \delta_{m2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \delta_{m2} \end{pmatrix}.$$

Next we calculate the matrix D^{-1} as:

$$D^{-1} = \bar{D}^{-1} + D_2^{-1} \Delta_m^{-1} D_1^{-1}$$

$$= D_2^{-1}(D_1^{-1}\bar{D}D_2^{-1} + \Delta_m)^{-1} D_1^{-1}$$

Using the Matrix Inversion Lemma, we obtain

$$\left(D_1^{-1}\bar{D}D_2^{-1} + \Delta_m\right)^{-1} = D_2\bar{D}^{-1}D_1 - D_2\bar{D}^{-1}D_1\Delta_m(D_2\bar{D}^{-1}D_1\Delta_m$$

where, I_{6*6} is the unit matrix. Therefore, we have:

$$D^{-1} = \bar{D}^{-1} - \bar{D}^{-1}D_1\Delta_m(D_2\bar{D}^{-1}D_1\Delta_m + I_{6*6})^{-1}D_2\bar{D}^{-1}$$

We show the matrix D^{-1} in LFT format:

$$D^{-1} = F_u(Q_m, \Delta_m) = Q_{m_{22}} + Q_{m_{21}}\Delta_m\left(I_3 - Q_{m_{11}}\Delta_m\right)^{-1}Q_{m_{12}}$$

such that

$$Q_m = \begin{pmatrix} -D_2\bar{D}^{-1}D_1 & D_2\bar{D}^{-1} \\ -\bar{D}^{-1}D_1 & \bar{D}^{-1} \end{pmatrix}.$$

In similar manner we carry out these proceeds for Q:

$$Q = \bar{Q} + \Delta_Q$$

where the elements of $\bar{Q}$ are determined by the nominal values of the moments of inertia,

$$\bar{Q} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -f1 & 0 \\ 0 & 0 & -f2 \end{pmatrix}.$$

The matrix Δ_Q may be represented as

$$\Delta_Q = K_1\Delta_n K_2$$

where,

$$K_1 = \begin{pmatrix} 0 & 0 & 0 \\ -0.5gL_1\overline{m_1}s_1 & -gL_1\overline{m_2}s_2 & 0 \\ 0 & 0 & -0.5gL_2\overline{m_2}s_2 \end{pmatrix}$$

$$K_2 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\Delta_Q = \begin{pmatrix} \delta_{m0} & 0 & 0 \\ 0 & \delta_{m1} & 0 \\ 0 & 0 & \delta_{m2} \end{pmatrix}.$$

In turn, the matrix $Q = \bar{Q} + K_1\Delta_n K_2$ can be re-written as an upper LFT:

$$Q = F_u(Z_Q, \Delta_Q) = Z_{Q_{22}} + Z_{Q_{21}}\Delta_Q\left(I_3 - Z_{Q_{11}}\Delta_Q\right)^{-1}Z_{Q_{12}}$$

where,

$$Z = \begin{pmatrix} 0_{6*6} & K2 \\ K1 & \bar{Q} \end{pmatrix}.$$

To represent the pendulum model as an LFT of the real uncertain parameters δ_{m0}, δ_{m1}, δ_{m2}, we extract out the uncertain parameters as shown in FIG 3.

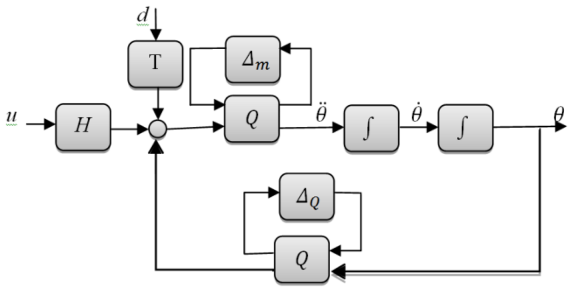

FIG 3 BLOCK DIAGRAM OF SYSTEM WITHUNCERTAIN PARAMETERS

Now we can see themain uncertain matrix:

$$\Delta_{main} = \begin{bmatrix} \Delta_m & 0 \\ 0 & \Delta_Q \end{bmatrix}$$

Controller Design

By assuming $m_0 = 1.8$; $m_1 = 0.8$; $m_2 = 0.5$; $L_1 = 0.3$; $L_2 = 0.65$; $g = 9.8$, the controller is designed. The design setup is based on the nominal setup described earlier. Combining the nominal design setup with the uncertain model description gives the complete design setup shown in FIG 4, where Wp is the weight matrix for the performance specification. The objectives of controller design are robust stability and good tracking performance in presence of disturbances, despite the limited control effort. These objectives can be optimized by the solution of a mixed-sensitivity problem formulated on the generalized plant illustrated in FIG 4.

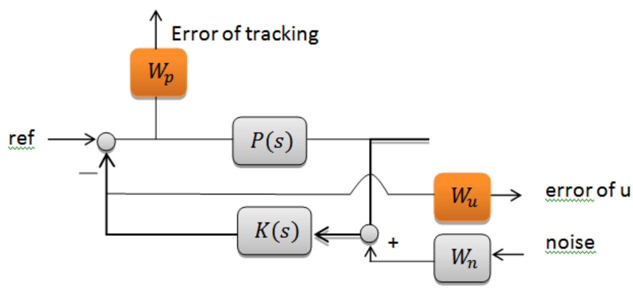

FIG 4BLOCK DIAGRAM REPRESENTATION OF MIXED-SENSITIVITY SOLUTION FOR DIPC SYSTEM

In the given case the matrix Wn is chosen as: $W_n(s) = w_n(s).I_{3*3}$, where the weighting transfer function $w_n = 2*10^{-5}\frac{15s+1}{0.15+1}$ is a high pass filter, which shapes the noise spectral density. The weighting performance functions are chosen in the forms of:

$$W_u(s) = w_u(s) = 3*10^{-6}$$

$$W_p(s) = \begin{bmatrix} w_{p1}(s) & 0 & 0 \\ 0 & w_{p2}(s) & 0 \\ 0 & 0 & w_{p3}(s) \end{bmatrix}$$

$$w_{p1}(s) = .4\frac{+13s^3+79s^2+80s+65.1}{15s^4+1500s^3+13000s^2+12000s+0.005}$$

$$w_{p2}(s) = w_{p3}(s) = 5*10^{-7}*\frac{s^4+13s^3+79s^2+80s}{15s^4+1500s^3+13000s^2+12000s}$$

Fig.5shows the magnitude of these weighting functions throughout the frequency:

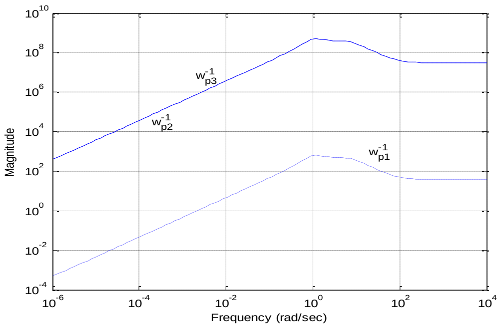

FIG. 5 INVERSE OF DIPC PERFORMANCEWEIGHTING FUNCTIONS

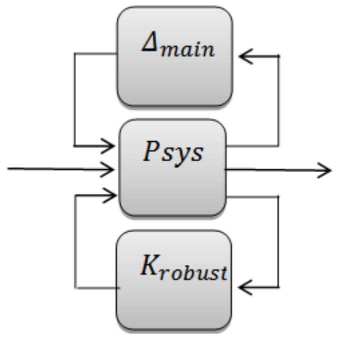

FIG. 6 THE SYSTEM SETUP FORDESIGN OF ROBUST FEEDBACKCONTROLLER

Let us denote the transfer function matrix of the nineteen-input, sixteen-output as open loop system consisting of the DIPC model plus the weighting functionsby P(s), and let the block structure $\Delta_P$of uncertainties be defined by:

$$\Delta_p = \left\{\begin{pmatrix} \Delta_{main} & 0 \\ 0 & \Delta_F \end{pmatrix} : \Delta \in R^{9*9}, \Delta_F \in R^{9*4}\right\}$$

The first block of the matrix Δ_P, the uncertainty block Δ_{main}, corresponds to the parametric uncertainties modeled in the DIPC system. The second block, Δ_F, is a fictitious uncertainty block, introduced to include the performance objectives in the framework of the μ-approach.

To reach the design objectives a stabilizing controller K is to be found such that, at each frequency $\omega \in (0,\infty)$, the following condition is satisfied.

$$\mu_{\Delta_p}[F_L(P,K)(jw)] < 1.$$

The fulfillment of this condition guarantees robust performance of the closed loop system, i.e.

$$\left\|\begin{pmatrix} W_p S(G) \\ W_u KS(G) \end{pmatrix}\right\|_\infty < 1$$

where, $G = FU(Gsys, \Delta)$, for all stable perturbations Δ with $\|\Delta\|_\infty < 1$.

The progress of the D-K iteration is shown in TABLE (II), it can be seen from the table that after the 6th iteration the maximum value of μ is equal to 0.963, which indicates that the robust performance has been achieved. The final controller obtained is of 82th order. It's clear that this controller is stable.

TABLE 2 D-K ITERATIONS RESULTS IN iterations results in M-SYNTHESIS

Number of iterations	Maximum value of μ	Controller order
3	1.996	70
4	1.473	90
5	1.171	84
6	0.963	82
7	0.871	82

And we select the 6th iteration as our controller.

Controller Analysis

The maximum value of μ for the case of robust stability is 0.90565 which means the stability of the system is preserved under perturbations which satisfy $\|\Delta\|_\infty < 1$. And the closed-loop system achieves robust performance, since the maximum value of μ is equal to 0.92847.

The transient response of the controlled system is shown in Fig.7 .

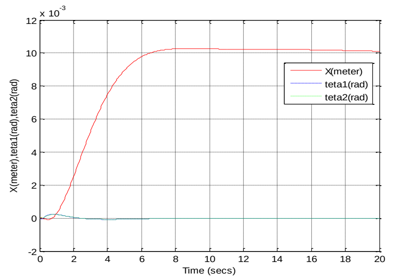

FIG 7 CLOSED-LOOPTRANSIENT RESPONSE OF CONTROLLER

The disturbance rejection of the closed-loop system is shown in Fig.8 which shows the response of the system against the disturbances applied in form of three step signals with gain 0.1 in all three input channels.

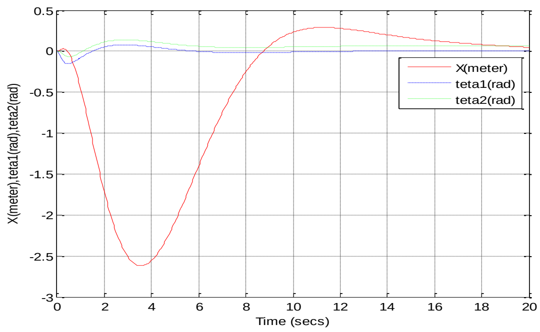

FIG 8 DISTURBANCE ATTENUATION

Reduced Order Control &Nonlinear Plant

As mentioned earlier, the controller obtained by μ synthesis is initially of 82th order, which makes its implementation in practice difficult. Therefore, it's necessary to reduce the controller order. So we use the model reduction algorithm based on system balancing followed by optimal Hankel approximation.

Fig.9 shows us the frequency responses of the full order and reduced order (order 8) μ controllers.

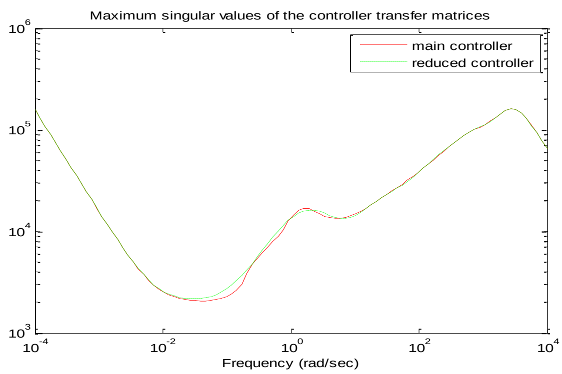

FIG 9 FREQUENCY RESPONSEES: FULL&REDUCED ORDER CONTROLLERS

The closed-loop transient responses with the full order controller and those with the reduced order controller are practically very similar. Fig. 10 shows the nonlinear simulation block diagram of closed-loop system in Simulink environment of MATLAB [8].

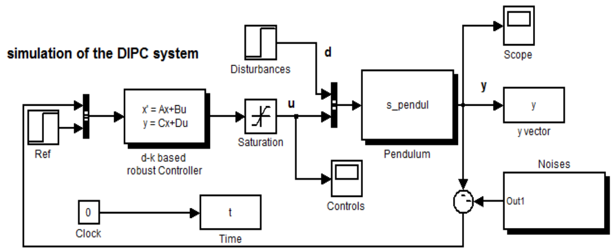

FIG 10 SIMULATION MODEL OF THE NONLINEAR DIPC SYSTEM

Conclusions

A complete design and implementation of robust controller for an unstable system has been described in this paper. A linear model of a double inverted pendulum system together with a description of the model uncertainties has been derived. A complete nonlinear model was used for implementing the designed proposed control which was simulated by Matlab toolbox. Relative parametric uncertainties were considered for every masses. It was shown that the proposed controller was capable to do the position tracking control for different masses. The effectiveness of the proposed control was shown by simulation results.

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