

Performance of DFT and Discrete Hilbert Transform based Approach in PMU for Three Phase Power Transmission Network

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Abstract

The proposed Phasor Measurement Unit (PMU) simulator can play key role in power transmission as well as distribution networks to measure amplitude and phase angle of voltage signal at fundamental time period (T_0) and its integer multiples. To measure waveform phasors, fast and accurate estimation techniques have to be developed in order to face the challenges of next-generation smart and micro grids. The main purpose of the paper is to highlight and compare the performance of both recursive DFT and proposed Discrete Hilbert Transform (DHT) estimation techniques under the condition of high frequency sampling. In order to evaluate the performance of these techniques in terms of total vector error (TVE) in compliant with the standard IEEE Std C37.118.1-2011, various faults such as Line to ground (LG), Line to Line(LL) and Line to line to line (LLL) under static condition and dynamic condition are considered for power transmission line in IEEE9 bus system. However the DHT estimation technique not only require lesser execution time in comparison to recursive DFT technique but also it is more accurate. Finally the proposed PMU simulator built in Matlab/Simulink based on DHT technique may be used by the researchers to achieve the phasor estimation at faster rate in large interconnected power transmission and distribution networks.

Keywords: DFT; Discrete Hilbert Transform; PMU; Power Transmission Network; Phasor Estimation Techniques

1. Introduction

Phasor Measurement Unit (PMU) is one of the integral component of wide-area measurement system (WAMS) for dynamic monitoring and online diagnostics of the power grid and thus make it smarter. PMU can measure phasor values of voltage and current using GPS timing information at any instant in a power transmission system. The voltages, currents and power flow at each and every bus can be measured using PMU in real time to ensure stable operation of power grid. The data gathered by different PMUs at PDC are time stamped which is used at load dispatch centre to check the status of a large interconnected power system [1-4]. In PMU, techniques used for phasor estimation plays an important role so that system analysts can use accurate and fast data provided by it to identify the malfunctions due to which catastrophic failure of the power system occurs and thus blackout condition may be avoided [5, 8]. In the reference [3], algorithms employed for frequency estimation include Prony method, Polynomial Fitting (PF), Complex Valued Least Square, and Phase Locked Loop (PLL) but Discrete Fourier Transform (DFT), PLL, and Smart Discrete Fourier Transform (SDFT) are used for phasor estimation. The time varying amplitude and frequency signals are considered to compare these different algorithms. Some other methods used for estimation of phasor

and frequency in PMU are zero crossing, wavelet transform, hilbert transform modified zero crossing, signal demodulation, Taylor weighted least square (TWLS), the optimization methods such as least mean square methods, Kalman filter and Newton's method [9-17].

In this paper, amplitude and phasor estimation using recursive DFT and proposed Discrete Hilbert transform (DHT) techniques with 6.4kHz sampling rate is presented. These techniques are employed for amplitude and phasor estimation in measurement unit of PMU and tested on power transmission line in IEEE9 bus system to verify their effectiveness in terms of total vector error (TVE) in compliant with the standard IEEE Std C37.118.1-2011. This standard guides the operation, design, & implementation of a PMU. This synchrophasor standard is also intended to define various PMU performance parameters which standardizes quality of PMUs measurements [18]. Different faults such as LG, LL, LLL on power transmission line between bus 7 and bus 8 of IEEE9 bus system are considered to analyse the performance of estimation techniques. The computational speed has also measured while using both the phasor estimation techniques in PMU. These discrete techniques can estimate the phasor and amplitude of voltage signal in power transmission system accurately in real-time.

2. Phasor Estimation Techniques

2.1 Recursive Discrete Fourier Transform (DFT)

DFT phasor estimation algorithm is based on steady state signal model. The voltage signal produced by the three-phase source in power transmission network at nominal frequency under steady state can be represented as:

$$v(t) = V_m \cos(2\pi f_0 t + \varphi) \quad (1)$$

Where V_m is the peak amplitude of the signal, φ is the phase angle & f_0 is the on nominal fundamental frequency. Whenever the signal is sampled at the rate of N samples per cycle (sampling frequency $F_s = Nf_0$) then equation (1) becomes:

$$v(n) = V_m \cos(2\pi n/N + \varphi) \quad (2)(2)$$

Also, fundamental phasor of the given signal can be expressed as:

$$V = \frac{V_m}{\sqrt{2}} e^{j\varphi} \quad (3)(3)$$

The N point DFT of the signal in (2) is [17]:

$$V(k) = \sum_{n=0}^{N-1} v(n) e^{-j2\pi k \frac{n}{N}} \quad (4)(4)$$

Where k is the harmonic index whose value is taken as 1 to estimate the phasor of the fundamental frequency component. Therefore, the fundamental phasor of the signal given in (3) estimated over a window of N samples, is given as [18]:

$$V = \frac{\sqrt{2}}{N} \sum_{n=0}^{N-1} v(n) e^{-j2\pi \frac{n}{N}} \quad (5)$$

To update the phasor with each new input data sample acquired, the observation window is shifted by each sample. Recursive DFT algorithm is used to estimate Phasors for the successive windows. It reduces the computational complexity in comparison to non-recursive DFT technique by performing recursive update of the old phasor obtained from the previous window. The phasor using Recursive DFT algorithm for window of N samples with first data sample as $v(r+1)$ is given as [18]:

$$V^{r+1} = V^r + \frac{\sqrt{2}}{N} (v(N+r) - v(r)) e^{-j(r)\theta} \quad (6)(6)$$

Where V^r is the estimated phasor of the preceding window; $r=0, 1, 2, \dots$

In three phase power transmission networks adopted in paper, input voltage signal is a constant sinusoid at nominal frequency and thus the phasor estimated using recursive DFT from new data window is same as the phasor estimated for the previous window.

2.1.2. Discrete Hilbert Transform

The Hilbert transform of a function $f(t)$ is defined by [19]

$$\hat{f}(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{f(\tau)}{t - \tau} d\tau \quad (7)$$

The Hilbert transform on a signal imparts a phase shift of $\pi/2$ to the signal $f(t)$, then signal $f(t)$ and its Hilbert transform $\hat{f}(t)$ relate to each other to form strong analytic signal which would have parameters as an amplitude and a phase. Hence the analytic signal is given by:

$$z(t) = f(t) + i\hat{f}(t) \quad (8)$$

The analytic signal can also be defined as a rotating vector with an instantaneous phase $\phi(t)$ and an instantaneous amplitude $A(t)$ as [19]

$$z(t) = A(t) e^{i\phi(t)} \quad (9)(9)$$

where,

$$A(t) = \sqrt{f^2(t) + \hat{f}^2(t)} \quad (10)$$

$$\phi(t) = \arctan \frac{\hat{f}(t)}{f(t)} \quad (11)$$

The Hilbert transform can be obtained for continuous as well as discrete signal. But in the paper, proposed Hilbert approach used continuous signal to compute Hilbert transform of voltage signal at any phase generated by three phase source in IEEE 9 bus system and thus complex-valued function called discrete analytic signal is created as shown in figure1.

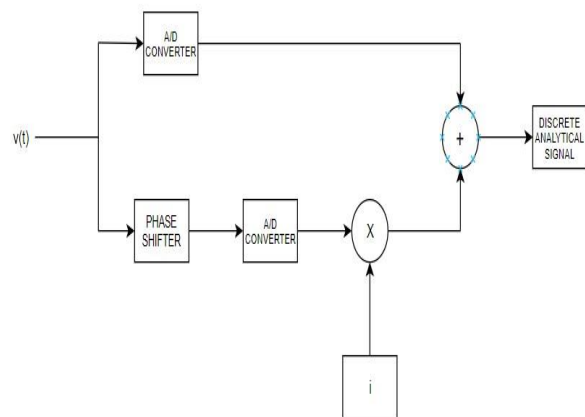


Figure1. Block Diagram of Proposed Discrete Hilbert Transform

In the proposed approach, phase shifter is implemented in MATLAB environment to shift the phase of incoming signal by 90° . Then incoming signal and its shifted version is converted into discrete form using analog to digital converter which consists of sample and hold circuit and quantizer. Thus discrete analytic signal is created using equation 8 in discrete domain which in turn used to get amplitude and phase of signal at specified observation intervals.

2.2. TVE for Performance Analysis of Techniques

In the paper, the Total Vector Error (TVE) is used as a performance metric to characterize the accuracy of amplitude and phase angle measurements. It is defined as the difference between the phasor estimated from the unit under test & a theoretical synchrophasor, with both the phasor samples having same time tag. The value of TVE is normalized & is expressed in terms of percentage of the theoretical synchrophasor as follows [18]:

$$TVE(t) = \frac{|X_{estimate}(t) - X_{theoretical}(t)|}{|X_{theoretical}(t)|} \times 100 \% \quad (12)$$

Or

$$TVE(t) = \sqrt{\frac{(\hat{X}_r(t) - X_r(t))^2 + (\hat{X}_i(t) - X_i(t))^2}{(X_r(t))^2 + (X_i(t))^2}} \times 100 \% \quad (13)$$

The estimated values of amplitude & phase angle by proposed PMU simulator can differ from the theoretical values. In the paper, the amplitude and phase angle errors are used separately as performance metrics. A maximum acceptable limit of TVE under static and dynamic conditions should have maximum value of 1% as specified in IEEE std C37.118 [18].

2.3. Proposed PMU Simulator

The main application of PMU is to estimate phasor measurements from electrical power transmission network across different locations in power grid. The phasor measurement techniques as discussed in the section II are used in PMU simulator whose internal structure is shown in figure2. The simulator consists of Sample and Hold circuit, Quantizer, Pulse generator and DFT & DHT based Measurement unit to measure voltage amplitude and phase angle at the fundamental time period and its integer multiples which can help researchers to understand the measurement process in PMU.

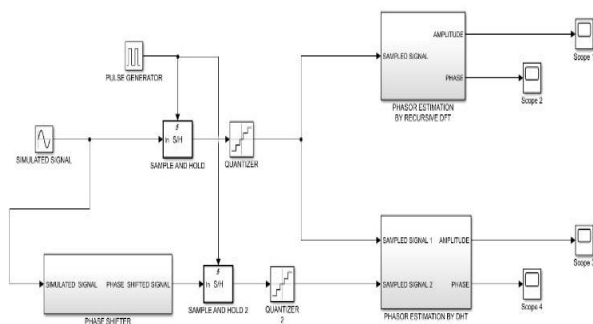


Figure 2. PMU Simulator using recursive DFT & Discrete Hilbert Transform (DHT)

The sample and hold block are used to sample the incoming voltage signal at high sampling rate provided by pulse generator and then quantized by the use of quantizer of

appropriate quantization interval. Thereafter measurement unit of PMU simulator takes the discrete output of ADC to compute amplitude and phase angle at the specified observation intervals. Then proposed PMU simulator is implemented on IEEE 9 bus system of power transmission network as shown in figure 3 in order to evaluate the appropriateness of these techniques at on nominal frequency under steady state condition.

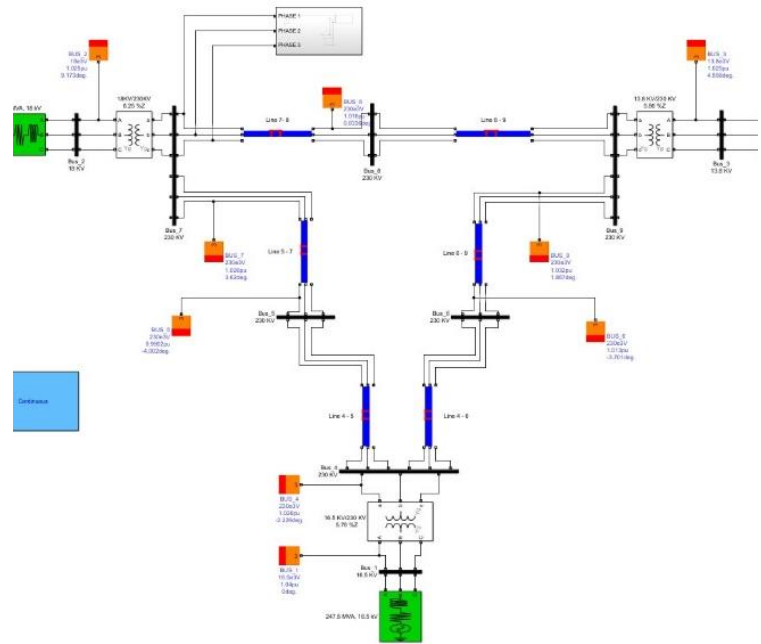


Figure3. Implementation of Proposed PMU Simulator on IEEE 9 Bus System

A disturbance in considered IEEE bus system occurs whenever a transmission line is disconnected due to faults such as LG, LL and LLL.

3. Results

In order to test the performance of phasor estimation techniques used in proposed PMU simulator, static and dynamic conditions have been considered as presented in following section:

3.1 Static condition

The simulation conditions taken into account to measure amplitude and phase angle of voltage of each phase at bus 7 in adopted IEEE 9 bus system are:

Sampling frequency (f_s) = 6.4 kHz

Data window length (N) = 128

Quantization Interval = 0.00000001

On Nominal fundamental frequency (f_0) = 50 Hz.

The voltage signals are synthesized with RMS nominal voltages of 230kV for total duration of 1 second during the implementation of both estimation techniques on IEEE power transmission system for healthy and faulty conditions. The measurements are taken at the rate of 50 readings per second which is the highest rate for 50 Hz power systems. In this regard, maximum amplitude and instantaneous phase angle of line to neutral voltages of each phase in IEEE 9 bus system under no fault conditions is depicted in figure4. It can be found that the amplitude and phase angle of each phase voltage signal would remain same if results are taken at fundamental time period and its integer multiples. These phasor measurements so obtained can be further used in power system state estimation, fault analysis and for the monitoring and control of power system efficiently.

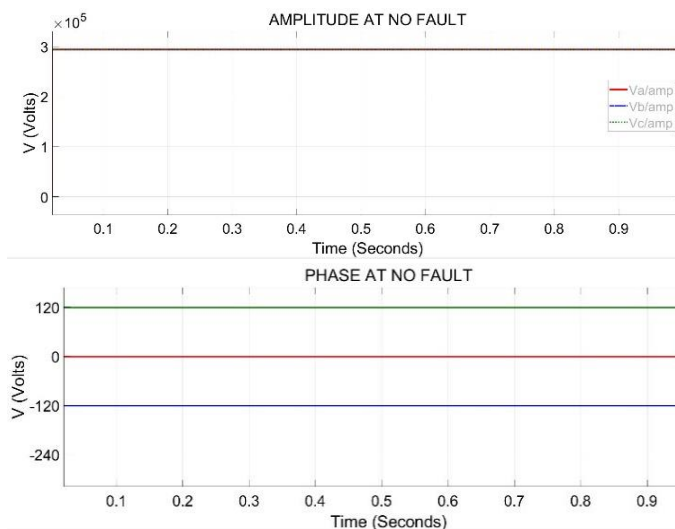


Figure4. Phasor Estimation at No fault

Thereafter faults such as LG (between phase a and ground), LL (between phase a and phase b) and LLL (between three phases) occur on transmission line between bus 7 and bus 8 of IEEE 9 bus system. It can be observed that amplitude and phase angle of line to neutral voltage of respective phase under faulty conditions changes with respect to healthy line as presented in figure5-7.

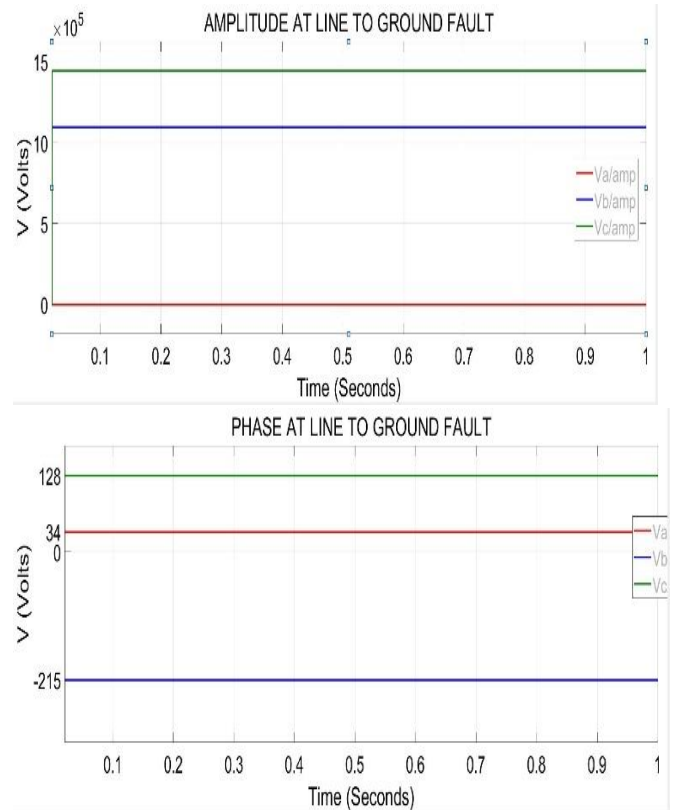


Figure5. Phasor Estimation under LG fault on Phase a

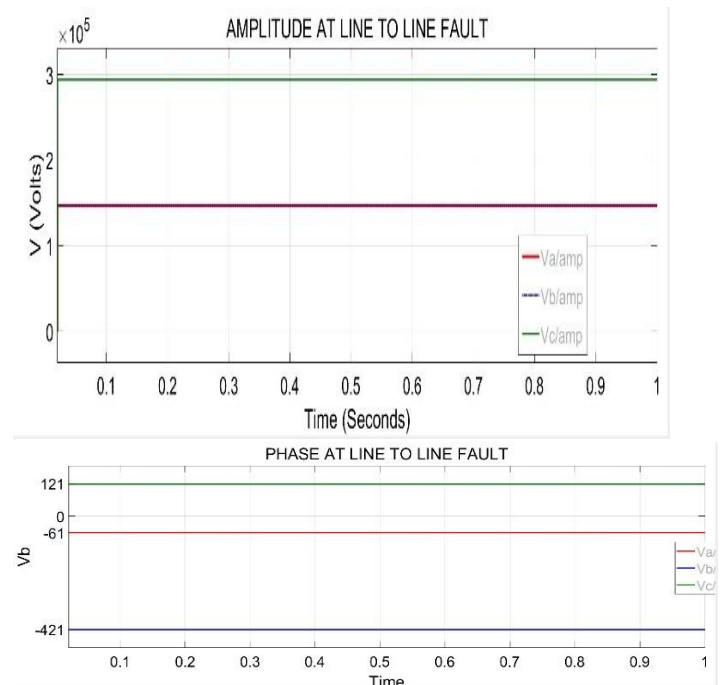


Figure6. Phasor Estimation under LL fault between Phase a & Phase b

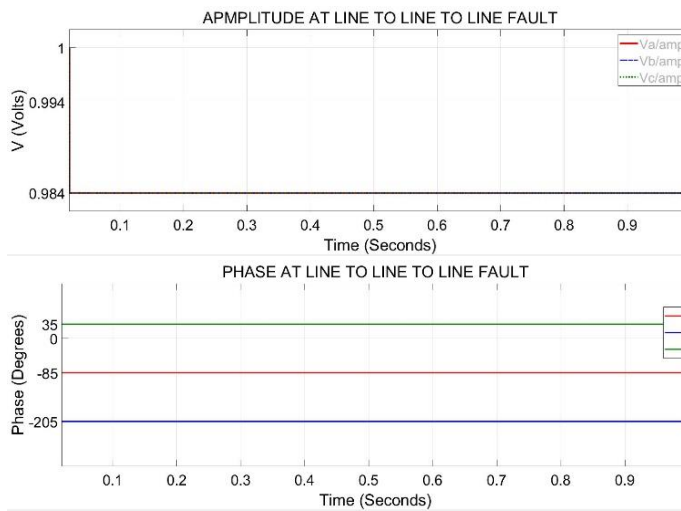


Figure 7. Phasor Estimation under LLL fault between three phases

3.2 Dynamic condition

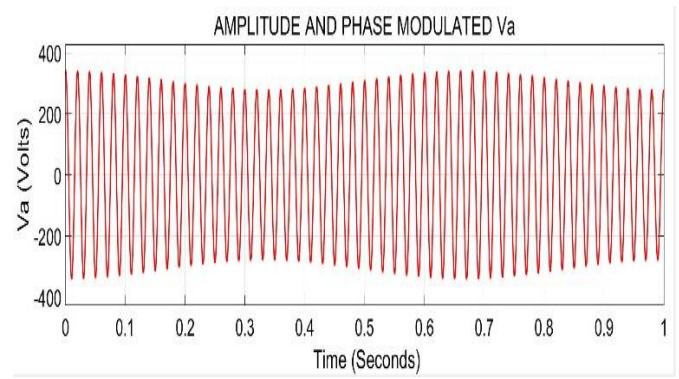
In order to take into account, the effect of dynamic condition, the case in which both amplitude and phase modulation of voltage signal in power system have been taken into consideration. During the phasor measurements, proposed DHT technique fails to give correct amplitude and phase angle due to ripples introduced in the desired signal whenever amplitude and both amplitude and phase of voltage signal changes. Thus, the desired operation is carried out using Discrete FIR filter in direct form of order 9.

Consider the amplitude and phase modulated voltage signal modelled as:

$$V_1(t) = V_m (1 + K_p \cos(2\pi f_m t)) (\cos(2\pi f_o t + \cos(2\pi f_m t - \pi))) \quad (14)$$

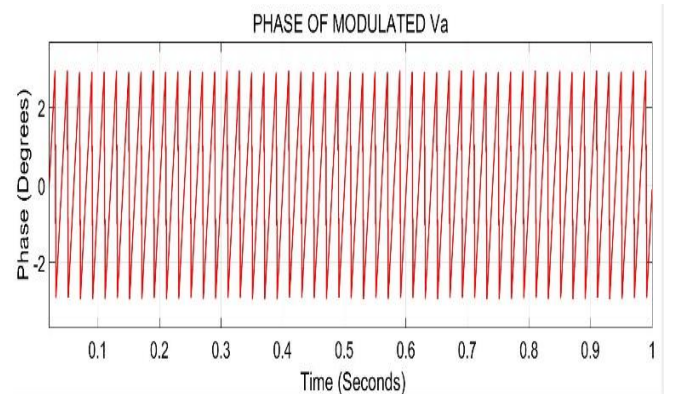
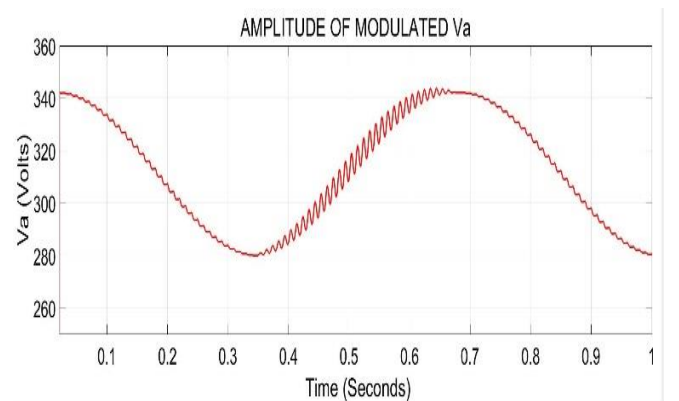
The amplitude of this signal is modulated by cosine with frequency much smaller than fundamental frequency.

This signal is characterized by $f_m = 1.5\text{Hz}$ the modulation frequency and $K_p = 0.1$ the modulation depth. Figure 8 shows that DFT does not enable to interpret the signal correctly thus proposed DHT has given better phasor computation at faster rate.

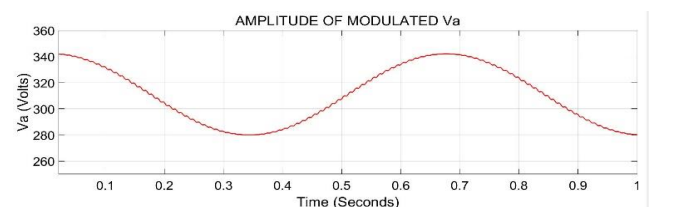


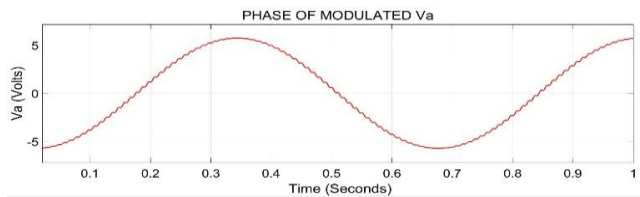
(A) INPUT

SIGNAL



(B) USING PROPOSED DHT





(C) USING RECURSIVE DFT

Figure8. Phasor Estimation under Amplitude and Phase Modulation

It has been observed that the amplitude and phase of amplitude modulated signal at $t = 0.4s$ comes out to be 284.07 V and 5.68° in DFT and 258.19 V and 5.71° in proposed DHT at first integer multiple of fundamental frequency for actual amplitude and phase angle of 284.88 V and 5.72° respectively.

It can be analysed that proposed DHT gives accurate results in comparison to DFT.

3.3 Performance Analysis of Proposed DHT

The numerical values of TVE obtained for input voltage signal at phase a and execution speed during implementation of PMU simulator using recursive DFT and proposed DHT under static and dynamic condition on IEEE9 bus system are summarized in Table I. It can be observed that TVE for cases under consideration are within limits specified in the standard IEEE Std C37.118.1-2011. It has been found that TVE achieved using proposed DHT has lesser value as compared to recursive DFT. It has also analysed that proposed DHT takes lesser computation time in comparison to recursive DFT. Thus, proposed PMU simulator using DHT technique in its measurement unit may be used for phasor estimation at higher speed.

Table 1. Performance and Speed Analysis of DFT & Proposed DHT

ESTIMATION ALGORITHM	TOTAL VECTOR ERROR (%)					COMPUTATIONAL TIME (S)
	STEADY STATE CONDITION				DYNAMIC CONDITION	
	NO FAULT	LG FAULT	LL FAULT	LLL FAULT	AMPLITUDE AND PHASE MODULATION	
RECURSIVE DFT	0.0022	0.0179	0.0030	0.0001	0.1088	18.42
DHT	0.0020	0.0168	0.0027	0.0001	0.2843	19.53

4. Discussion

The main focus of investigation in presented simulation scenario is to provide fast and accurate phasor estimation technique which can be used in PMU for monitoring and control of power grid efficiently using fault analysis at faster speed. In order to compare performance and speed of phasor estimation techniques, PMU simulator based on recursive DFT and DHT techniques as discussed in section III has been implemented on IEEE9 bus system of power transmission network as shown in figure 3. After successful implementation of PMU simulator on transmission line under various faults such as LG,LL and LLL at static condition and also under the

condition in which both amplitude and phase modulation of incoming voltage signal takes place, it has been analysed that proposed DHT provided accurate results in comparison to DFT as presented in table 1. However DHT technique was unable to give correct amplitude and phase angle at fundamental time period and its integer multiples, thus discrete FIR filter has been used to make the phasor parameters accurate

as can be seen in figure8. It can also be observed that PMU simulator based on proposed DHT has given output at fast speed so that fault on power transmission line can be predicted in short time period.

5. Conclusions

In the proposed work, No fault and faulty conditions such as LG,LL and LLG on power transmission line between bus 7 and bus 8 have been considered in order to demonstrate the performance of the recursive DFT and proposed DHT at sampling rate of 128 samples per cycle of fundamental frequency. The proposed PMU simulator is capable of estimating amplitude and phase angle in accordance with TVE requirement stated in the IEEE Standard C37.118.1. Unlike other works presented in the available literature, the proposed work is based on analysing the performance and computational speed of both the phasor estimation techniques used in PMU. The measurements are obtained at the rate of 50 readings per second which is highest reporting rate for 50Hz power system. It can be concluded that DHT phasor estimation technique gives better performance and is faster as compared to recursive DFT under similar sampling rate of 6.4 kHz. The estimation techniques can also be easily implemented into FPGAs, DSPs, or embedded systems in real time.

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