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INFLUENCE OF CHEMICAL REACTION ON MHD CASSON FLUID FLOW OVER A VERTICAL POROUS SURFACE

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Abstract:

In this paper, we analyze the mass transfer of MHD Casson fluid flow in presence of chemical reaction over a porous stretching surface. The boundary layer equations are transformed into ordinary differential equations by using suitable similarity transformations. The resulting equations are solved using an implicit FDM known as the Keller Box method. The results are discussed graphically for the effect of various physical parameters like magnetic parameter, Casson parameter, Grashof number, Schmidt number etc. for the velocity and concentration profiles.

Key Words: MHD, Casson parameter, Mass transfer, Keller Box method, Grashof number.

Introduction:

In view of industrial applications in industries like polymer industry, drilling of petroleum, food manufacturing etc., non-Newtonian fluids are more important than Newtonian fluids [1]. Also, the stretching sheet problem applications in industry motivated many authors to do research in this direction. P.S.Gupta and A.S.Gupta [2] analyzed the similarity solutions for the heat and mass transfer of a boundary layer over a stretching sheet subject to suction or blowing and an incompressible second order fluid past a stretching sheet occur in extrusion of a polymer sheet from a die as mentioned by K. R. Rajgopal et al. [3].

Casson fluid is a non-Newtonian fluid, as human blood and many other important fluids like jelly's, printer ink etc., are casson fluids recently lot of research is focused on this type of fluids. Casson fluid has an infinite viscosity at zero rate of shear and has an yield stress below which no flow occurs[4]. Many experiments are conducted to study the behavior of human blood, a casson fluid[5].

Analytical solutions are also obtained for some casson fluid problems, Fredrickson[6] considered steady flow through tube and Mustafa et. al [7] and Casson[8] studied the problem of unsteady boundary layer flow and heat transfer over a moving flat plate with parallel free stream.

Recently study of magneto hydrodynamic flow gained lot of importance as they have wide range of theoretical and practical applications in designing cooling systems and the quality of the final product depends on cooling process and is effected much by the application of magnetic field. Properties of electrically conducting fluid in presence of transverse magnetic field over a stretching sheet was studied by A.Chakrabarti and A.S.Gupta [9], H.I. Anderson [10] and Ming-I Char [11] obtained solutions for the heat and mass transfer over an exponentially stretching sheet in presence of transverse magnetic field. The study of stretching sheet concept has been extended to casson fluid in view of its vast application. Hayat et. al [12] discussed the problem of mixed convection flow of a casson fluid at stagnation point. The boundary layer flow problem of a casson fluid over a permeable stretching or shrinking sheet in presence of external magnetic field was presented by Bhattacharyya et. al [13].

In this chapter study of magneto hydrodynamic flow of Casson fluid over a vertical porous surface with chemical reaction is considered. Numerical solutions are obtained with the help of implicit finite difference numerical technique called Keller Box method discussed in Cebeci et. al [14]. Similarity transformations are used to convert governing partial differential equations into set of ordinary differential equations. The effect of various flow parameters like Casson parameter, magnetic parameter, suction parameter, Schmidt number, Grashof number etc., on velocity and concentration are displayed through graphs and discussed in detailed.

Equations of motion:

Consider a two-dimensional steady incompressible Casson fluid flow over a vertical porous stretching surface at $y = 0$ in the presence of a transverse magnetic field of strength B_0, as shown in **Figure 1**. Let the *x*-axis be taken along the direction of the plate and *y*-axis normal to it. The fluid occupies the half space $y > 0$. The mass transfer phenomenon with chemical reaction is also retained. The tangential velocity u_w, due to the stretching surface is assumed to vary proportionally to the distance x so that $u_w = ax$, where a is a constant. Let the applied magnetic field is zero.

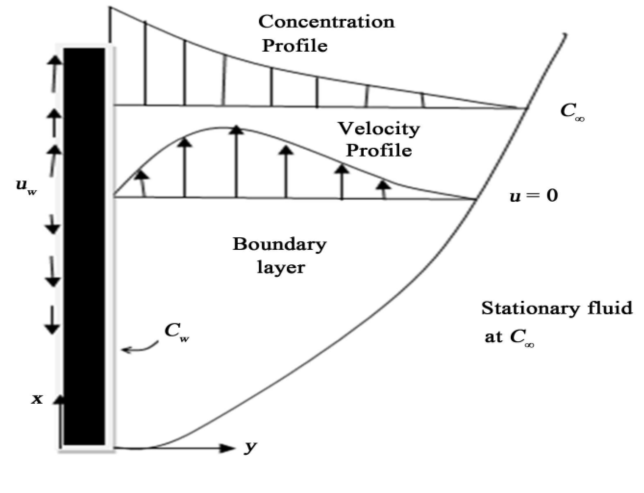

Fig.1 Schematic diagram

The rheological equation of state for an isotropic flow of a Casson fluid [8] can be expressed as**:**

$$\tau_{ij} = \begin{cases} 2(\mu_B + P_y/\sqrt{2\pi})e_{ij}, \pi > \pi_c \\ 2(\mu_B + P_y/\sqrt{2\pi_c})e_{ij}, \pi < \pi_c \end{cases} \quad (1)$$

Here $\pi = e_{ij}e_{ij}$ and e_{ij}, is the (i,j)th component of the deformation rate, π is the product of the component of the deformation rate with itself, π_c is a critical value of this product based on the non-Newtonian model, μ_B is plastic dynamic viscosity of the non-Newtonian fluid, and P_y is the yield stress of the fluid, C is the concentration of the fluid.

The equations governing the steady boundary layer flow of the Casson fluid are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\left(1 + \frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} + g\beta_c(C - C_\infty) - \sigma\frac{B_0^2}{\rho}u \quad (3)$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2} - \gamma(C - C_\infty) \quad (4)$$

Subject to the following boundary conditions:

$u(x,0)=u_w$, $v(x,0)=-v_0(x)$, $C(x,0)=C_w$, $u(x,\infty)=0$, $C(x,\infty)=C_\infty$ (5)

where, u and v are the components of velocity respectively in the x and y directions, υ is the kinematic viscosity, D_m is the mass diffusion, $\beta = \mu_B\sqrt{2\pi_c}/P_y$ is the non-Newtonian parameter of the Casson fluid, γ is the reaction rate, $v_0(x)$ is the suction velocity from the surface, C_w is the concentration at the surface, C_∞ is the free stream concentration, β_c is the solutal expansion coefficient, σ is electrical conductivity of the fluid, ρ is the fluid density, g is gravitational acceleration. Introducing the following dimensionless quantities:

$$u = u_w f'(\eta) \quad (6)$$

$$v = -\sqrt{\frac{\upsilon u_w}{x}}f(\eta) \quad (7)$$

$$\eta = y\sqrt{\frac{u_w}{\upsilon x}} \quad (8)$$

$$\phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \quad (9)$$

upon substitution, the governing partial differential equations are converted into ordinary differential equations

$$\left(1 + \frac{1}{\beta}\right)f''' + ff'' - f'^2 + Gc\,\phi - Mf' = 0 \quad (10)$$

$$\phi'' + Sc.f\phi' - ScB\phi = 0 \quad (11)$$

and the boundary conditions take the following form

$$at\ \eta = 0, f = S, \phi = 1, f' = 1 \quad (12)$$

$$as\ \eta \to \infty, f' \to 0, \phi \to 0 \quad (13)$$

The prime denotes differentiation with respect to the similarity variable η, where $B = \gamma / a$ is the chemical reaction parameter, $M = \frac{2\sigma B_0^2 l}{\rho U_0 e^{x/l}}$ is the magnetic parameter, $S = v_0 x/\sqrt{a\upsilon}$ is the suction parameter, $Sc = \upsilon / D_m$ is the Schmidt number, $G_c = g\beta_c(C_w - C_\infty)x/u_w^2$ is the local solutal Grashof number.

Numerical Procedure:

Equations subject to boundary conditions are solved numerically using an implicit-finite difference scheme known as Keller box method, as described by Cebeci and Bradshaw[14]. The steps followed are

1. Reduce equations(10)-(11) to a first order equations
2. Write the difference equations using central differences
3. Linearize the resulting algebraic equation by Newton's method and write in matrix vector form
4. Use the block tri-diagonal elimination technique to solve the linear system.

Introduce $f' = p$, (14)

$p' = q$, (15)

$g' = t$ $(g = \phi)$ (16)

equations (9) and (10) reduces to

$$\left(1 + \frac{1}{\beta}\right)q' + fq - p^2 - Gcg - Mp = 0 \quad (17)$$

$$t' + Scft - ScBg = 0 \quad (18)$$

consider the segment η_{j-1}, η_j with $\eta_{j-1/2}$ as the mid-point $\eta_0 = 0$, $\eta_j = \eta_{j-1} + h_j$, $\eta_J = \eta_\infty$ (19)

where h_j is the $\Delta\eta$ spaces and j=1,2,……..J is a sequence in number that indicates the coordinate locations.

$$\frac{f_j - f_{j-1}}{h_j} = \frac{p_j + p_{j-1}}{2} = p_{j-1/2} \quad (20)$$

$$\frac{p_j - p_{j-1}}{h_j} = \frac{q_j + q_{j-1}}{2} = q_{j-1/2} \quad (21)$$

$$\frac{g_j - g_{j-1}}{h_j} = \frac{t_j + t_{j-1}}{2} = t_{j-1/2} \quad (22)$$

$$\left(1 + \frac{1}{\beta}\right)\frac{q_j - q_{j-1}}{h_j} + \left(\frac{f_j + f_{j-1}}{2}\right)\left(\frac{q_j + q_{j-1}}{2}\right) - \left(\frac{p_j + p_{j-1}}{2}\right)^2 - Gc\left(\frac{g_j + g_{j-1}}{2}\right) - M\left(\frac{p_j + p_{j-1}}{2}\right) = 0 \quad (23)$$

$$\frac{t_j - t_{j-1}}{h_j} + Sc\left(\frac{f_j + f_{j-1}}{2}\right)\left(\frac{t_j + t_{j-1}}{2}\right) - .ScB\left(\frac{g_j + g_{j-1}}{2}\right) = 0 \quad (24)$$

Newton's method

To linearizing the non linear system of equations (7) to (11) introduce,

$$f_j^{(k+1)} = f_j^{(k)} + \delta f_j^{(k)}$$
$$p_j^{(k+1)} = p_j^{(k)} + \delta p_j^{(k)}$$
$$q_j^{(k+1)} = q_j^{(k)} + \delta q_j^{(k)}$$
$$g_j^{(k+1)} = g_j^{(k)} + \delta g_j^{(k)}$$
$$t_j^{(k+1)} = t_j^{(k)} + \delta t_j^{(k)} \quad (25)$$

Substitute equations (12) in (7) to (11), write

$$\delta f_j - \delta f_{j-1} - \frac{h_j}{2}\left(\delta p_j + \delta p_{j-1}\right) = (r_1)_{j-\frac{1}{2}} \quad (26)$$

$$\delta p_j - \delta p_{j-1} - \frac{h_j}{2}\left(\delta q_j + \delta q_{j-1}\right) = (r_2)_{j-\frac{1}{2}} \quad (27)$$

$$\delta g_j - \delta g_{j-1} - \frac{h_j}{2}\left(\delta t_j + \delta t_{j-1}\right) = (r_3)_{j-\frac{1}{2}} \quad (28)$$

$$(a_1)_j \delta q_j + (a_2)_j \delta q_{j-1} + (a_3)_j \delta f_j + (a_4)_j \delta f_{j-1} + (a_5)_j \delta p_j + (a_6)_j \delta p_{j-1} = (r_4)_{j-\frac{1}{2}} \quad (29)$$

$$(b_1)_j \delta t_j + (b_2)_j \delta t_{j-1} + (b_3)_j \delta f_j + (b_4)_j \delta f_{j-1} + (b_5)_j \delta p_j + (b_6)_j \delta p_{j-1} + (b_7)_j \delta g_j + (b_8)_j \delta g_{j-1} = (r_5)_{j-\frac{1}{2}} \quad (30)$$

where

$$(a_1)_j = 1 + \frac{\beta h_j}{4(\beta+1)}(f_j + f_{j-1})$$
$$(a_2)_j = (a_1)_j - 2.0$$
$$(a_3)_j = \frac{\beta h_j}{4(\beta+1)}(q_j + q_{j-1})$$
$$(a_4)_j = (a_3)_j$$
$$(a_5)_j = -\frac{\beta h_j}{(\beta+1)}((p_j + p_{j-1}) + M)$$
$$(a_6)_j = (a_5)_j \quad (31)$$

$$(b_1)_j = 1 + \frac{\Pr h_j}{4}(f_j + f_{j-1})$$
$$(b_2)_j = (b_1)_j - 2.0$$
$$(b_3)_j = \frac{\Pr h_j}{2}(t_j + t_{j-1})$$
$$(b_4)_j = (b_3)_j$$
$$(b_5)_j = -\frac{\Pr h_j}{2}(g_j + g_{j-1})$$
$$(b_6)_j = (b_5)_j$$
$$(b_7)_j = -\frac{\Pr h_j}{2}(p_j + p_{j-1})$$
$$(b_8)_j = (b_7)_j \quad (32)$$

and

$$(r_1)_j = f_{j-1} - f_j + \frac{h_j}{2}\left(p_j + p_{j-1}\right)$$
$$(r_2)_j = p_{j-1} - p_j + \frac{h_j}{2}\left(q_j + q_{j-1}\right)$$
$$(r_3)_j = g_{j-1} - g_j + \frac{h_j}{2}\left(t_j + t_{j-1}\right)$$
$$(r_4)_j = q_{j-1} - q_j - \frac{\beta h_j}{4(\beta+1)}\left(f_j + f_{j-1}\right)\left(q_j + q_{j-1}\right) + \frac{\beta h_j}{2(\beta+1)}\left(p_j + p_{j-1}\right)^2 + M\frac{\beta h_j}{2(\beta+1)}\left(p_j + p_{j-1}\right)$$
$$(r_5)_j = t_{j-1} - t_j - \frac{\Pr h_j}{4}\left(f_j + f_{j-1}\right)\left(t_j + t_{j-1}\right) + \frac{\Pr h_j}{4}\left(p_j + p_{j-1}\right)\left(g_j + g_{j-1}\right) \quad (33)$$

Taking j=1,2,3…, The system of equations becomes

$[A_1][\delta_1]+[C_1][\delta_2]=[r_1]$

$[B_2][\delta_1]+[A_2][\delta_2]+[C_2][\delta_3] = [r_2]$

$[B_{J-1}][\delta_{j-2}]+[A_{J-1}][\delta_{j-1}]+[C_{J-1}][\delta_j] = [r_{J-1}]$

$[B_J][\delta_{J-1}]+[A_J][\delta_J]= [r_J]$ (34)

where

$$A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ d & 0 & 0 & d & 0 \\ 0 & d & 0 & 0 & d \\ (a_2)_1 & 0 & (a_3)_1 & (a_1)_1 & 0 \\ 0 & (b_2)_1 & (b_3)_1 & 0 & (b_1)_1 \end{bmatrix}$$

$$A_j = \begin{bmatrix} d & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & d & 0 \\ 0 & -1 & 0 & 0 & d \\ (a_6)_j & 0 & (a_3)_j & (a_1)_j & 0 \\ (b_6)_j & (b_8)_j & (b_3)_j & 0 & (b_1)_j \end{bmatrix}$$

$$B_j = \begin{bmatrix} 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & d & 0 \\ 0 & 0 & 0 & 0 & d \\ 0 & 0 & (a_4)_j & (a_2)_j & 0 \\ 0 & 0 & (b_4)_j & 0 & (b_2)_j \end{bmatrix}$$

$$\boldsymbol{C}_j = \begin{bmatrix} \boldsymbol{d} & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ (\boldsymbol{a}_5)_j & 0 & 0 & 0 & 0 \\ (\boldsymbol{b}_5)_j & (\boldsymbol{b}_7)_j & 0 & 0 & 0 \end{bmatrix} \qquad (35)$$

The Block Elimination Method:

The linearized differential equations of the system has a block diagonal structure. This can be written in tri diagonal matrix form as

$$\begin{bmatrix} [A_1] & [C_1] & & & \\ [B_2] & [A_2] & [C_2] & & \\ & & \vdots & & \\ & & [B_{J-1}] & [A_{J-1}] & [C_{J-1}] \\ & & & [B_J] & [A_J] \end{bmatrix} \begin{bmatrix} [\delta_1] \\ [\delta_2] \\ \vdots \\ [\delta_{J-1}] \\ [\delta_J] \end{bmatrix} = \begin{bmatrix} [r_1] \\ [r_2] \\ \vdots \\ [r_{J-1}] \\ [r_J] \end{bmatrix} \qquad (36)$$

This is of the form $\mathbf{A}\,\boldsymbol{\delta} = \mathbf{r}$ (37)

To solve the above system

write $[A] = [L]\,[U]$ (38)

where

$$L = \begin{bmatrix} [\alpha_1] & & & & \\ [\beta_2] & [\alpha_2] & & & \\ & & \ddots & & \\ & & & [\alpha_{j-1}] & \\ & & & [\beta_J] & [\alpha_J] \end{bmatrix}$$

and

$$U = \begin{bmatrix} [I] & [\Gamma_1] & & & \\ & [I_2] & [\Gamma_2] & & \\ & & \ddots & & \\ & & & [I] & [\Gamma_{J-1}] \\ & & & & [I] \end{bmatrix} \qquad (39)$$

Where [I] is the identity matrix

$[\alpha_i][\Gamma_i]$ are determined by the following equations

$[\alpha_1] = [A_1]$

$[A_1][\Gamma_1] = [C_1]$

$[\alpha_j] = [A_j]-[B_j][\Gamma_{j-1}]$ j=2,3,. ....,J

$[\alpha_j][\Gamma_j] = [C_j]$ j=2,3,..........,J-1

Substituting (38) in (37)

$LU\delta = r$

Let $U\delta = W$

then $LW = r$

where $W=\begin{bmatrix} [w_1] \\ [w_2] \\ \vdots \\ [w_{j-1}] \\ [w_J] \end{bmatrix}$

Now

$[\alpha_1]\,[w_1] = [r_1]$

$[\alpha_j]\,[w_J] = [r_J]-[B_J][W_{j-1}]$ for $2 \le j \le J$

Once the elements of W are found, substitute in $L\delta=W$ and solve for δ

$[\delta_J] = [W_J]$

$[\delta_J] = [W_J]-[\Gamma_J][\delta_{J+1}]$, $1 \le j \le J-1$

These calculations are repeated until convergence criterion is satisfied and we stop the calculations when $|\delta g_0^{(i)}| \le \varepsilon$, where ε is very small prescribed value taken to be ε = 0.0001.

Results and Discussion:

From the process of numerical computation, the plate surface temperature, the local skin-friction coefficient, the local

Nusselt number and the local Sherwood number, which are respectively proportional to $-f''(0)$ and $-\phi'(0)$ are computed and their numerical values are presented through a tabular form. From Table 1 and Table 2 comparison of work of [16] to [18], we can notice that the results obtained by present work are in good agreement with the results available in literature.

Table.1 Comparison of values of $-\phi'(0)$ for different values of B with $S = Gc = 0$ as $\beta \to \infty$.

B	[16] Andersson et. al	[17] Shehzad et. al	[18]Arthur et. al	Present Study
0.1	0.66902	0.66898	0.668983	0.668992
0.5	--	--	--	0.932248
1	1.17649	1.17650	1.176439	1.176505
10	3.23122	3.23175	3.231228	3.231244

For Newtonian case the local skin friction and Sherwood number are shown in Table.2

Table 2. Numerical results of skin friction coefficient and the Sherwood number

B	Gc	Sc	S	B	-f ”(0)	-ϕ‘(0)
0.5	0.1	0.6	0.1	0.3	0.571798	0.691393
1					0.699787	0.675804
1.5					0.766503	0.668280
0.5	0.5				0.483050	0.699964
	1				0.376545	0.709572
	1.5				0.273907	0.718263
		0.5			0.570030	0.622040
		1			0.576205	0.928675
		1.5			0.579180	1.172916
			0.5		0.645503	0.845481
			1		0.748123	1.058821
			1.5		0.861368	1.290755
				0.5	0.573100	0.777377
				1	0.575405	0.958031
				1.5	0.576977	1.108648

MATLAB has been used for numerical computations and to plot velocity & concentration profiles to demonstrate the effect of various flow parameters.

Fig..1, Fig..2, Fig.3, represents the change in velocity with respect to change in Casson parameter β, Suction Parameter S and Magnetic parameter M respectively. From these graphs it is observed that velocity is decreasing with the increase of these parameters. This is because of the fact that increase in these parameters offers resistance to flow. Fig.4 shows that increase in Grashof number Gc results in increase in velocity.

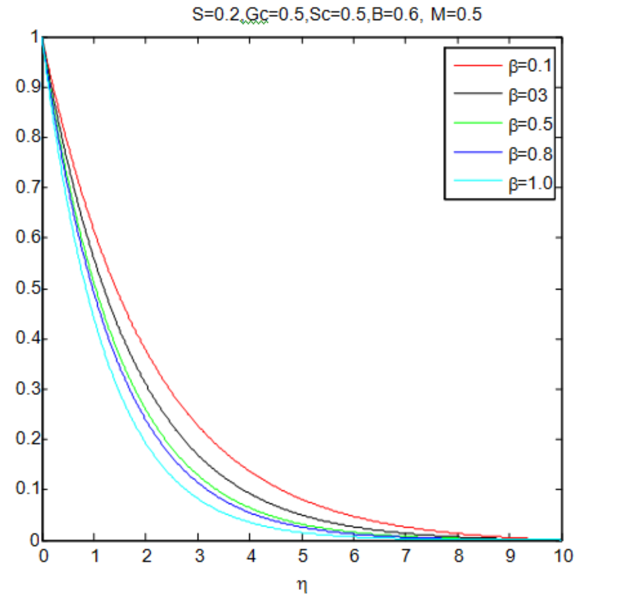

Fig.1. **Velocity Profiles with variations in Casson Parameter β**

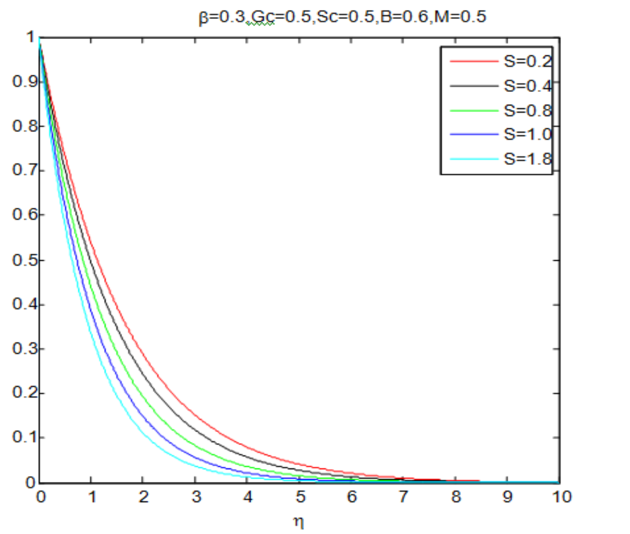

Fig.2 Velocity profiles with change in Suction S

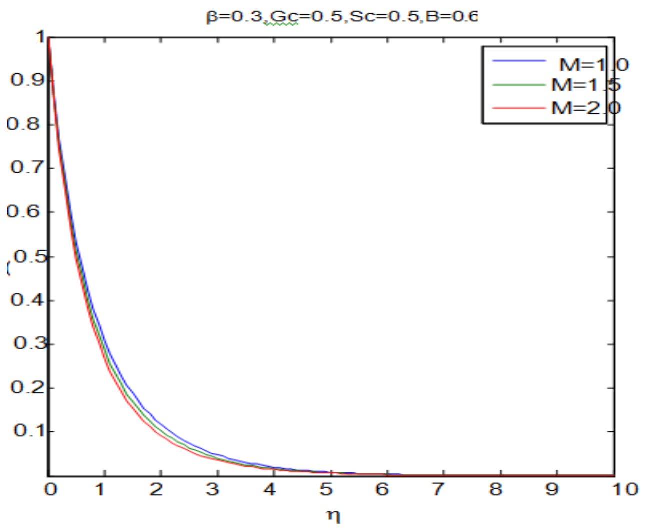

Fig.3 Velocity profiles with change in magnetic parameter

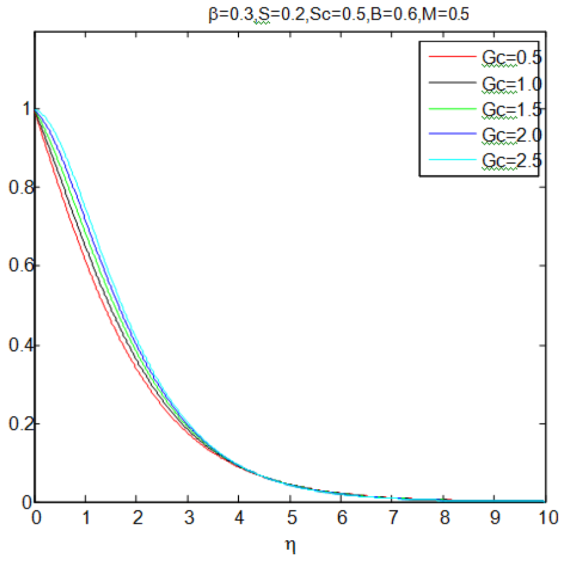

Fig.4 Velocity profiles with variations in Grashof Number Gc

Fig..5, Fig..6 presents the variation of concentration with casson parameter β and magnetic parameter M respectively. It is noticed that increase in casson parameter and Magnetic parameter results in increase in concentration.

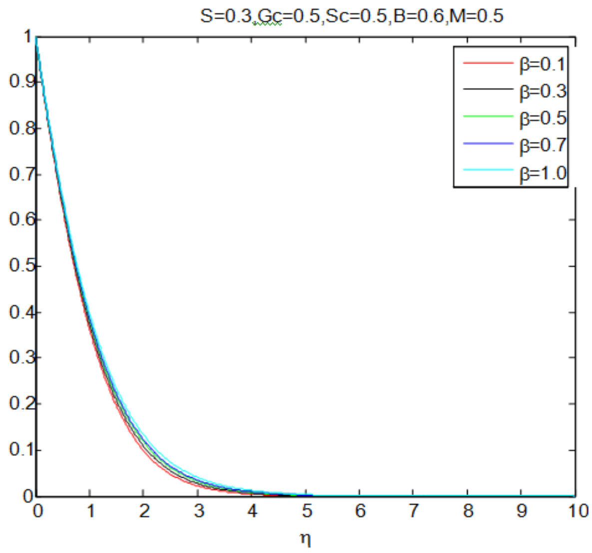

Fig. 5 Concentration profiles with variations in Casson Parameter β

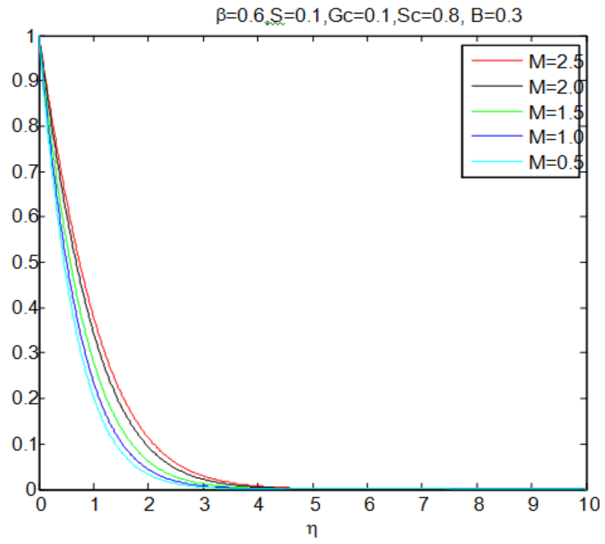

Fig. 6 Concentration profiles with variations in magnetic parameter M

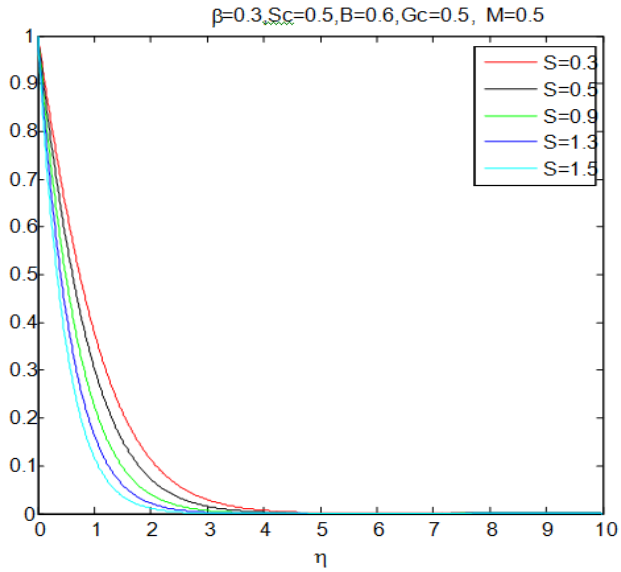

Fig.7 Concentration profiles with variations in Suction S

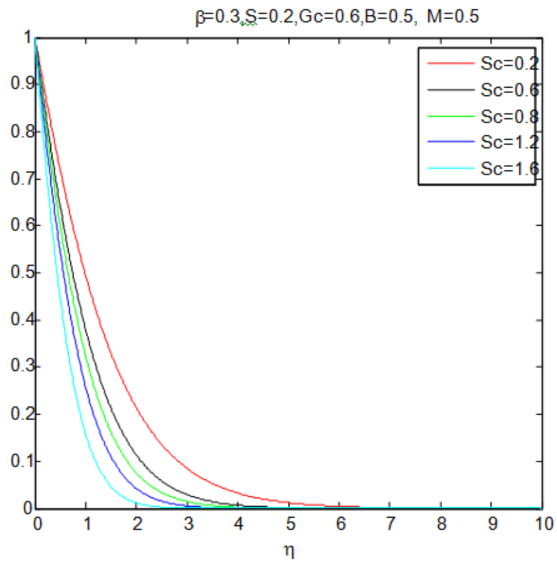

Fig.8 Concentration profiles with variations in Schmidt number Sc

Fig..7, Fig.8 shows the change in concentration with suction parameter S, Schmidt number Sc. From graphs clearly it is observed that concentration decreases with the increasing values of suction and Schmidt number.

Conclusions:

Results derived by this method are compared with available results in literature and observed they are in good agreement.

It is also observed that:

a) The velocity decreases with the increase in S, M and βand increases with Gc.

b) The concentration boundary layer decreases with the increasing values of S, Sc and increases with the increasing values of β and M

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