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Analyzing the Applications of Graph Theory in Mechanical Engineering through 2-odd Graphs

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Abstract –The foundations for contemporary analysis can be traced back to the early research in the discipline of linear graph theory applied to engineering problems. Many researchers clearly showed the relationship between linear graphs and physical systems. Among the continuum of mechanical system models, one can distinguish labeling graphs that are used for formulating and formalizing mechanical tasks. Mechanical coupled systems are effectively modeled using hybrid graphs which combine characteristics of polar and flow graphs. Similarly, a hybrid graph is used to describe a model of a mechanical system that depends on the selected algebraic method, and applying matrices is the most convenient method to determine the dynamic characteristics of the system. A graph$\mathbf{G}(\mathbf{V}(\mathbf{G}), \mathbf{E}(\mathbf{G}))$ is said to be a $\mathbf{2-odd}$ graph if there exists an injective function $\mathbf{h}: \mathbf{V}(\mathbf{G}) \rightarrow \mathbf{Z}$such that for any two adjacent nodes$\mathbf{s}$and $\mathbf{t}$, the integer$|\mathbf{h}(\mathbf{s}) - \mathbf{h}(\mathbf{t})|$ is either odd or exactly$\mathbf{2}$. This paper discusses$\mathbf{2-odd}$ labeling of degree splitting graph of some special graphs such as triangular snake graph$\mathbf{Tn}$, double triangular snake graph $\mathbf{DTn}$, triple triangular snake graph $\mathbf{TTn}$,and alternate triangular snake graph $\mathbf{ATn}$.Moreover, a few interesting applications of graph theory in mechanical engineering are also highlighted.

Keywords: Graphlabeling; 2-odd graph; Degree splitting graph of a graph; Triangular snake graph, Mechanical engineering.

1. Introduction

All the graphs considered here are finite, simple, undirected, and connected. Let G be with the node-set $V(G)$ and the line set $E(G)$. According to Laison et al. [6], G is $2-odd$ if there exists a one-to-one labeling $f: V(G) \rightarrow Z$ such that for any two adjacent nodes u and v, the integer $|f(u) - f(v)|$ is either odd or exactly 2. They also defined that $f(uv) = |f(u) - f(v)|$ and called f a $2-odd$ labeling of G. So G is said to be a $2-odd$ graph if and only if there exists $2-odd$ labeling of G. Furthermore, by this definition $h(uv)$ may still be either 2 or odd if uv is not a line of G. For more results on 2-odd labeling, see [1, 6]. Let P_n denote a path on n nodes, namely $v_1, v_2, \dots, v_n$.

2. Preliminaries

Definition: [2]A triangular snake T_n is obtained from $P_n$by joining$v_i$andv_{i+1} to a nodeu_i for $i = 1,2, \dots\dots n-1$. i.e., every line of P_n is replaced by C_3.

Definition:[2]A double triangular snake DT_n is obtained from two T_n with a common P_n.

Definition:[2] A triple triangular snake TT_n is obtained from P_n, by joiningv_i and v_{i+1} to three nodesu_i, w_i and t_i for $1 \leq i \leq n-1$. i.e., TT_n consists of three T_n that have a common P_n.

Definition:[10] An alternate triangular snake AT_n is obtained from $P_n$by joiningv_i and v_{i+1} alternatively $(i = 1, 3, 5, \dots)$ to a nodeu_i. i.e., every alternate line of P_n is replaced by C_3.

Definition:[8] A graph $H$with $V = S_1 \cup S_2 \cup \ldots \cup S_i \cup T$where each $S_i$is a set of nodes having at least two nodes of the same degree and $T = V \cup S_i$.The degree splitting graph of $H, DSG(H)$is obtained from $H$by adding nodes$w_i: 1 \leq i \leq t$and connecting to each node of $S_i$for $1 \leq i \leq t$.

This paper derives $2 - odd$ labeling of degree splitting graph of triangular snake Tn, double triangular snake DTn, triple triangular snake graph TTn, and alternate triangular snake ATn, besides highlighting the applications of graph theory in mechanical engineering.

3. Main Results

Theorem 1. $DSG(T_n)$ admits $2 - odd$ labeling for $n \geq 2$.

Proof.Let T_n be the given triangular snake graph on $2n - 1$ nodes and $3(n - 1)$ lines. Obtain $DSG(T_n)$ by introducing two nodes, x,y and joining x to the nodes of degree 2 and y to the nodes of degree 3 (see Figure 1). Note that $|V(DSG(T_n))| = 2n + 1$. Define a 1-1 function $f: V(DSG(T_n)) \to Z$ as follows: Without loss of generality, let $f(v_1) = 1$, $f(v_i) = 2i; 2 \leq i \leq n - 1$, $f(v_n) = f(v_{n-1}) + 3$, and $f(x) = 2$. Again, let $f(u_i) = 2i + 1; 1 \leq i \leq n$, and $f(y) = t$, where t is a sufficiently large unused odd number. Therefore, $DSG(T_n)$ admits $2 - odd$ labeling for $n \geq 2$.

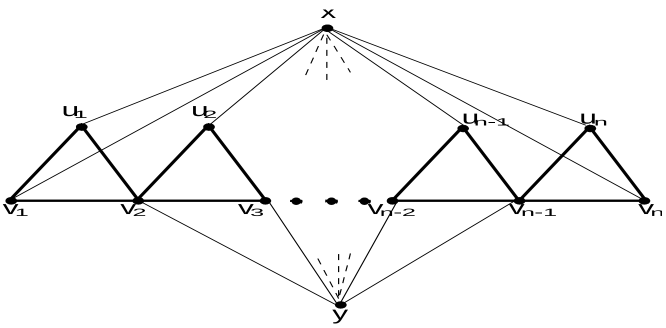

Figure 1:$DSG(T_n)$

Theorem2.$DSG(DT_n)$ admits 2-odd labeling for $n \geq 2$.

Proof.Let DT_n be the given double triangular snake graph on $3n - 2$ nodes. Obtain $DSG(DT_n)$ by introducing three nodes say, x, y, z, and joining x to the nodes of degree 2, y to the nodes of degree 6 and z to the nodes of degree 3 (see Figure 2). Note that $|V(DSG(DT_n))| = 3n + 1$. Define a 1-1 function $f: V(DSG(DT_n)) \to Z$ as follows: Without loss of generality, let $f(v_i) = 2i; 1 \leq i \leq n$, $f(u_i) = 2i + 1; 1 \leq i \leq n$, $f(w_1) = f(u_n) + 2$, $f(w_i) = f(w_{i-1}) + 2, 2 \leq i \leq n$, and $f(x) = 2s$, where 2s is sufficiently large unused even number. Again, let $f(y) = t$, $f(z) = k$, where t and k are the sufficiently large unused odd number. Therefore, $DSG(DT_n)$ admits $2 - odd$ labeling for $n \geq 2$.

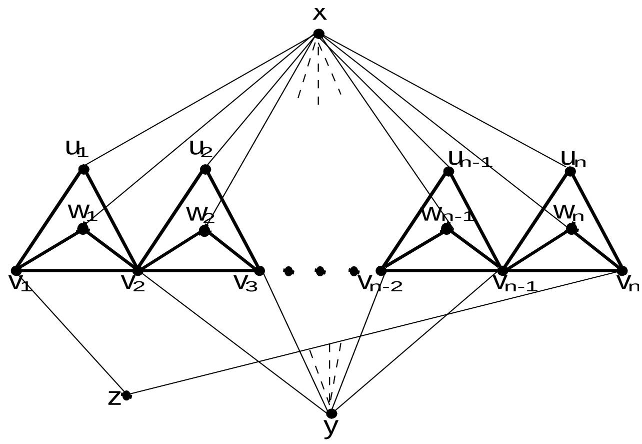

Figure 2:$DSG(DT_n)$

Theorem 3.$DSG(TT_n)$ admits 2-odd labeling for $n \geq 2$.

Proof.Let TT_n be the given triple triangular snake graph on $4n - 3$ nodes. Obtain $DSG(TT_n)$ by introducing three nodes say, x, y, z, and joining x to the nodes of degree 2, y to the nodes of degree 8, and z to the nodes of degree 4 (see Figure 3). Note that $|V(DSG(TT_n))| = 4n$. Define a 1-1 function $f: V(DSG(TT_n)) \to Z$ as follows: Without loss of generality, let $f(v_i) = 2i; 1 \leq i \leq n$,$f(w_i) = 2i + 1; 1 \leq i \leq n$, $f(u_1) = f(w_n) + 2$, $(u_i) = f(u_{i-1}) + 2, 2 \leq i \leq n$, $f(w'_i) = -f(w_i)$, $1 \leq i \leq n$ and $f(x) = 2s$, where 2s is a sufficiently large unused even number. Also let, $f(y) = t$ and $f(z) = k$, where t, k are the sufficiently large unused odd numbers. Therefore, $DSG(TT_n)$ admits $2 - odd$ labeling for $n \geq 2$.

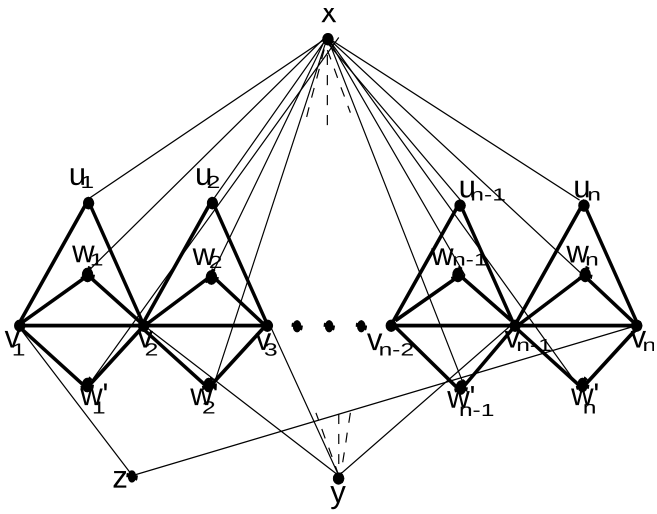

Figure.3:$DSG(TT_n)$

Theorem 4.$DSG(AT_n)$ admits $2 - odd$ labeling for $n \geq 2$.

Proof.Let AT_n be the given alternate triangular snake graph on $2n - 3$ nodes. Obtain $DSG(AT_n)$ by introducing two nodes say, x, y, and joining x to the nodes of degree 2, y to the nodes of degree 3 (see Figure 4). Note that $|V(DSG(AT_n))| = 2n - 1$. Define a 1-1 function $f: V(DSG(AT_n)) \to Z$ as follows: Without loss of generality let, $f(v_i) = 2i + 2; 1 \le i \le n-1$, $f(v_n) = f(v_{n-1}) + 3$, and $f(x) = 2$. Again, let $f(u_i) = f(v_j) + 1; 1 \le i \le n$, and $j = 1, 3, \dots n-1$ and $f(y) = t$, where t is a sufficiently large unused odd number. Thus, $DSG(AT_n)$ admits $2-odd$ labeling for $n \ge 2$.

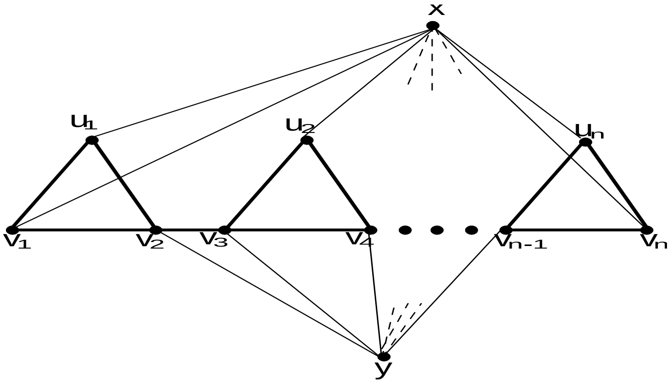

Figure 4: $DSG(AT_n)$

4. Applications of Graph Theory in Mechanical Engineering

Graphs are used as models of versatile technical systems such as electrical and electronic systems, railways and road networks, phone networks, and mechanical systems. The most essential introduction to the graph representation of mechanical systems can be seen in books [7, 9,5]. Graph transformations are used in civil engineering [3, 4] but recently this is also been used in mechanical engineering [9]. The graphs' application in civil engineering focuses mainly on the layout of civil engineering structures like trusses, buildings, or floor arrangements in buildings. In [9], the synthesis of mechanisms based upon an application of graph grammar is presented. The task of synthesis and enumeration of all possible designs of a particular mechanical artifact can also be performed by means of the graph-based approach [3]. Mainly, graphs are used for encoding a functional scheme of a planetary gear or a geometrical structure of a truss. Some adequate transformations are connected with the changes of drives and the related changes in the passage of a rotational movement and power throughout a gear. Simultaneously these transformations cause simplification of adequate equation systems in an automatic way. The whole process makes it possible to analyze simplified functional schemes and it allows for the derivation of relevant simplified kinematic equation systems for consecutive considered work modes in the case of the automatic gearboxes.

The possibility of representation of a mechanical system M by means of G consists in simplification and representation of M by means of relations between its elements. These relations can be then turned into graphs where elements of relations are presented as lines with adequate weights. A review of some more frequently used approaches is given in [13]. Other graph-based methods of modeling mechanical systems are described in works [11, 12]. Even a wider range of mechanical tasks can be done using graph-based models [11, 12]. Further, graph transformations are also used to find the degenerate structures of gears and search for the redundant geared wheels or other redundant elements in the considered gear structures.

5. Conclusion

In this paper, we have proved that $DSG(Tn)$, (DTn), $D(TTn,)$ and $DSG(ATn)$ admit 2-odd labeling, besides highlighting the applications of graph theory in mechanical engineering. Discovering the complete characterization of 2-odd graphs and exclusive applications of 2-odd graphs in mechanical engineering are the open problems of high interest and they are for future work.

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